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BAYESIAN ACCELERATED FAILURE TIME MODEL WITH SPATIAL DEPENDENCY: APPLICATION TO UNDER FIVE MORTALITY RATE

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ABSTRACT

Nigeria is one of the top five in the world with the highest under-five mortality (U5M) rate. The risk of U5M in Nigeria is assumed to vary from one state to another due to diversity in socio-economic and even environmental factors. Thus, this study aimed to quantified the hazard of U5M using Bayesian Accelerated Failure Time (AFT) with spatial dependency. The data for the study were obtained from 2018 Nigerian Demographic and Health Survey (NDHS). The study utilized Bayesian technique based on Markov Chain Monte Carlo (MCMC) technique to obtained the posterior estimates of the parameters. The Loglogistic, Weibull and Lognormal AFT models with and without spatial dependency were considered in this study. Out of these models used, the log-logistic AFT model with Intrinsic Conditional Autoregressive (ICAR) spatial prior performed better than the other models considered in the study. The findings revealed that the survival of under-five children (U5C) was not homogeneous across the states ($\tau^2 = 6.358,$ 95% CI: 2.6330, 11.4630). The subject-specific factors such as maternal age at birth, duration of breast feeding, preceding birth intervals, maternal educational qualification, wealth index, region, number of antenatal visits, duration of pregnancy, gender of child, twin status, contraceptive used and toilet facility were the significant risk factors of U5M. Based on these findings, it was recommended among others that the disparities observed across states should be taken into account at the policy level in order to meet the Sustainable Development Goal (SDG) 2030 targets.

KEYWORDS: Accelerated Failure Time Model, Bayesian technique, Spatial Dependency, Survival, Under-Five Mortality.

INTRODUCTION

The global under-five mortality rate continues to be an ongoing issue. In 2009, it was estimated that 5.2 million children under five died worldwide (Egbon *et al.*, 2022). The most recent projections from the United Nations Inter-Agency Group for Child Mortality Estimation (UN IGME, 2023) show that this figure has decreased to 4.9 million in 2022. There are differences between nations even if the U5M rate has recently significantly decreased globally. According to the World Health Organization (WHO), 2021, the Sub-Saharan Africa region continues to have the largest burden of under-five mortality worldwide, with one in thirteen children dying before turning five. This puts the region at significant risk of U5M.

In 2019, Nigeria was one of the top five in the world with the highest U5M rates. Nigeria recorded 117 deaths for every 1,000 live births in 2019. Despites the effort of government, stakeholders, and non-governmental organizations working together to combat U5M and enhance children's well-being, the U5M index in Nigeria remains quite high (Adeyinka *et al*, 2020). In order to achieve the SDG of ending preventable deaths of children under five by 2030, the rate's gradual decline may prove to be a hindrance. According to the United Nations International Children's Emergency Fund (UNICEF) and national Bureau of Statistics (NBS) (2018), Nigeria has to reduce early childhood mortality by at least 50% in order to meet the 2030 SDG target of reducing U5M rate to at least as low as 25 per 1000 live births.

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As a result, quick action is needed to reduce Nigeria's under-five mortality burden and get it below the average death rate worldwide.

Recently, there has been growing interest in the application of survival analysis technique in study of U5M rate. The survival analysis technique differs from other statistical technique as the dependent variable is always the time until the occurrence of the event (Isaac, 2019). This event, can be death, failure of an equipment and so on. Three method of survival analysis models are frequently employed in literatures. The non-parametric method such as the Kaplan Meir, log-rank and life table that have no distributional assumption; the semi-parametric method such as the cox proportional Hazard model that the baseline hazard is left unspecified and the parametric methods that the baseline hazard follows a distribution such as Exponential, Log-logistics, log normal, Weibull among others. The Proportional Hazard (PH), Accelerated Failure Time (AFT) and the Proportional Odd (PO) are survival models frequently employed (Muse et al, 2022). These models are unique in their applications. For instance, in the AFT model, the covariate either accelerate or decelerate the survival time.

Many studies have investigated the determinants of under-five mortality using survival analysis methods. For instance, Daniel (2021) assessed the spatial variation in under-five mortality in Kenya using spatial survival approaches. To estimate the hazard at the county level, ICAR models were fitted, taking into account spatial dependency and clustering. A cox-proportional hazard model was also fitted to quantify the risk factors linked to child death. The best fit model was found to be the spatial Cox proportional hazard model. According to the model, there is a spatial structure to the risk of U5M in the Kenyan counties. The central Kenyan counties of Laikipia, Nyandarua, Nyeri, Kiambu, Machakos, and Makueni have the highest fatality risk, however other counties have a similar risk. Western Kenyan counties and Nyanza have the least amount of danger.

Ahmed et al. (2020) looked at the risk variables of U5M in Sudan using the K-M, Cox proportional, and frailty models. According to the study's findings, there were statistically significant factors that influenced U5M in Sudan, including twin vs single births, family wealth index, previous birth intervals of less than two years, and residency location.

Using data from the Ethiopia demographic health survey, Yalew et al. (2022) evaluated the time to U5M and its factors in rural Ethiopia. The Cox's-gamma shared frailty and K-M models were used in the investigation. The study's conclusions showed that the following factors were associated with a lower risk of U5M: female sex, having more than five children at birth, being extremely large or very little at birth, twin pregnancies, never breastfeeding, an outdated latrine, and health insurance coverage.

Singh and Singh (2023) examined U5M in Manipur, India, using a multilevel model to determine the important contributing factors. The National Family Health Survey (2019–21) data were used in the study. The fixed effect model does not fit the data as well as the three-level mixed-effects Weibull parametric survival model with primary sampling units nested within the districts. It was discovered that the risk factors for U5M were the number of births in the previous five years, the mother's age at her first delivery, the use of contraception, and the size of the kid at birth.

The parametric multilevel survival strategy was utilized by Jaiswal et al. (2024) to investigate the survival rate of children under five in India. The fourth Demographic and Health Survey (2015–2016) was used in the study. The study made the assumption that people (level 1) are nested within "districts" (level 2), and that "states" (level 3) surround districts. The results of the study showed that women with a birth interval of less than two years have the greatest under-five death rate. In terms of parity, women with third and higher order parity have about 4% U5M rate. Additionally, the chance of dying rose sharply throughout the first year of life, then gradually until the age of three, at which point it stabilized.

Yemane et al. (2024) used data from the 2019 Ethiopia Mini Demographic and Health Survey to evaluate the survival status of U5M and its factors in rural Ethiopia. The Cox's gamma shared frailty model and the K-M model were the survival analysis techniques used in the study. The research findings showed that U5M was significantly predicted by the mothers' age, the sex of the household, breastfeeding, birth types, the child's sex, the mothers' educational attainment, the wealth index, the number of children they had, their marital status, and their water supply.

In Nigeria, Wegbom *et al.* (2019) evaluate the impact of socioeconomic, bio-demographic, and health-related factors on mortality risk among Nigerian children under the age of five using 2013 NDHS data. The Cox proportional hazard model and non-parametric K-M survival were used in this study. Significant influences were the mother's educational attainment, wealth index, marital status, place of residence, child's sex, area, mother's age at childbirth, number of children born, interval between births, and kid size at birth. Also, Kunnuji *et al.* (2022) employed the cox model to determine the factors influencing infant mortality in Nigeria using the 2018 NDHS survey data. Their findings revealed that residing in the North West or South East zone; having a mother with more education as compared to having no education; and belonging to a household in the richest quintile or the highest quintile compared to the lowest wealth quintile are the predictors of infant survival.

Biradar *et al.* (2019) investigated the relationship between childhood mortality and wealth and birth interval.

The 2013 NDHS data were used. The data was analyzed using the bi-variate and cox proportional hazard models in this study. The findings revealed that mothers who were poor, illiterate, and employed in agriculture or as professionals or technicians had greater rates of U5M. Children with mothers who had a birth interval of fewer than two years also had the greatest U5M rates. Also, mothers under the age of twenty experienced a higher rate of child loss.

Egbon *et al.* (2022) quantified the survival probabilities and the impact of socioeconomic and demographic factors, proximate and biological determinants, and environmental factors on the risk of U5M in Nigeria using 2018 NDHS data set. The Exponential, Gamma, Log-normal, Weibull, and Cox hazard models in a Bayesian mixed effect hierarchical hazard modeling framework with spatial components were considered in this study. The findings of the study revealed that log-normal model with spatial components performed better than other model considered in this study. Also, gender, maternal education, household wealth status, source of water and toilet facility, residence, mass media, frequency of antenatal and postnatal visits, marital status, place of delivery, multiple births, who decide healthcare use, use of bed net are significant risk factors of child mortality in Nigeria.

Okoli *et al.,* (2022) investigates inequalities in geographic and socioeconomic factors influencing survival time of children under-five in Nigeria using 2018 NDHS data set. The K–M survival estimates, Log-rank test statistics, and the Cox proportional hazards were used as tools for data analyses. The results from the cox model indicate that children born to fathers with no formal education, primary education and secondary education had higher risk of under-five mortality compared to children born to fathers with tertiary education. In addition, children born to mothers living in the North-West region of Nigeria had higher risk of under-five mortality as compared to those in the South West region of Nigeria.

Musa *et al.* (2020) used the Cox proportional hazards model to investigate several factors that significantly affect U5M using 2013 and 2018 NDHS data. According to the study, there is a substantial correlation between the use of contraceptives by mothers, their state of residency, the birth weight of their kid, and the type of toilet facility their household uses and the survival of children under five in the North Central Region of Nigeria.

From the empirical literature reviewed, it was observed that there is paucity of existing literatures that have examined the factors of U5M using the parametric survival model most especially in Nigeria. This study intends to examined the factors influencing under five mortality using Bayesian Parametric Accelerated Failure Time model with spatial dependency.

1.0 Materials and Methods

2.1 Source of Data

The data used in this study were sourced from 2018 NDHS. The survey was implemented by National Population Commission (NPC) in collaboration with the National Malaria Elimination Programme (NMEP) of the Federal Ministry of Health, Nigeria.

2.2 Variable Selection

The outcome variable of the study is the “Time to death of Under-Five Children” The explanatory variables used were selected based on existing literatures on the determinants of U5M. The study focused on the existence of spatial variations and the disparities in U5M in Nigeria. The brief description of the data was provided in table 1.

Table 1: Description of Variables

Covariates	Category and Code
Status	Censored = 0; Dead = 1
Time	Time to Dead in months
Maternal Age at Birth	Age of Mothers at birth in Years
Duration of Breastfeeding	Duration of Breastfeeding in Months
Preceding Birth Intervals	< 24 months = 0; 24 – 33 months = 1; More than 33 months = 2
Maternal Education Qualification	High Education = 0; Secondary = 1; Primary = 2; No education = 3
Wealth Index	Poorest = 0; Poorer = 1; Middle = 2; Richer = 3; Richest = 4
Region	NC = 0; NE = 1; NW = 2; SE = 3; SS = 4; SW = 5
Number of Antenatal Visits	None = 0; 1 - 8 times = 1; 9 and above = 2
Duration of Pregnancy	≤ 9 months = 0; More than 9 Months = 1
Gender of Child	Male = 0; Female = 1
Child Twin	Singleton = 0; Twin = 1
Place of Residence	Urban = 0; Rural = 1
Use of Mosquito Net	No = 0; Yes = 1
Conceptive Use	No = 0; Yes = 1
Source of Drinking Water	Unimproved = 0; Improved = 1
Toilet Facility	Unimproved = 0; Improved = 1

Source: Author’s Compilation

STATISTICAL TECHNIQUES

Accelerated Failure Time (AFT) Model Formulation

In AFT model, the covariates are assumed to act multiplicatively on survival time and additively on logarithm of survival time (Collett, 2015). Thus, the general log-linear AFT model is given by:

$$Log(T) = \beta_0 + \sum_{j=1}^{p} \beta_j M_j + \sigma\varepsilon \quad (1)$$

Taking the exponent of both side of equation (1), this may result to:

$$T = \exp(\beta_0 + \sigma\varepsilon)\exp\left(\sum_{j=1}^{p} \beta_j M_j\right) = T_0 \exp\left(\sum_{j=1}^{p} \beta_j M_j\right) \quad (2)$$

Where:

$M_j, j = 1, 2, \dots, p$ are the covariates, $\beta_j, j = 0, 1, \dots, p$ are the regression coefficient, $\sigma(> 0)$ is the scale parameter and ε is a random error which has a specified distribution.

The AFT model in terms of hazard (Muse, 2022) is defined through the individuals hazard function as follows:

$$h(t_{ij}|M_{ij}) = \exp(\beta^T M_{ij}) h_0(t_{ij}\exp(\beta^T M_{ij}) \quad (3)$$

Where $h_0(.)$ is a baseline hazard, $\beta = (\beta_1, \dots, \beta_p)^T$ is the vector of regression parameters affecting the time and hazard scale, M_{ij} *is* the vector of covariates affecting the time and hazard scale.

In order to study the heterogeneity in the survival times, an additional dimensional random effect is included in the AFT model. The model is defined in terms of hazard function as given below:

$$h(t_{ij}|M_{ij}, n_i) = \exp\{\beta^T M_{ij} + n_i\} h_0(t_{ij} exp\{\beta^T M_{ij} + \bar{n}_i\}) \quad (4)$$

The AFT model with random effect in log-linear form is given as:

$$Log(T) = \beta_0 + \sum_{j=1}^{p} \beta_j M_j + n_i + \sigma\varepsilon \quad (5)$$

where n_i is the random effect which independently and identically distributed with mean 0 and variance σ_u^2 that is, $n_i \sim N(0, \sigma_n^2)$. The random effect termed "frailty" is to account for heterogeneity after adjusting for covariates.

The AFT model can also be extended to include a spatial random effect that will have effect on the hazard and time scale. The model is defined in terms of hazard function as given below:

$$h(t_{ij}|M_{ij}, w_i) = \exp\{\beta^T M_{ij} + w_i\} h_0(t_{ij} exp\{\beta^T M_{ij} + \bar{w}_i\}) \quad (6)$$

The AFT model with spatial dependency in log-linear form is given as:

$$Log(T) = \beta_0 + \sum_{j=1}^{p} \beta_j M_j + w_i + \sigma\varepsilon \quad (7)$$

$$w_i = n_i + v_i$$

Where the frailty term w_i incorporate the effect of both heterogeneity via the non-spatial frailty n_i and spatial dependency through the spatial frailty v_i.

2.3.2 Baseline Distributions

Weibull Distribution

The probability density function, survival function and hazard function for a two parameters Weibull distribution are presented respectively as

$$f(t|\varphi, \gamma) = F'(t{:}\, \varphi, \gamma) = \frac{\gamma}{\varphi}\left(\frac{t}{\varphi}\right)^{\gamma-1} \exp[-\left(\frac{t}{\varphi}\right)^{\gamma}] \quad (8)$$

$$S(t) = 1 - F(t) = \exp[-\left(\frac{t}{\varphi}\right)^{\gamma}] \quad (9)$$

$$h(t|\varphi, \gamma) = \frac{\varphi}{\gamma}\left(\frac{t}{\gamma}\right)^{\varphi-1} \quad (10)$$

Where $t \geq 0, \varphi > 0$ *and* $\gamma > 0$

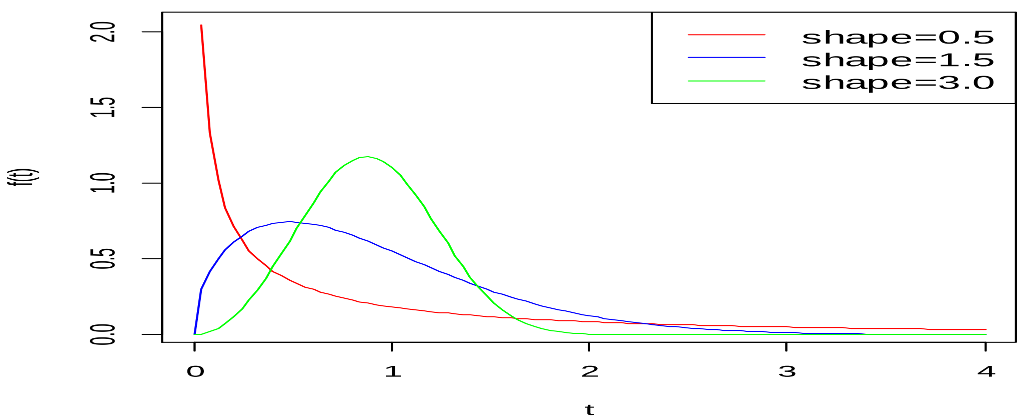


Figure 1: Weibull Density function for different values of shape and scale is unity.

LOGLOGISTIC DISTRIBUTION

Suppose that survival time T follows log-logistic distribution with shape parameter $\varphi > 0$ and scale parameter $\gamma > 0$, then the pdf $\mathrm{f(t)}$, survival function $\mathrm{S(t)}$ and hazard function $\mathrm{h(t)}$ is given respectively as follows:

$$\mathrm{f}(\mathrm{t}|\varphi,\gamma) = \left(\frac{\varphi}{\gamma}\right)\left(\frac{t}{\gamma}\right)^{\varphi-1}\left[1+\left(\frac{t}{\gamma}\right)^{\varphi}\right]^{-2}, t > 0 \quad (11)$$

$$\mathrm{S}(\mathrm{t}|\varphi,\gamma) = \left[1+\left(\frac{t}{\varphi}\right)^{\varphi}\right]^{-1} \quad (12)$$

$$\mathrm{h}(\mathrm{t}|\varphi,\gamma) = \left(\frac{\varphi}{\gamma}\right)\left(\frac{t}{\gamma}\right)^{\varphi-1}\left[1+\left(\frac{t}{\gamma}\right)^{\varphi}\right]^{-1} \quad (13)$$

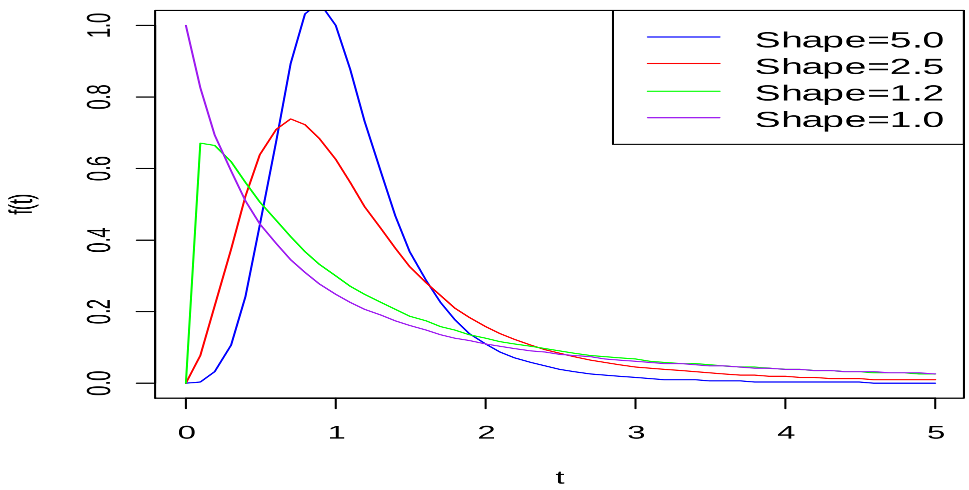


Figure 2: Loglogistic Density function for different values of shape parameters and scale is unity

Lognormal Distribution

Suppose that the life T is such that $Y = \log(T)$ follows a normal distribution with mean $\mu = \varphi$ and variance $\sigma = \gamma$, then T follows a log-normal distribution with location parameter $\varphi(-\infty < \varphi < \infty)$, scale parameter $\gamma > 0$ having a pdf, survival function and hazard function given respectively as follows:

$$f(t|\varphi,\gamma) = \frac{1}{t\gamma\sqrt{2\pi}} e^{\left\{-\frac{1}{2}\left(\frac{\log(t)-\varphi}{\gamma}\right)^2\right\}} \quad (14)$$

$$S(t|\varphi,\gamma) = 1 - \Phi\left(\frac{\log(t)-\varphi}{\gamma}\right) \quad (15)$$

$$h(t|\varphi,\gamma) = \frac{f(t|\varphi,\gamma)}{S(t|\varphi,\gamma)} \quad (16)$$

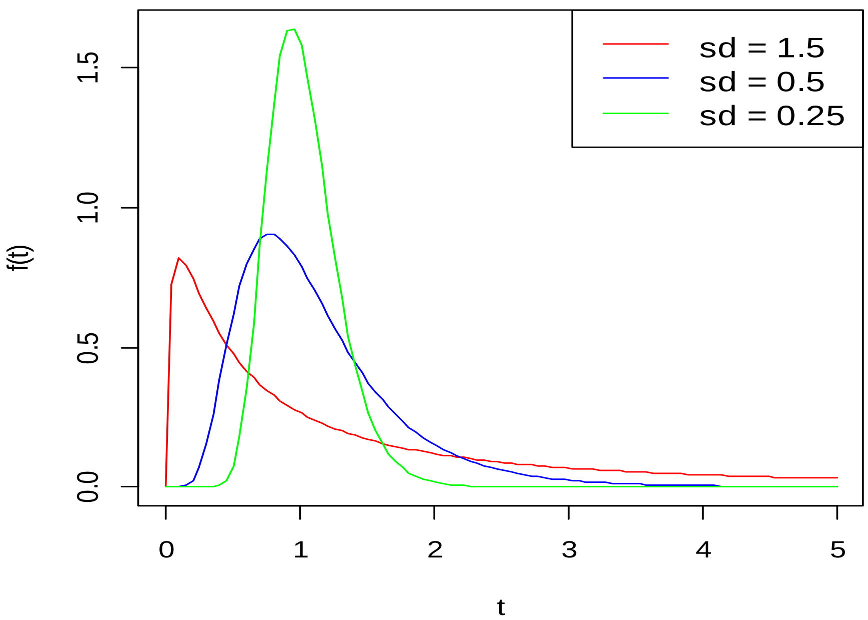


Figure 3: Lognormal Density function for different values of scale and location is zero.

BAYESIAN INFERENCE

A typical Bayesian workflow includes three major steps. That is specifying the Prior distribution $\pi(\vartheta)$, Likelihood Function $L(\vartheta)$ and Posterior distribution $\pi(\vartheta|K)$.

Likelihood Function off Right Censored data

Supposed that $t=(t_1,t_2,\dots,t_n)^T$ are independently observed survival times with event or censored, each having a survival model where $\delta=(\delta_1,\delta_2,\dots,\delta_n)^T$is a censoring indicator with $\delta_i=1$ indicating occurrence of event and $\delta_i=0$ indicating censored observations; $M_i=\left(M_{i1},M_{i2},\dots,M_{ip}\right)^T$is the vector of covariates for the i^{th}individuals and $K=(t,\delta,Y)$ denote the observed data for the model. Then the likelihood function for the parameters $\vartheta=(\alpha,\beta)=(\alpha,\beta_1,\beta_2,\dots,\beta_n)$ where $\alpha=\varphi,\gamma$ and β are the distributional and regression coefficient respectively for the right censored data (Ashraf-Ul-Alam, M., & Ali Khan, 2021) is given:

$$L(\alpha,\beta|K)=\prod_{i=1}^{n} f(t_i|\alpha,\beta)^{\delta_i}\, S(t_i|\alpha,\beta)^{1-\delta_i} \quad (17)$$

$$=\prod_{i=1}^{n}\left(\frac{f(t_i|\alpha,\beta)}{S(t_i|\alpha,\beta)}\right)^{\delta_i} S(t_i|\alpha,\beta) \quad (18)$$

$$=\prod_{i=1}^{n} h(t_i|\alpha,\beta)^{\delta_i}\, S(t_i|\alpha,\beta) \quad (19)$$

The log likelihood of (18) and (19) can be written respectively as:

$$\log[L(\alpha,\beta|K)]=\sum_{i=1}^{n}(\delta_i[\log f(t_i|\alpha,\beta)-logS(t_i|\alpha,\beta)]+log\, S(t_i|\alpha,\beta) \quad (20)$$

$$\log[L(\alpha,\beta|K)]=\sum_{i=1}^{n}[\delta_i\,(logh(t_i|\alpha,\beta)+logS(t_i|\alpha,\beta)] \quad (21)$$

Prior Distributions for the Parameters

The Gamma distributions are flexible and can include non-informative priors (uniform) and the marginal prior distribution for each regression coefficient $M=1,\dots,15$. These priors are taken into consideration in numerous study publications in the literature, including (Alvares et al., 2021; Muse, et al., 2022; Khan & Basharat, 2022). The prior distributional parameters are given as below:

$$\pi(\varphi)\sim G(c_1,d_1)=\frac{d_1^{c_2}}{\Gamma(c_1)}\varphi^{c_1-1}e^{-d_1\varphi};\ c_1,d_1,\varphi>0 \quad (22)$$

$$\pi(\gamma)\sim G(c_2,d_2)=\frac{d_2^{c_2}}{\Gamma(c_2)}\gamma^{c_2-1}e^{-d_2\gamma};\ c_2,d_2,\gamma>0 \quad (23)$$

The prior distribution for the regression coefficient is given as:

$$\pi(\beta)\sim G(c_3,d_3) \quad (24)$$

The joint prior distribution of all unknown parameters has a pdf given by:

$$\pi(\varphi,\gamma,\beta)=\pi(\varphi)\pi(\gamma)\pi(\beta) \quad (25)$$

Where φ,γ are the distributional parameters while β is the vector of regression coefficient.

PRIOR DISTRIBUTION FOR THE SPATIAL RANDOM EFFECT

This study aimed at incorporating spatial effects in the AFT model by incorporating the neighborhood structure into the distribution of the random effects. The random effect is defined based on two approaches: defining the random effect purely based on "independent and identically distributed (IID) and defining the random effect based on "Intrinsic Conditional Autoregressive (ICAR) model".

i) Independent and identically distributed (IID) Model

Frailties in survival analysis have been frequently used to induce correlation among related survival times in models which have a linear predictor. The linear predictor is augmented:

$$\eta_i=M'\beta+w_i,\quad w_i=n_i+v_i. \quad (26)$$

Where the non-spatial frailty n_i is a random effect which is defined purely based independent and identically distributed (IID) that is $n_i\sim N(0,\sigma^2)$ the random effect account for heterogeneity after adjusting for subject specific covariates. This type of frailty model has random effect within each group e.g. states as used in this study to which observations i belongs. In this study, n_i is considered as non-spatial random effects which are independent of neighboring regions. Therefore, the uncorrelated random effect n_i is modeled using a Normal prior with mean 0 and variance σ^2. Thus, the IID prior is defined as:

$$\pi(IID\_Prior)\ =\ n_i\sim N(0,\sigma^2) \quad (27)$$

ii) Intrinsic Conditional Autoregressive (ICAR) model.

An intrinsic autoregressive model, also known as the conditional autoregressive (CAR) model, was proposed by Besag *et al*. (1991) to explain the spatial dependencies. The spatial frailty term in (26) is denoted by v_i. Let $e_{ij}=1$ if area $e_i\ and\ e_j$ share a nontrivial border and $e_{ij}=0$ otherwise. Set $e_{ij}=0$, then the $G\times G$ matrix $E=[e_{ij}]$ is called the adjacency matrix for the region D. The ICAR prior is defined through the set of all conditional distributions as given below:

$$\pi(ICAR\_Prior)=v_j|\{v_i: i\neq j\}\sim N\left(\bar{v}_j,{}^{\vartheta^2}\!/_{e_{j+}}\right), j=1,\dots,G \quad (28)$$

Equation 28 denoted $v\sim ICAR\left({}^{1}\!/_{\vartheta^2}\right)$, where $e_{j+}$is the number of neighbors of area e_j, $\bar{v}_j=\frac{1}{e_{j+}}\sum_{i:e_{ij=1}} v_i$ is the sample mean of the e_{j+}values of the neighboring areal unit frailties, and ${}^{\vartheta^2}\!/_{e_{j+}}$is the conditional variance.

In this study, the spatial parameter v_i is a 37×1 vector of spatial effects to account for heterogeneity between states in Nigeria that is $v = (v_1, \dots, v_{37})$.

Posterior Distribution

The joint posterior density function using the IID prior is expressed as the multiplication of the likelihood function in Equation (17), the prior distribution in Equation (25) and the IID prior in equation (27):

$$p(\alpha, \beta|t) = L(\varphi, \gamma, \beta|K) \times \pi(\varphi, \gamma, \beta) \times \pi(IID\_Prior) = \prod_{i=1}^{n} f(t_i|\alpha, \beta)^{\delta_i} S(t_i|\alpha, \beta)^{1-\delta_i} \times \pi(\varphi)\pi(\gamma)\pi(\beta) \times \pi(IID\_Prior) \quad (28)$$

The joint posterior density function using the ICAR prior is expressed as the multiplication of the likelihood function in Equation (17), the prior distribution in Equation (25) and the ICAR prior in equation (28):

$$p(\alpha, \beta|t) = L(\varphi, \gamma, \beta|K) \times \pi(\varphi, \gamma, \beta) \times \pi(ICAR\_Prior) = \prod_{i=1}^{n} f(t_i|\alpha, \beta)^{\delta_i} S(t_i|\alpha, \beta)^{1-\delta_i} \times \pi(\varphi)\pi(\gamma)\pi(\beta) \times \pi(ICAR\_Prior) \quad (29)$$

The joint posterior density is analytically intractable because of the challenge of integrating it. Thus, the inference can be based on MCMC algorithms. The analysis was carried out in R software using spBayesSurv package.

MODEL SELECTION CRITERION

The performance of each model will be evaluated using the Deviance Information Criterion (DIC) (Spiegelhalter *et al.,* 2002), the Watanabe Akaike Information Criterion (WAIC) (Watanabe & Opper, 2010) and Log Pseudo Marginal Likelihood (LPML) (Geisser & Eddy, 1979). Generally, the model with least values of DIC and WAIC and large values of LPML is considered as the best model (Egbon et al., 2021; Pérez-Castro et al., 2024).

RESULTS AND DISCUSSION

Table 2: AFT Model Selection Criteria

Baseline Distributions	Spatial Prior Distributions	DIC	WAIC	LPML
Loglogistic	ICAR	59380.54	59371.11	-31685.56
	IID	66880.11	66838.59	-33419.29
	None	65915.78	65903.25	-32951.63
Weibull	ICAR	76534.76	76486.81	-38243.40
	IID	80549.94	80493.55	-40246.75
	None	80849.25	80839.81	-40419.9
Lognormal	ICAR	69956.63	69899.15	-34949.56
	IID	65456.80	65363.04	-32681.36
	None	67689.83	67537.17	-33768.68

Source: Authors Compilation

Table 2 presents the model selection criteria. The Deviance Information Criteria (DIC), Watanabe Akaike information criterion (WAIC) and Log Pseudo Marginal Likelihood (LPML) were used. Model with smaller values of DIC and WAIC show good fit, while model with larger LPML values indicate a high predictive power. Using these criteria, it was concluded that the log-logistic AFT model with ICAR spatial prior have the least values of DIC, WAIC and large value of LPML indicating that it performed better and have good predictive power as compared to others models considered in this study.

Table 3: Posterior Parameter Estimates of Log-logistic AFT Model with ICAR Prior

Covariates	Mean	Medan	Std Dev.	95% CI-L	95% CI-U
Maternal Age at Birth	-0.0076	-0.0062	0.0051	-0.0215	-0.0012
Duration of Breast Feeding	-0.0648	-0.0619	0.0224	-0.1120	-0.0314
Preceding Birth Intervals					
< 24	***Ref***				
24 – 33	0.0404	0.0297	0.0549	-0.0275	0.1322
> 33	-0.2183	-0.2236	0.0763	-0.3811	-0.1088
Maternal Educ. Qualification					
Higher Education	***Ref***				
Secondary	0.4063	0.4424	0.0892	0.1517	0.4842
Primary	0.5895	0.6438	0.1339	0.2573	0.7327
No education	0.6115	0.6422	0.1069	0.3471	0.7308
Wealth Index					
Poorest	***Ref***				
Poorer	-0.0958	-0.0911	0.0552	-0.1793	-0.0078
Middle	-0.0225	-0.0319	0.0344	-0.0825	0.0699
Richer	-0.1109	-0.0982	0.0501	-0.2232	-0.0175
Richest	-0.0994	-0.0861	0.0576	-0.2483	-0.0221
Region					
North Central	***Ref***				
North East	0.3843	0.4249	0.2539	-0.1273	0.6890
North West	-0.2767	-0.2348	0.1631	-0.6147	-0.0545
South East	-0.6085	-0.5444	0.4856	-1.7115	-0.0216
South-South	2.9022	3.0895	0.8764	0.8571	3.8715
South West	0.2387	0.2911	0.5168	-0.9108	0.8649
Number of Antenatal Visits					
None	***Ref***				
1 – 8	-4.3768	-4.3990	0.1012	-4.0832	-4.4705
>9	-0.0148	-0.0212	0.2023	-0.3124	0.3300
Duration of Pregnancy					
≤ 9	***Ref***				
> 9	0.4005	0.3831	0.2391	0.0628	0.8338
Gender of Child					
Male	***Ref***				
Female	0.1123	0.1178	0.0272	0.0591	0.1666
Child Twin					
Singleton	***Ref***				
Twin	0.1774	0.1761	0.0430	0.0732	0.2634
Place of Residence					
Urban	***Ref***				
Rural	-0.0659	-0.0791	0.0618	-0.1423	0.0639
Use of Mosquito Net					
No	***Ref***				
Yes	-0.1046	-0.1079	0.0230	-0.0673	0.1425
Conceptive Use					
No	***Ref***				
Yes	-0.2700	-0.2784	0.0389	-0.3364	-0.1629
Source of Drinking Water					
Unimproved	***Ref***				
Improved	-0.0029	-0.0121	0.0349	-0.0846	0.0450
Toilet Facility					
Unimproved	***Ref***				
Improved	-0.1216	-0.1151	0.0333	-0.1959	-0.0769
CAR Frailty Variance (τ^2)	6.358	6.093	2.331	2.6330	11.4630

Source: Author's Compilation

Table 3 presents the posterior parameter estimates of log-logistic AFT model with ICAR prior distribution. The posterior mean and median for the maternal age at birth is -0.0076 and -0.0062 respectively and is significant with a 95% CI of (-0.0215, -0.0012) indicating that maternal age at birth accelerates the survival of U5C. The posterior mean and median for duration of breast feeding is -0.0648 and -0.0619 respectively and is significant with a 95% CI (-0.1120, -0.0314) implying that the duration of breast feeding accelerates the survival of U5C.

The posterior mean and median for preceding birth interval between 24 to 33 months were 0.0404 and 0.4424 respectively and is statistically insignificant with a 95% CI (-0.0275, 0.1322). However, posterior mean and median for preceding birth interval greater than 33 months were -0.2183 and -0.2236 respectively and is significant at 95% CI (-0.3811, -0.1088) indicating that under-five children with preceding birth interval greater than 33 months accelerates the survival of U5C as compared with those that have less than 24 months of preceding birth interval.

Maternal educational qualification was a significant predictor of U5M. The posterior mean and median of a child whose maternal educational qualification is secondary level is 0.4063 and 0.4424 respectively and is significant at a 95% CI (-0.1793, -0.0078); primary is 0.5895 and 0.6438 respectively and is significant at 95% CI (0.2573, 0.7327); No formal education is 0.6115 and 0.6422 respectively and is significant at 95% CI (0.3471, 0.7308). The positive mean values indicates that children whose maternal education is secondary, primary and no education decelerates the survival of U5C as compared to those whose mothers have higher education qualification. This implies that children whose mother have higher education qualification have higher probability of survival.

The posterior mean and median for wealth index: poorer are -0.0958 and -0.0911; richer is -0.1109 and -0.00982 and richest is -0.0994 and -0.0861 were significant predictors of U5M with a 95% CI: (-0.1793, -0.0078), (-0.2232, -0.0175) and (-0.2483, -0.0221) respectively. This implies that wealth index (poorer, richer and richest) accelerates the survival of U5C as compared to the reference category (poorest). However, wealth index (middle) whose mean and median is -0.0225 and -0.0319 respectively with a 95% CI (-0.0825, 0.0699) was not a significant predictor of U5M.

The posterior mean and median estimates of under-five children who were from the North-West region is -0.2767 and -0.2348, South-East region is -0.6085 and -0.5444 and South-South region is 2.9022 and 3.0895 and were significant at 95% CI (-0.6147, -0.0545), (-1.7115, -0.0216) and (0.8571, 3.8715). These results indicate that North-West and South-East regions accelerates the survival of U5C as compared to North-Central. However, South-South region decelerates the survival of U5C as compared to children from North-Central (reference category). The posterior mean and median of children from South-West region were 0.2387 and 0.2911 and was insignificant with a 95% CI (-0.9108, 0.8649).

The posterior mean and median for number of antennal visits between 1 – 8 times were -4.3768 and -4.3990 and was significant in predicting U5M with a 95% CI (-4.0832, -4.4705). This implies that children whose parents had antennal visits of 1 – 8 times accelerates the survival of U5C as compared to the reference category (none visit). The posterior mean and median for number of antennal visits greater than or equal to 9 is 0.4005 and 0.3831 and is insignificant with a 95% CI (-0.3124, 0.3300). The duration of pregnancy was a significant predictor of U5M. The posterior mean and median for duration of pregnancy (> 9 months) is 0.405 and 0.3831 respectively which is significant with a 95% CI (0.0628, 0.8338). This implies that mothers whose last pregnancy duration was greater than 9 months decelerates the survival times of U5C as compared to the reference category (≤ 9 months).

Also, the sex of a child was significant predictor of U5M. The posterior mean and median for the sex of the child is 0.1123 and 0.1178 respectively and was significant with a 95% CI (0.00591, 0.1666) indicating that the covariates sex(females) decelerates the survival of U5C as compared to their male counterpart. This implies that the probability of survival is less in female children as compared to male. The posterior mean and median for twin U5C were 0.1774 and 0.1761 and is significant with a 95% CI (0.0732, 0.2634) implying that twin status decelerates the survival of U5C as compared to singleton. This suggest that singleton have high probability of survival as compared to twin.

The place of residence was not a significant predictor of U5M. The posterior mean and median of place of residence is -0.0659 and -0.0791 respectively with a 95% CI (-0.1423, 0.0639). Similarly, use of mosquito net was insignificant in predicting U5M. The posterior mean and median were -0.1046 and -0.1079 with a 95% CI (-0.0673, 0.1425).

The contraceptive used was a significant predictor of U5M. The posterior mean and median were -0.2700 and -0.2784 respectively and is significant at 95% CI (-0.3364, -0.1629). This implies that use of contraceptive accelerates the survival time of U5C as compared to the reference category (not using). Source of drinking water was not a significant predictor of under-five mortality. The posterior mean and median were -0.0029 and -0.0121 respectively with a 95% CI (-0.0846, 0.0450). The posterior mean and median for children from homes with improved toilet facility is -0.1216 and -0.1151 and is significant at 95% CI (-0.1959, -0.0769) indicating that improved toilet facility significantly accelerates the survival time of U5C as compared to the reference category (unimproved).

The mean and median of variance of the spatial term v which was represented as $\tau^2 = 6.358\ and\ 6.093$ is significant at a 95% CI (2.6330, 11.4630). This suggest therefore that the inclusion of the random effect was relevant, implying that the risk of U5M was not homogeneous across the thirty-seven states in Nigeria.

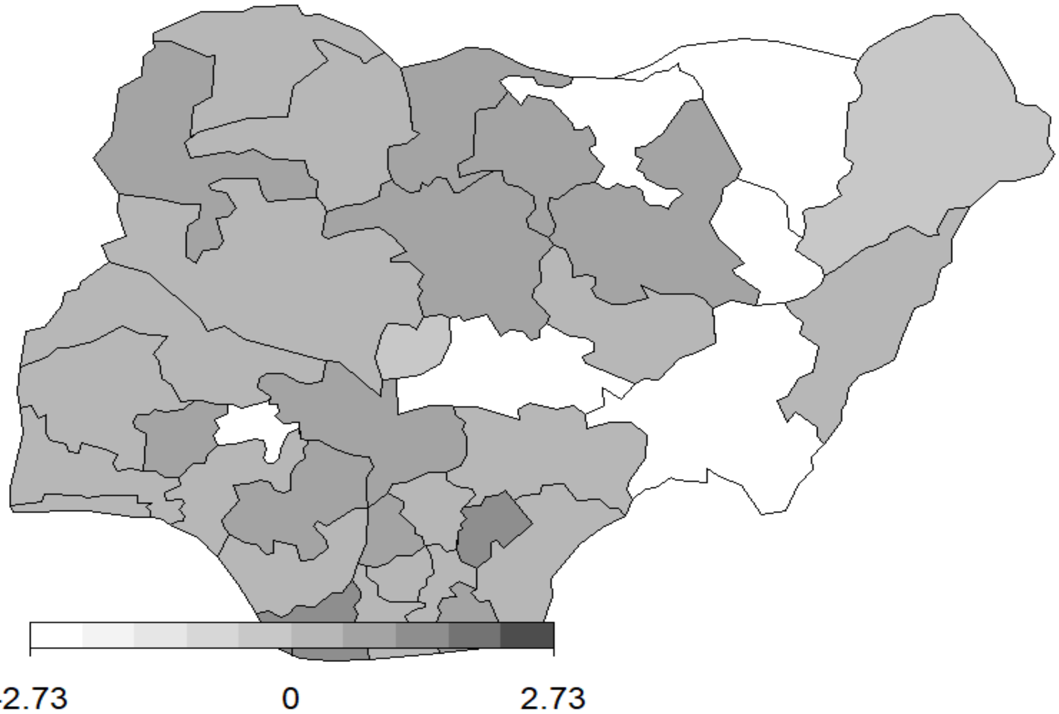
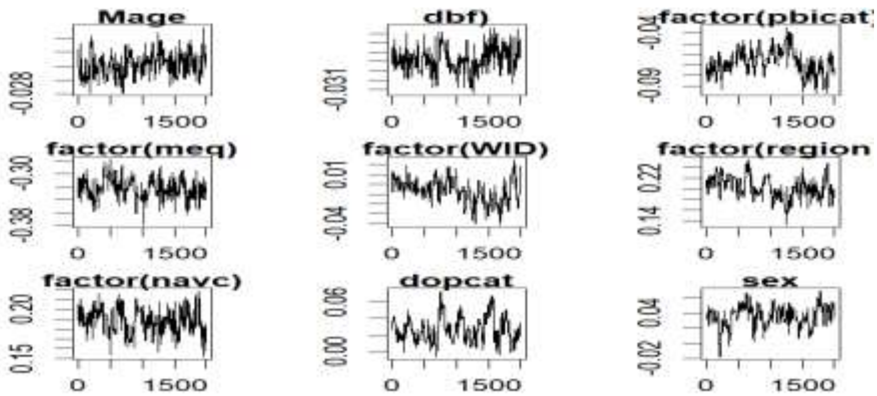


Figure 4: Posterior mean (Average) Spatial Effect of State Based Survival Risk of Under Five Mortality

From figure 4 the inhabitants located in the grey states indicate regions with high risk of U5M compared to the overall adjusting for the effect of sub-specific covariates.



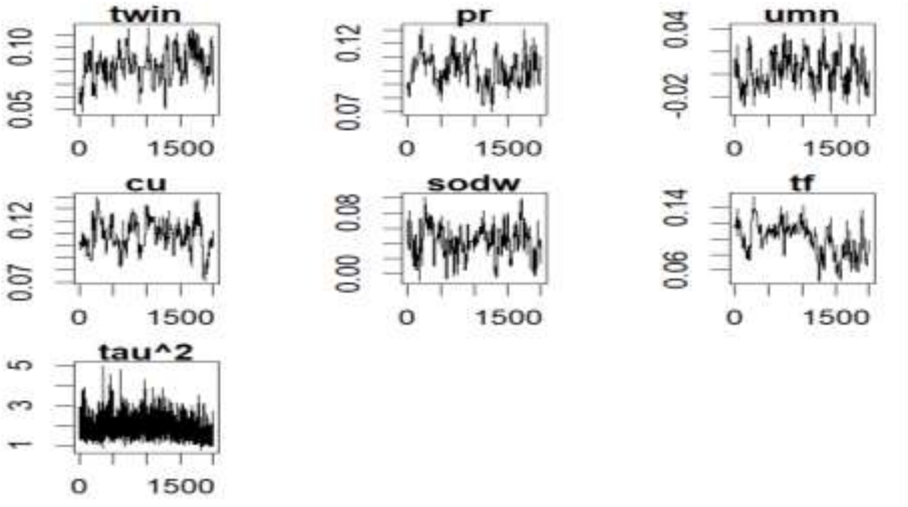


Figure 5: Trace Plot of the Parameters in the log-logistic AFT model with ICAR spatial prior

Figure 5 presents the trace plot of the parameter estimate in the Log-logistic AFT model with ICAR spatial prior of the parameters $\beta_1, \ldots, \beta_{15}$ and for the mixing of τ (the variance of the spatial frailty term v). The mixing is stationary thus indicating a good model performance.

4.3 DISCUSSION OF FINDINGS

In this study, the time to event (death) outcome of under -five children in Nigeria were modelled using the Bayesian Accelrtaed Failure Time Modelling framework. AFT model with three different baseline distributions were considered including Log-logistic, Weibull and Lognormal each with ICAR, IID priors and the without prior were fitted. In order to select the best model that fitted the data under study, three criteria DIC, WAIC and LPML were used. Among the nine models considered in this study, the log-logistic AFT model with ICAR spatial prior stood out to be the best model to explain the variations in the time to death outcome variable.

The findings showed that maternal age at birth, duration of breast feeding, preceding birth intervals, maternal educational qualification, wealth index, region, number of antenatal visits, duration of pregnancy, gender of child, twin status, contraceptive used and toilet facility were the significant risk factors of under-five mortality. The findings also indicated disparity in geographical regions as represented in the map. The findings revealed that maternal age at birth accelerates the survival time of under-five children. This implies that a unit increase in maternal age at birth tends to decrease under-five mortality. This finding is consistent with Singh and Singh (2023); Yemane et al. (2024) and Wegbom *et al.* (2019). The study revealed further that duration of breast feeding accelerates the survival time of under-five children. This implies that increase in the duration of breast feeding tends to decrease the hazard of under-five mortality. This finding is in line with findings of Yalew et al. (2022) and Yemane et al. (2024).

The preceding birth interval between 24 to 33 months was also a significant predictor of under-five mortality. Women with preceding birth interval between 24 to 33 months accelerates the survival time of under-five children as compared to the reference group (less than 24 months). This suggests a decrease survival process of under-five children in women with preceding birth interval less two years (less than 24 months). This finding is similar to that of Ahmed et al. (2020); Jaiswal et al. (2024) and Wegbom *et al.* (2019).

Maternal educational qualification was a significant predictor of under-five mortality. Under-five children whose mothers have secondary level of education, primary level and no formal education decelerate the survival process of under-five children as compared to those with higher educational qualification. This implies that children whose mother have higher educational qualification have higher probability of survival. This finding is similar to findings of Yemane et al. (2024); Kunnuji *et al.* (2022) and Wegbom *et al.* (2019). Furthermore, women with wealth index: poorer, richer and richest accelerate the survival of under-five children as compared to those whose family wealth index is poorest (reference category). This finding is similar to findings from a study conducted by Ahmed et al. (2020) and Wegbom *et al.* (2019) whose study found wealth index as significant predictor of under-five mortality.

The under-five children who were from the North-West and South-East have less risk of dying before their fifth birth-day as compared to child from North-Central. However, children from South-South region have high risk of dying before their fifth-birthday as compared to children from North-Central (reference category). This finding is similar to findings of Kunnuji *et al.* (2022) and Okoli et al., (2022).

The number of antennal visits was also a significant predictor of U5M. Under-five children whose parents had antennal visits of 1 – 8 times accelerate the survival of under-five children as compared to the reference category (none visit). This finding is in line with the finding of Egbon et al. (2022) whose study found frequency of antenatal visits as a significant predictor of under-five mortality. The duration of pregnancy was a significant predictor of U5M. Women whose last pregnancy duration was greater than 9 months decelerate the survival of under-five children as compared to the reference category (≤ 9 months). Also, the sex of a child was significant predictor of under mortality. The sex (female) decelerates the survival of under-five children as compared to their male counterpart. This implies that the probability of survival is less in female children as compared to male. This finding is similar to finding of Egbon et al. (2022).

The study revealed further that under-five children that were twin decelerate the survival of under-five as compared to singleton. This suggest that singleton have high probability of survival as compared to twin. This finding is in line with the findings of Egbon et al. (2022) and Yalew et al. (2022). The study revealed further that the use of contraceptive accelerates the survival of under-five as compared to the reference category (not using). This implies that under-five children whose parent used contraceptive have high survival as compared to those that their parent do not use. This finding is similar to finding of Musa et al. (2020). In addition, it was discovered that improved toilet facility accelerates the survival of under-five as compared to those with unimproved toilet facility. This finding is consistent with findings of Musa et al. (2020) and Egbon et al. (2022).

4.0 CONCLUSION AND RECOMMENDATIONS

In this study, the Bayesian Accelerated Failure Time model with ICAR spatial prior was examined. The AFT model with three different baseline distributions were considered including Log-logistic, Weibull and Lognormal each with ICAR, IID priors and the model without prior were fitted. The models were used to quantified the risk factors of under-five mortality. Using the DIC, WAIC and LPML as the model selection criteria, it was found that log-logistic AFT model with ICAR spatial prior stood out to be the best model to explain the variations in the time to death of under-five children. Arising from the best model, maternal age at birth, duration of breast feeding, preceding birth intervals, maternal educational qualification, wealth index, region, number of antenatal visits, duration of pregnancy, gender of child, twin status, contraceptive used and toilet facility were the significant risk factors of under-five mortality. The findings also indicated disparity in geographical regions as represented in the map.

The considerable disparities observed across states and regions should be taken into account at the policy level in order to meet the SDG 2030 targets.

In order to protect a child from death, the value of education cannot be overstated. This guarantees that women are properly informed on, among other things, breastfeeding, vaccinations, meals with higher nutritional value, and hygiene. Compared to women without any education, mothers with some education also have a higher likelihood of having a source of income. The probability of a child's survival is greatly increased when a mother with a certain level of education marries a man who has a comparable or better degree of education. This boosts the income of the household as a whole.

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