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ON THE CALCULATION OF THE AVERAGE VALUE OF THE STRONG COUPLING CONSTANT α_s, USING THE BEST LINEAR UNBIASED ESTIMATOR (BLUE) METHOD.

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ABSTRACT

Experiments to measure a single physical quantity often produce several estimates based on the same data, and which are hence correlated. We describe how to combine these correlated estimates in order to provide the best single answer, and also how to check whether the correlated estimates are mutually consistent.
We discuss the properties of our technique, and illustrate its application by using it for a specific experiment which measured the strong coupling constant α_s.In this work, we computed the mean value of the strong coupling constant by relying on the three measurement values of ATLAS.
We present a method to calculate the mean value of the strong coupling constant and the uncertainty about this value at a centre of-mass energy $\sqrt{s}$ =7Tev, 8Tev and 13Tev based on the results obtained in ref[1].
We get the result $\alpha_s = 0.116 \pm 0.0035$, we will compare it with the result obtained by the authors of ref[1], which is $0.117^{+0.0034}_{-0.0036}$.

KEYWORDS: ATLAS, CMS,parton distribution function (PDF), correlation matrix, covariance matrix, Next-to-Leading-Order(NLO),Next-to-Next-to-Leading-Order (NNLO).

1 INTRODUCTION

The strong coupling constant is one of the fundamental parameters of standard theory of particle physics.
Our study in this article is a statistical study, it is based on finding the average value of the strong coupling constant, α_s, as well as the error committed in measuring this value.
We have divided the work into two parts:
The first part was devoted to calculating the average value of the alphas, where 79 average values of α_s were extracted through different experimental values from the ATLAS experiments by applying the Monte Carlo Box Muller method [2] to extract the trekking curves containing the mean values of the Alphas and the error committed around These values.
The second part is based on the calculation of the average value of α_s for the experiments of Atlas 7 Tev, Atlas 8 Tev and Atlas 13 Tev , where we will compare our result obtained with the result obtained by the authors of ref [1],using the data in Table7 for Atlas measurements , where we used these data by aplying the BLUE (Best Linear Unbiased Estimate) method, which gave us the final result 0.116 ± 0.0035.

2 CMS detector

The Compact Muon Solenoid (CMS) experiment is one of two large general-purpose particle physics detectors built on the Large Hadron Collider (LHC) at CERN in Switzerland and France.
CMS is 21 metres long, 15 m in diameter, and weighs about 14,000 tonnes.[8]
The central feature of the CMS apparatus is a superconducting solenoid, of 6m internal diameter, providing a magnetic field of 3.8 T. Within the field volume are a silicon pixel and strip tracker, a crystal electromagnetic calorimeter (ECAL), and a brass and scintillator hadron calorimeter.[9]

3 ATLAS detector

ATLAS is the largest general-purpose particle detector experiment at the Large Hadron Collider (LHC), a particle accelerator at CERN (the European Organization for Nuclear Research) in Switzerland. The experiment is designed to take advantage of the unprecedented energy available at the LHC and observe phenomena that involve highly massive particles which were not observable using earlier lower-energy accelerators. ATLAS was one of the two LHC experiments involved in

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the discovery of the Higgs boson in July 2012. It was also designed to search for evidence of theories of particle physics beyond the Standard Model.[10]

4 Extraction of α_s and averages

4.1 The experimental data

The measured inclusive isolated prompt photon production cross sections $(d\sigma/dE_T^\gamma)_{exp}$ are presented as a function of the photon transverse energy E_T^γ, for each pseudorapidity intervals.

The first 24 data points are given in [3],were measurement spans from $E_T^\gamma = 15Gev$ to $E_T^\gamma = 100Gev$ in eight E_{bins}^T for the $\eta^\gamma \leq 0.6,\ 0.6 \leq \eta^\gamma < 1.37$ and $1.52 \leq \eta^\gamma < 1.81$ regions.

32 supplementary data points are reported in [4], in eight E_{bins}^T between 45 and 400 GeV in the four pseudorapidity intervals $\eta^\gamma \leq 0.6,\ 0.6 \leq \eta^\gamma < 1.37,\ 1.52 \leq \eta^\gamma < 1.81$ and $1.81 \leq \eta^\gamma < 2.37$.

Measurements are completed with 23 new data points extending significantly the measured kinematic range to 1TeV [5].We have a total of 79 experimental data points with asymmetric errors of the form:

$$[(d\sigma/dE_T^\gamma)_{exp}]_{-\Delta_n}^{+\Delta_p} \quad (1)$$

Dealing with asymmetric errors requires special care. After the symmetrization following the prescriptions in [6], making the results approximately symmetric and Gaussian (Fig1):

$$(d\sigma/dE_T^\gamma)_{exp} = \sigma_{exp} \pm \Delta_{exp} \quad (2)$$

we propagate the experimental uncertainties by means of a series of pseudo-experiments using Monte Carlo technique as we will see in next section.

4.2 Monte-Carlo generation

We used a Monte Carlo Box Muller method to generate Gaussians, using results of experimental measurements for cross sections, for each transverse impulse $\boldsymbol{P_t}$ on has the measurement of cross section and the corresponding error, in total we have 79 Gaussian. Box-Muller method is described in Ref.[2].

4.3 The nominal values

We have estimated the 79 × 79 "nominal" covariance matrix using samples α_{ik}:

$$Cov_{ij} = \frac{1}{N}\sum_{k=1}^{N} (\bar{\alpha}_i - \alpha_{ik}), for i \neq j \quad (3)$$
$$Cov_{ii} = \sigma^2$$

where the subscript k denotes the Monte Carlo experiment number and i refers to the method used to determine α_s (i.e i goes from 1 to 79, and k from 1 to 10000000), $\bar{\alpha}_i$ is the Monte Carlo (Box-Muller) average for the i^{th} method i.e.

$$\bar{\alpha}_i = \frac{1}{N}\sum_{k=1}^{N} \alpha_{ik} \quad (4)$$

Fig1 below represents the $\frac{d\sigma}{dE_T^\gamma}$ distribution obtained by the Monte Carlo Box-Muller method.

As for Fig 2, we extracted from $d\sigma/dE_T^\gamma$ the values of α_s, which are represented in the attached graphs.

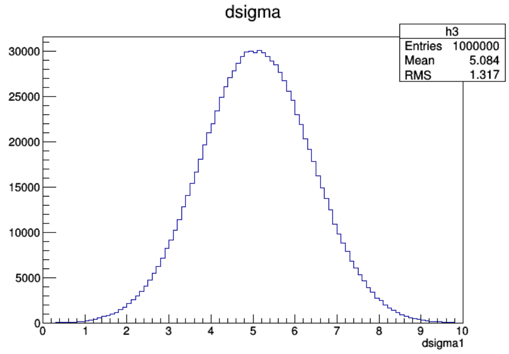


Fig 1: An example of Gaussian distributions generated by the Monte Carlo(Box-Muller) method corresponding to the first data point (5.09± 1.36) (nb/GeV) in the range $15 < E_T^\gamma < 20GeV$ and $\eta^\gamma < 0.6$. The original (asymmetrical) value is $5.24_{-1.4}^{+1.3}(total) \pm 0.58(luminosity)$. The x-axis represents $d\sigma/dE_T^\gamma$ in (nb/GeV) and the y-axis the number of entries

.

5 The averaging procedure

5.1 The BLUE metod

The Best Linear Unbiased Estimate of the observable is obtained with the following meaning: Best: the combined result for the observable obtained this way has the smallest variance; Linear: the result is constructed as a linear combination of the individual estimates; Unbiased Estimate: when the procedure is repeated for a large number of cases consistent with the underlying multi-dimensional PDF(probability density function), the mean of all combined results equals the true value of the observable.

5.2 Calculation of α_s using The BLUE metod

To obtain the average value of the strong coupling constant α_s from nominal values extracted in individual $|\eta^\gamma| - E_T^\gamma$,we must take into account their correlations. For this purpose we used the Best Linear Unbiased Estimate (BLUE) method described in Ref [7] (see Equations 3, 4 and 5). where the unbiased estimate α is a linear combination of the individual estimates,

$$\alpha = \sum_{i=1}^{79} w_i \bar{\alpha}_i \quad (5)$$

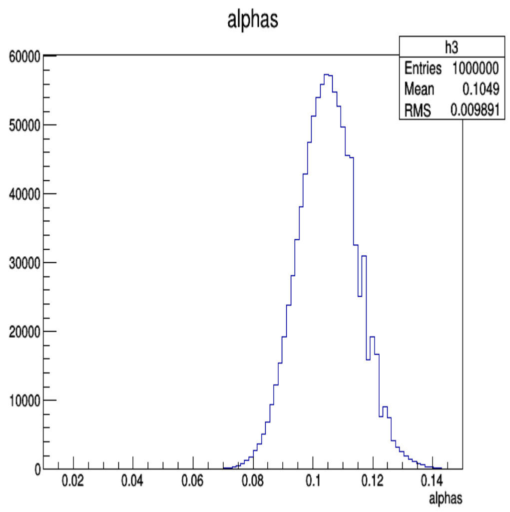
Here w_i denotes the weights and $\bar{\alpha}_i$ the mean of strong coupling constant α_s with the constraint :

$$\sum_{i=1}^{79} w_i = 1 \quad (6)$$

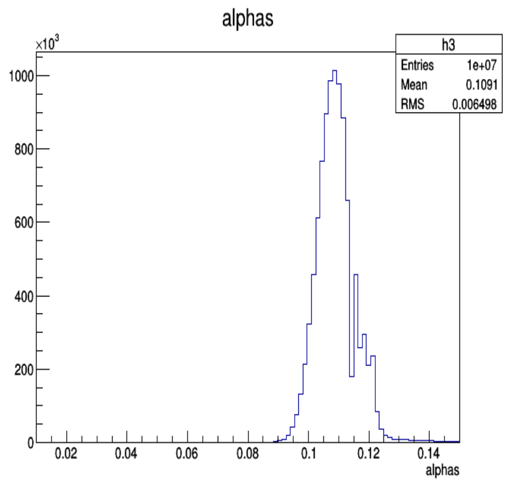
having the minimum variance σ^2:

$$\sigma^2 = \sum_{i,j=1}^{79} w_i Cov_{ij} w_j \quad (7)$$

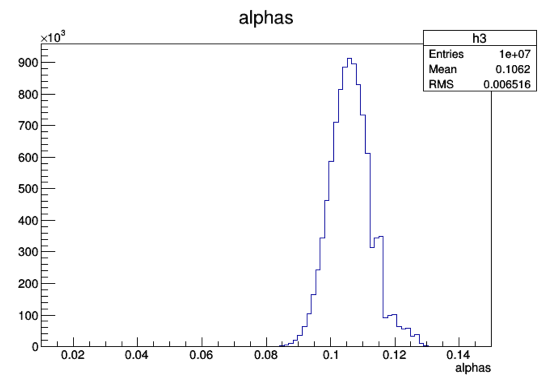
Cov_{ij} represents the covariance matrix elements.



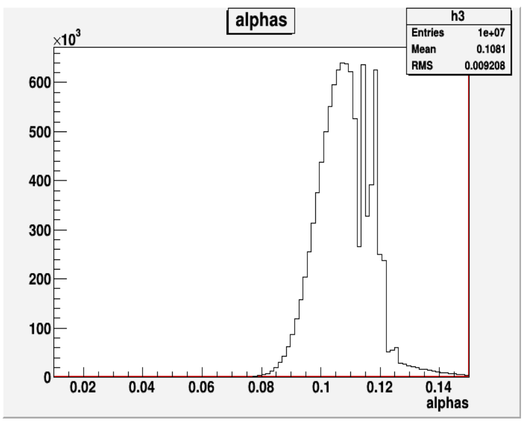
$a)$ $15 < E_T^\gamma < 20, |\eta^\gamma| < 0.6$



$c)$ $25 < E_T^\gamma < 30, |\eta^\gamma| < 0.6$



$b)$ $20 < E_T^\gamma < 25, |\eta^\gamma| < 0.6$



$d)$ $30 < E_T^\gamma < 35, |\eta^\gamma| < 0.6$

Fig 2: Examples of α_s probability distributions constructed from data generated by pseudo-experiments in different kinematic range. The x-axis represents $\alpha_s(M_Z^2)$ and the y-axis the number of entries

Application of BLUE method for calculating α_s

Table 1: This table is extracted from Table 7 of Ref [1]

	Center	Stat	Syst	Lumi	E_{beam}	PDF	Scale	m_t	Total
ATLAS (7Tev)	0.1190	+0.0009 -0.0009	+0.0012 -0.0012	+0.0010 -0.0011	+0.0001 -0.0001	+0.0025 -0.0026	+0.0016 -0.0015	+0.0012 -0.0013	+0.0036 -0.0037
ATLAS (8Tev)	0.1152	+0.0004 -0.0004	+0.0012 -0.0013	+0.0011 -0.0012	+0.0001 -0.0001	+0.0024 -0.0025	+0.0018 -0.0017	+0.0013 -0.0014	+0.0037 -0.0037
ATLAS (13Tev)	0.1168	+0.0007 -0.0007	+0.0022 -0.0023	+0.0015 -0.0016	+0.0002 -0.0002	+0.0019 -0.0020	+0.0022 -0.0020	+0.0015 -0.0015	+0.0043 -0.0043

1. Statistical uncertainties are considered uncorrelated for all experimental inputs.
2.Systematic uncertainties are considered fully correlated only for measurements obtained with the same detector. This concerns the measurements performed by CMS and ATLAS at different centre-of-mass energies.
3. Uncertainties due to beam energy are fully correlated between ATLAS and CMS and are taken to be correlated across energies.
4.The uncertainties on the predicted cross sections (due to the parton distribution function (PDF), the top-quarkmass and the renormalisation and factorisation scale) are generally strongly correlated.

The combination result strongly depends on the assumed correlation structure of these theoretical uncertainties if included in the combination, which is usually not known precisely in particular for the scale uncertainty.

We therefore adopt a different procedure: The individual results are simultaneously shifted up and down by their respective total theory uncertainties, and the combination is re-evaluated. The difference between the upper and lower bounds and the original combination is taken to be the (asymmetric) theoretical uncertainty.

5.The m_t uncertainties are considered fully correlated for all measurements.

First,we make the errors symmetric using the method described in Ref. [6],so we get the results shown in Table2.

Table 2: Results for the strong coupling evaluated at the Z-boson mass scale and individual uncertainty contributions.These are based on cross sections calculated at NNLO+NNLL using the CT14nnlo series of PDFs.

	Center	Stat	Syst	Lumi	E_{beam}	PDF	Scale	m_t	Total
ATLAS (7Tev)	0.1190	± 0.0009	± 0.0012	± 0.00105	± 0.0001	± 0.00255	± 0.00155	$+0.00115$	± 0.00365
ATLAS (8Tev)	0.1152	± 0.0004	± 0.00125	± 0.00125	± 0.0001	± 0.00245	± 0.00175	± 0.00135	± 0.0037
ATLAS (13Tev)	0.1168	± 0.0007	± 0.00225	± 0.00155	± 0.0002	± 0.00195	± 0.0021	± 0.0015	± 0.0043

From the previous table, we extract three different values for the three experiments : ATLAS 7Tev, ATLAS 8Tev and ATLAS 13Tev.

$$\boldsymbol{\alpha_{s1} = 0.119 \pm 0.0009(stat) \pm 0.0012(syst) \pm 0.00105(lumi) \pm 0.0001(E_{beam}) \pm 0.00255(PDF) \pm 0.00155(Scale) \pm 0.00125(M_t)}$$

$$\boldsymbol{\alpha_{s2} = 0.1152 \pm 0.0004(stat) \pm 0.00125(syst) \pm 0.00125(lumi) \pm 0.0001(E_{beam}) \pm 0.00245(PDF) \pm 0.00175(Scale) \pm 0.00135(M_t)}$$

$$\boldsymbol{\alpha_{s3} = 0.1168 \pm 0.0007(stat) \pm 0.00225(syst) \pm 0.00155(lumi) \pm 0.0002(E_{beam}) \pm 0.00195(PDF) \pm 0.0021(Scale) \pm 0.0015(M_t)}$$

Applying the BLUE method in $C++$ program and based on the information given in the previous paragraph we obtain the following correlation matrix :

$$\boldsymbol{\rho = \begin{pmatrix} 1 & 0.9593 & 0.897 \\ 0.9593 & 1 & 0.9351 \\ 0.897 & 0.9351 & 1 \end{pmatrix}}$$

The covariance matrix of the input 3x3 matrix is as follows :

$$\boldsymbol{cov = \begin{pmatrix} 0.00001273 & 0.00001216 & 0.897 \\ 0.00001216 & 0.00001262 & 0.00001131 \\ 0.897 & 0.00001131 & 0.00001606 \end{pmatrix}}$$

This error matrix determines the weights of the various estimates as :

$$\boldsymbol{w_1 = 0.4399, w_2 = 0.8905, w_3 = -0.3304}$$

Together with the results of table, these give a best estimate :

$$\boldsymbol{\alpha_s 0.116 \pm 0.0005806(stat) \pm 0.0008976(syst) \pm 0.000067(E_{beam}) \pm 0.00266(PDF) \pm 0.001546(scale)(Scale) \pm 0.001256(M_t)}$$

$$\boldsymbol{\alpha_s = 0.116 \pm 0.0006(stat) \pm 0.0034(sys) = 0.116 \pm 0.0035(tot)}$$

CONCLUSION

We have in our work extract the value of the strong coupling constant α_s from several experiments of the ATLAS detector in which different values of α_s are related to the values of the cross section. We obtained different mean values for the strong coupling constant using the root program .

On the other hand we applied the Best Linear Unbiased Estimate (BLUE) method through the $C++$ program for calculating the mean values of α_s and the error made in the calculation of this average value by extracting from a table of three different values for the ATLAS experiments[i] which are ATLAS 7 Tev, ATLAS 8 Tev and ATLAS 13 Tev, where we get the result with symmetric errors:

$$\boldsymbol{\alpha_s = 0.116 \pm 0.0006(stat) \pm 0.0034(syst) = 0.116 \pm 0.0035(total)}$$

which is in good agreement with the most recent world average value $0.117^{+0.0034}_{-0.0036}$.

It is important to note that the theoretical uncertainties are mostly coming from terms beyond NLO (Next-to-Leading-Order).

The calculations of prompt photon production cross sections to NNLO (Next-to-Next-to-Leading-Order) are necessary to overcome this deficiency, especially they will minimize the sensitivity of the result to the scale parameters and will improve accuracy in α_s determination.

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[i] Table 7 of ref [1]