



STOCHASTIC ANALYSIS OF STOCK PRICE CHANGES AS MARKOV CHAIN IN FINITE STATES

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ABSTRACT

In this work, stochastic analysis of Markov chain model used to examine stock price formation in finite states. The data was subjected to 5-step transition matrix for independent stocks where transition matrix replicated the use of 3-states transition probability matrix. This enables us proffer precise condition of obtaining expected mean rate of return of each stock. Out of the four stocks studied, stock (1), stock (2), stock (3) and stock (4), it was also discovered that stock (1) has the highest mean rate of return:4.0548 and Stock (4) has the best probability of price increasing in the near future:21%. This informs the investor about the behavior of the stocks for the purpose of decision making. From the stochastic analysis, it is revealed that stock price changes are memory-less satisfying the properties of Markov chain. i.e., it converges to a point or becomes stationary at $n=5$ ie $S1:0.1967-0.2354, S2:0.2053-0.1913, S3:0.1972-0.2051$ and $S4:0.2023-0.1835$. Also all states of the transition communicate and are all time dependent.

KEYWORDS: Stock market price, Markov Chain, Transition matrix, Stochastic Analysis.

INTRODUCTION

It is obvious that changes in stock price changes over long or short trading period is seen as a stochastic process, which can be modeled for the purpose of decision making. The Nigerian stock Exchange (NSE) plays vital roles in raising capital funds and also acts as a medium between firms and the investors. Therefore, empirical studies of stocks on the NSE show that useful results can be obtained for the investor when the stock price dynamics are well known Agwuegbo et al.(2010)[1]. Hence stock market performance can be used to assess investment viability within the financial market. Many scholars have extensively written on stock market price viability such as Amadi et al.(2022),Osu et al.(2014) etc.

The price evolution of risky assets is usually modeled as the trajectory of a Markov process defined on some underlying transition probability state space. From the stochastic point of view the method of Markov chain stipulates a system of transition matrix of an element beginning from one state to another.

It ascertains the transition as a random process, and show-cases the memory-less property of Markov chain. That is to say that the future state of any process strictly depends on its current state but not on its past series acquired over time. Markov chain is one of the most well developed theories of stochastic process with its applications in various fields of science and technology. Many scholars have written extensively on the modeling of stock price using Markov chain and results obtained in diverse ways. For instance, Mettle et al.(2014) considered stochastic analysis of share prices. His results showed the precise condition of determining expected mean return time for stock price. Investment decision can be improved based on highest transition probabilities. In the same manner, Agwuegbo et al.(2010) examined stock market prices, its fluctuations, influences on financial lives and economic health of a country. Their findings showed that stock price is a random walk and no investor can alter the fairness and unfairness of a stock price as defined by expectation. More so, Bairagi and Kakaty (2015) studied the behavior of stock market using Markov chain. The study reveals

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that regardless of bank's current share price steady state probabilities of share price remain the same all through the iteration. Zhang and Zhang (2009) Introduced a Markov chain model for stock market trend forecasting. The study revealed the Markov chain model was more effective to analyze and predict the stock market index and closing stock price under the market mechanism. Eseoghene (2011) Studied long run prospects of security prices in Nigeria where the data were collected from the randomly selected banks from the banking sector of Nigeria. The analysis suggested that the price level of stocks of Nigerian banks were likely to remain relatively stable in the long run irrespective of the current situations. Christain and Timothy(2014) examined the long run behavior of the closing price of shares of eight Nigerian banks using Markov chain model. They computed limiting distribution transition probability matrix of share price and found that despite the current situation in the market for Nigerian bank stocks would produce viable returns in the future. It was concluded that the results derived from the study will be useful to investors.

However, based on the fact that stock market prices changes over time and can be viewed as a stochastic process, the method of Markov chain was studied using historical data of four different stocks represented as S1, S2, S3 and S4 respectively. The data was subjected to 5-step transition matrix for independent stocks where transition matrix replicated the use of 3-states transition probability matrix. This enables us proffer precise condition of obtaining expected mean rate of return of

each stock and also shows that stock price formation converges to a point or becomes stationary at a particular point. This implies that stock price has no memory of the past information.

The aim of this paper is first, to present the unstable nature of stock market prices using Markov chain, then determine the expected mean rate of returns for independent stocks in finite states, and obtain steady-state probabilities with respect to stock market prices in NSE.

The rest of this paper is arranged as follows: Section 2 presents mathematical frame work, the problem formulation is seen in Subsection 2.1, Results are presented in Section 3 and the paper is concluded in Section 4.

MATHEMATICAL FRAMEWORK

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and let T be an arbitrary set called (called the index set). Any collection of random variables $X = \{X: t \in T\}$ defined on $(\Omega, \mathcal{F}, \mathbb{P})$ is a stochastic process with index set T . For time, t $X(t)$

is called the state of the procedure at time t . In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

It can also be seen as a statistical event that evolves with time in accordance to probabilistic laws. For, a stochastic process the collection of random variables are ordered in time and defined at a set of time points which may be continuous or discrete.

Definition 1: A stochastic process X is said to be a Markov chain if the Markov property is satisfied:

$$P(X_{n+1} = j / X_0, X_1, \dots, X_n) = P(X_{n+1} = j / X_n) \quad (1.1)$$

For all $n \geq 0$ and $i, j \in S$ (state space).

It is sufficient to know that the Markov property given in (1.1) is equivalent to the following, for each $j \in S$.

$$P(X_{n+1} = j / X_{n_1}, X_{n_2}, \dots, X_{n_k}) = P(X_{n+1} = j / X_{n_k}) \quad (1.2)$$

(for any $n_1 < n_2 < \dots, n_k \leq n$)

Assuming $X_n = i$ implies that the chain is in the i th state at the n th step. it can also be said that the chain 'has the value i ' or 'is in state i '. The idea behind the chain is described by its transition probabilities:

$$P(X_{n+1} = j / X_n = i) \quad (1.3)$$

They are dependent on i, j and n .

Definition 2: The chain X is said to be homogeneous if the following holds

$$P(X_{n+1} = j / X_n = i) = P(X_1 = j / X_0 = i) \quad (1.4)$$

For all n, i, j .

The transition matrix $P = (P_{ij})$ is an $n \times n$ matrix of transition probabilities.

$$P_{ij} = P(X_{n+1} = j / X_n = i) \quad (1.5)$$

Hence, the transition probabilities with homogenous Markov chain are always stationary at a point.

Theorem 1: Suppose P is a stochastic matrix which implies the following:

(i) P has non-negative entries or $P_{ij} \geq 0$

(ii) $\sum_j P_{ij} = \sum_j P(X_{n+1} = j / X_n = i) = \sum_j P(X_1 = j / X_0 = i)$

which is stationary or point of convergence.

Proof : (i) each associated entry in P is a transition probability P_{ij} and being a probability $P_{ij} \geq 0$.

$$(ii) \sum_j P_{ij} = \sum_j P(X_{n+1} = j / X_n = i) = \sum_j P(X_1 = j / X_0 = i)$$

Which is stationarity.

$$P(X_i \in S / X_0 = i) = 1.$$

Assumptions

- Stock price must be a probability of going from one state to the other(ie transition matrix)
- Stock market prices satisfies Markov property such that , it has no memory: conditional upon the present; the future does not depend on the past.
- To determine the levels of changes of each stocks.

Theorem 2 :(Chapman-Kolmogorov Equations).

Assuming each of the stock market prices follows a Markov property such that the following condition holds:

$$P_{ij(m+n)} = \sum_{r=i}^n P_{ir(m)} P_{rj(n)} \text{ Since } P_{m+n} = P_m P_n \text{ and so on } P_n = P^n \text{ the } nth \text{ power of } P .$$

Where P is probability matrix of stock prices, ij is element of state space of each stock.

Proof:

$$\begin{aligned} P_{ij(m+n)} &= P(X_{m+n} = j / X_0 = i) \\ &= \sum_r P(X_{m+n} = j, X_m = r / X_0 = i) \\ &= \sum_r P(X_{m+n} = j / X_m = r / X_0 = i) P(X_m = r / X_0 = i) \end{aligned}$$

Using the following in probability rule:

$$P(A \cap B / C) = P(A / B \cap C) P(B / C) \text{ and setting } A = \{X_{m+n} = j\}, B = \{X_m = r\}, \text{ and } C = \{X_0 = i\}$$

Using Markov property yields

$$\begin{aligned} P_{ij(m+n)} &= \sum_r P(X_{m+n} = j / X_m = r) P(X_m = r / X_0 = i) \\ &= \sum_r P_{rj(n)} P_{ir(m)} \\ &= \sum_r P_{1r(m)} P_{r1(n)} \end{aligned}$$

Hence $P_{m+n} = P_m P_n$ and so $P_n = P^n$, the power of P .

PROBLEM FORMULATION

Here, let S_1, S_2, S_3 and S_4 of daily prices in naira of four different selected stocks be defined as five –state Markov process. Then a $N \times N$ data matrix associated with S_1, S_2, \dots, S_N be X_t which measures the change in stock prices at time t , where $t = 0, 1, 2, \dots, N$ and t is

measured in weekly intervals, $t \in \square$. Considering N stocks over N trading days, time horizon, for each of the three X_i is a row vector where the leading diagonal elements of the transition probability matrix will be determined. Therefore, the vector D_{it} is defined such that

$$D_{it} = (D_{i1}, D_{i2}, D_{i3}, \dots, D_{in}) \quad (1.6)$$

The estimates ,of the transition probability is obtained as follows:

$$\begin{aligned} P_{ij} &= P(X_t = j / X_{t-1} = i), \text{ for } j = 0, 1, 2, 3, \dots, N \\ P_{ij} &= \begin{cases} P & \text{if } j = 1 + i \\ q = 1 - P & \text{if } j = i - 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

where $k + 1$ is the number of states.

$$\left. \begin{aligned} n_{ij} &= \sum_{i=1}^n P_{ij} \text{ for } i, j=0,2,3 \\ \frac{n_{ij}}{n_i} &\text{ for } i, j = 0,1,\dots,k \end{aligned} \right\} \quad (1.7)$$

However, for $k = 4$ is an estimate of the transition matrix.

$$\hat{P} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{02} & \hat{P}_{03} & \hat{P}_{04} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} & \hat{P}_{13} & \hat{P}_{14} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} & \hat{P}_{23} & \hat{P}_{24} \\ \hat{P}_{30} & \hat{P}_{31} & \hat{P}_{32} & \hat{P}_{33} & \hat{P}_{34} \\ \hat{P}_{40} & \hat{P}_{41} & \hat{P}_{42} & \hat{P}_{43} & \hat{P}_{44} \end{pmatrix} \quad (1.8)$$

Setting $i, j = 0,1,2$ for $k = 3$

$$\hat{P} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{02} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix} \quad (1.9)$$

Assuming X_t has state space and transition probability matrix of (1.9) becomes

$$\hat{P} = \begin{pmatrix} \beta_1 & \phi & \theta \\ 1-\phi & \beta_2 & 1-\alpha \\ 1-\theta & \alpha & \beta_3 \end{pmatrix} \quad (1.10)$$

For simplicity and without loss of generality β_i will be determined following (1.6) such that:

$$\left. \begin{aligned} \beta_1 &= \min(a_1, a_2, \dots, a_n) \\ \beta_2 &= \min(b_1, b_2, \dots, b_n) \\ \beta_3 &= \min(c_1, c_2, \dots, c_n) \end{aligned} \right\} \quad (1.11)$$

Thus each minimum value will be used as estimates, in order to determine the value of an economic asset such as stock; consideration should be made to identify some aspect of random variability which is the expected rate of returns and price of the underlying asset, Osu et al.(2014). Hence to determine the expected rate of returns for each stock for the period of trading days the following are defined

$$\left. \frac{1-\phi}{\beta_1}, \frac{1-\theta}{\beta_2} \text{ and } \frac{1-\alpha}{\beta_3} \right\} \quad (1.12)$$

To show for stationary of (1.8) the chain will take into account the behavior of stock market, each is categorized as increase or decrease or remains the same.

RESULTS

To illustrate the stock market price performance in finite states using Markov chain model, the daily prices (in naira) of four (4) selected stocks for sixty (60) trading days on the Nigeria stock Exchange (NSE) (extracted from [5]) were used for the study. The 4 stocks are

represented as stock (1), stock (2), stocks (3) and stock (4) respectively. A random sample of twenty-five (25) stock prices were selected to form a 5x5 transition probability matrix for each of the companies stated below:

Stock(1): transition probability matrix

$$P_{s(1)} = \begin{pmatrix} 0.1990 & 0.2075 & 0.1982 & 0.1888 & 0.2065 \\ 0.1972 & 0.2075 & 0.2082 & 0.1896 & 0.1975 \\ 0.1863 & 0.1793 & 0.2141 & 0.2141 & 0.2063 \\ 0.2015 & 0.2015 & 0.2015 & 0.2015 & 0.1942 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix}$$

Stock (2): transition probability matrix

$$P_{S(2)} = \begin{pmatrix} 0.1961 & 0.1910 & 0.2206 & 0.1961 & 0.1961 \\ 0.1968 & 0.2110 & 0.2001 & 0.1921 & 0.2001 \\ 0.2118 & 0.1974 & 0.1974 & 0.2008 & 0.1928 \\ 0.1996 & 0.2015 & 0.1999 & 0.1905 & 0.1996 \\ 0.2218 & 0.2019 & 0.1921 & 0.1921 & 0.1921 \end{pmatrix}$$

Stock (3): transition probability matrix

$$P_{S(3)} = \begin{pmatrix} 0.2086 & 0.1987 & 0.1985 & 0.1917 & 0.2031 \\ 0.1838 & 0.2036 & 0.2036 & 0.2036 & 0.2053 \\ 0.2051 & 0.1954 & 0.1961 & 0.1869 & 0.2166 \\ 0.1960 & 0.1960 & 0.1960 & 0.1960 & 0.2161 \\ 0.1982 & 0.1982 & 0.1982 & 0.1982 & 0.2072 \end{pmatrix}$$

Stock(4): transition probability matrix

$$P_{S(4)} = \begin{pmatrix} 0.2081 & 0.2132 & 0.2132 & 0.1971 & 0.1684 \\ 0.2038 & 0.2038 & 0.2038 & 0.2141 & 0.1744 \\ 0.1996 & 0.1863 & 0.2147 & 0.2374 & 0.1624 \\ 0.1964 & 0.1964 & 0.2171 & 0.1927 & 0.1974 \\ 0.2039 & 0.1957 & 0.2029 & 0.1733 & 0.2243 \end{pmatrix}$$

Table 1: Transition matrices for selected stock market prices

Stock	P_{00} β_1	P_{01} ϕ	P_{02} θ	P_{10} $1-\phi$	P_{11} β_2	P_{12} $1-\alpha$	P_{20} $1-\theta$	P_{21} α	P_{23} β_3
S1	0.1990	0.2075	0.1982	0.7925	0.2075	0.8207	0.8018	0.1793	0.2141
S2	0.1961	0.1910	0.2206	0.809	0.2110	0.8026	0.7794	0.1974	0.1974
S3	0.2086	0.1987	0.1985	0.8013	0.2036	0.8046	0.8015	0.1954	0.1961
S4	0.2081	0.2132	0.2132	0.7868	0.2038	0.8137	0.7868	0.1863	0.2147

Table 1 gives a complete summary of state transition matrix probabilities, of stock market prices for four different companies. It is obtained by applying 3 state space and using (1.10) Following (1.12) gives the following conditions:

$$\beta_1 = \text{Min}(0.1990, 0.1961, 0.2086, 0.2081) = 0.1961$$

$$\beta_2 = \text{Min}(0.2075, 0.2110, 0.2036, 0.2038) = 0.2036$$

$$\beta_3 = \text{Min}(0.2141, 0.1974, 0.1961, 0.2147) = 0.1961$$

Hence we have $\beta_1 = 0.1961$, $\beta_1 = 0.2036$ and $\beta_3 = 0.1961$

Using (1.13) the expected mean rate of returns of each stock is obtained.

Table 2: Expected mean rate of return for each stock

Companies	$\frac{1-\phi}{\beta_1}$	$\frac{1-\theta}{\beta_2}$	$\frac{1-\alpha}{\beta_3}$	Mean
S1	4.0413	3.9381	4.1851	4.0548
S2	4.1254	3.8281	4.0928	4.0154
S3	4.0862	3.9366	4.1030	4.0419
S4	4.0122	3.8644	4.1494	4.0087

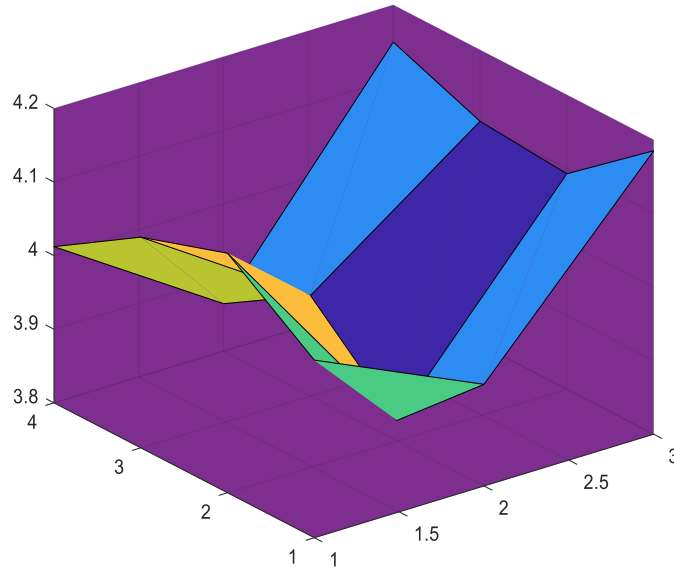


Figure 1: Combined stocks expected rate of returns

Table 3: Distribution of changes of stock market prices as a Markov chain in finite states

Companies	Trading days(N)	$P_{11}^{(N)}$	$P_{12}^{(N)}$	$P_{13}^{(N)}$	$P_{14}^{(N)}$	$P_{15}^{(N)}$
Stock(1)	1	0.1990	0.2075	0.1982	0.1888	0.2065
	2	0.08052	0.0843	0.0826	0.0769	0.0821
	3	0.1174	0.1199	0.1250	0.1178	0.1230
	4	0.1554	0.1579	0.1617	0.1558	0.1597
	5	0.1967	0.1992	0.203	0.1971	0.2384
	6	0.1967	0.1992	0.203	0.1971	0.2384
	7	0.1967	0.1992	0.203	0.1971	0.2384
Stock(2)	1	0.1961	0.1910	0.2206	0.1961	0.1961
	2	0.0760	0.0778	0.0815	0.0752	0.0767
	3	0.1227	0.1213	0.1250	0.1194	0.1145
	4	0.1618	0.1608	0.1642	0.1568	0.1536
	5	0.2053	0.2004	0.2019	0.1945	0.1913
	6	0.2053	0.2004	0.2019	0.1945	0.1913
	7	0.2053	0.2004	0.2019	0.1945	0.1913
Stock(3)	1	0.2086	0.1987	0.1985	0.1917	0.2031
	2	0.0800	0.0819	0.0819	0.0804	0.0832
	3	0.1189	0.1207	0.1208	0.1175	0.1236
	4	0.1569	0.1583	0.1584	0.1551	0.1630
	5	0.1972	0.1963	0.1987	0.1954	0.2051
	6	0.1972	0.1963	0.1987	0.1954	0.2051
	7	0.1972	0.1963	0.1987	0.1954	0.2051
Stock(4)	1	0.2081	0.2132	0.2132	0.1971	0.1684
	2	0.0868	0.0878	0.0878	0.0867	0.0722
	3	0.1293	0.1275	0.1336	0.1373	0.1068
	4	0.1680	0.1662	0.1764	0.1753	0.1457
	5	0.2023	0.1992	0.2106	0.2045	0.1835
	6	0.2023	0.1992	0.2106	0.2045	0.1835
	7	0.2023	0.1992	0.2106	0.2045	0.1835

Table 4: Transition probability matrix of limiting distribution of respective stocks.

	a_{11}	a_{12}	a_{13}
Stock(1)	0.1990	0.2075	0.1982
Stock(2)	0.1961	0.1910	0.2206
Stock(3)	0.2086	0.1987	0.1985
Stock(4)	0.2081	0.2132	0.2132

DISCUSSION OF RESULTS

In Table 1, it is clear that the following stock variables or quantities were obtained row-wise following probability concept, $1 - \phi$, $1 - \theta$ and $1 - \alpha$, which are seen in columns 5, 7 and 8 respectively. Thus, adding the following columns: 3 and 5, 4 and 8, 7 and 9 gives probability of 1 in all the corresponding entries and are all non-negative. This means that they are all sure events and can be used to improve performance and measure the reliability in stock trading. The occurrence of the entire entry shows that price formation is based on the sequence of past price changes.

It can be observed in Table 2, that the return rates are indexed in millions of naira which is profit maximizing for an investor. This portrays to investors, that high probabilities of little gains can be exchanged for low probabilities of big gains for the stipulated trading periods. However, for this short trading period, stock S1 gives the highest mean return on investment compared to stocks S2, S3 and S4 respectively. This analysis is an eye opener to investors on a particular stock, in order to maximize profit and minimize loss.

However, in terms of predicting future of the following investments based on the expected rate of returns; Stock (1): Has 404% chance of reducing its expected rate of return; 394% chance of increasing its price in future and 419% chance of no change in expected rate of return with mean rate of 405%.

Stock (2): Has 413% chance of reducing its expected rate of return; 383% chance of increasing in future and 409% chance of no change in expected rate with mean rate of 402%.

Stock (3): Has 409% chance of reducing its expected rate of return; 394% chance of increasing in future and 410% chance of no change in expected rate of return with mean rate of 404%.

Stock (4): Has 401% chance of reducing its expected rate of return; 386% chance of increasing in future and 415% chance of no change in expected rate of return with mean of 401%.

Figure 1 showed the original nature of the expected rate of return which seems to grow uniformly from a certain point in form of a V-shape. The plots signify that investments grow from one level to the other as far as stock market business is concerned.

In Table 3, the probability matrix of each partition (stocks) is a probability for change in the behavior of stock market prices. The Markov chain in finite state at one is given as follows:

$$P^{(1)} = (0.1990 \ 0.2075 \ 0.1982 \ 0.1888 \ 0.2065)$$

This means that at its initial state $P^{(0)}$, the distribution $P^{(n)}$ of changes of stock market price has same limiting vector.

From the trading days of 5-7 at each partition and columns gives the vector indicating that N does not have to be very big before the distribution of changes of stock market prices as a Markov chain in finite state becomes equivalent to the fixed vector. The chain converges or becomes stationary at $N = 5$ implying that the stock market price satisfies one of the properties of Markov chain; which is its steady state. This result is in consonance with the work of [1].

Clearly the following discussion holds for the matrix Table 4 above:

Stock (1): Has 20% chance of reducing its price; 20% chance of increasing its price in future and 20% chance of no change in price.

Stock (2): Has 20% chance of reducing its price; 19% chance of increasing its price in future and 22% chance of no change in price.

Stock (3): Has 21% chance of reducing its price; 20% chance of increasing its price in future and 20% chance of no change in price.

Stock (4): Has 21% chance of reducing its price; 21% chance of increasing its price in future and 21% chance of no change in price.

It is observed that Stock (4) has the best probability of price increasing in the near future. This result will be beneficial to investors because it presents an assessment of future returns, to enable better investment decision, especially when large amount of money is involved.

The names of the companies whose stocks were used for the work were not mentioned. This is because the work is not about assessing performance of stocks of these companies, but about presenting a finite space Markov chain approach; which provides the investor with a Mathematical analysis that helps with investment decision.

CONCLUSION

From the analysis of this paper Markov chain is represented as a Mathematical tool for taking better investment decisions in order to maximize profit and minimize loss. In this work, stochastic analyses of Markov chain were developed to examine stock price formation in finite states. The stochastic analysis reveals that stock price changes are memory -less which satisfies the properties of Markov chain; and all states of transition communicate and are all time dependent. It was also discovered that stock S1 has the highest mean rate of return and Stock (4) has the best probability of price increasing in the near future. These are very useful about behavior of stocks required before investment. Future studies should explore the uniqueness of finite

and infinite state space in analyzing stock market price formation as Markov chain.

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