



EVIDENCE OF DYNAMICAL TRANSITION AND MAXIMUM PREDICTABILITY OF AIR TEMPERATURE, RELATIVE HUMIDITY AND DEW POINT TEMPERATURE.

ABIDEMI E. ADENIJI AND ADEWOYIN D. ADEYINKA

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ABSTRACT

Monitoring and predicting the climatic phenomenon are the major global concern because of its devastating effects on people's lives and their environments. As a result of this, there is a need to understand the natural processes that control the dynamic evolution of the climatic phenomenon. Air temperature and relative humidity data collected from Nsukka station by the Centre for Atmospheric Research (CAR), measured in 5 minutes time steps from 1st January till 31st December, 2012 have been analysed. Dew point temperature was calculated from the actual readings of air temperature and relative humidity using appropriate empirical relation. In this paper, Average Mutual Information (AMI), False Nearest Neighbour (FNN) and Lyapunov Exponent methods were used to study changes and transitions in the dynamics of these meteorological parameters or temporal deviations from their overall dynamical regimes. The results show that the dynamic model needed to describe the data has 4-5 dimensions for air temperature, 4-6 for relative humidity and 4-5 for dew point temperature. Positive and negative Lyapunov exponents were observed in the air temperature, relative humidity and dew point temperature time series. This indicates that there exists periodicity inherent in the chaotic behaviour of these meteorological time series, causing a transition from chaoticity (positive Lyapunov exponent) to periodicity (negative Lyapunov exponent) and thereafter to chaoticity (positive Lyapunov exponent). The results, therefore, provide additional information about the climate transitions, maximum predictability and also, for formulating a weather prediction model.

KEYWORDS: Chaoticity, Climate transition, Dynamical evolution, Lyapunov exponent, Periodicity

INTRODUCTION

The Earth's tropospheric layer is the region of the Earth's atmosphere where most terrestrial climatic phenomena occur. The average weather conditions over a long period describe the climate of a particular location. Therefore, it's imperative to understand the behaviour of specific weather parameters that influence an area's atmospheric conditions. Some of these critical parameters, such as relative humidity, air temperature, and dew point temperature, significantly impact the environment, agriculture, industry and economy (Shrestha *et al.*, 2019). Interestingly, these parameters are interrelated such that they influence each other's behaviour (Chabane *et al.*, 2018). The relative humidity (RH) is the amount of water vapour in an air sample (Abu-Taleb *et al.*, 2007). It is one of the critical weather parameters that influence the amount of solar radiation in an area. RH has a strong effect on the formation of fog, smog, cloud, and atmospheric visibility. It can be expressed as the ratio of water vapour in the air to the maximum water vapour the air can hold at a given temperature (Nicholas *et al.*, 2018). Thus, the air temperature is a determinant of the capacity of air to

hold water vapour. Temperature is a measure of how hot or cold the atmosphere is. It is another crucial parameter that is widely used to determine the change in the weather. Also, it plays a significant role in controlling other elements of weather, such as the dew point temperature. The dew point temperature is the temperature at which water vapour in the atmosphere will condense into liquid water at the same rate at which it evaporates (Shrestha *et al.*, 2019).

The most preferred of these three main weather parameters is the dew point temperature. This is predicated upon the fact that meteorologist often uses it for forecasting weather and using it as an indicator of how comfortable and uncomfortable warm air will feel (Ukhurebor *et al.*, 2017). Generally, many studies have been carried out in different locations globally and various applications of weather parameters have been suggested in the literature. However, studies at more local scales are needed. Consequently, this research aims to study changes and transitions in air temperature dynamics, relative humidity, and dew point temperature or temporal deviations from their overall dynamical regimes in the tropospheric region of south-east Nigeria.

Abidemi E. Adeniji, Department of Physical Sciences, Bells University of Technology, Ota, Ogun State, Nigeria.

Adewoyin D. Adeyinka, Distance Learning Institute, University of Lagos, Akoka, Lagos State, Nigeria.

2 Materials And Methods

2.1 Evaluation of dew point temperatures from relative humidity and air temperature

It has been established that the dew point depends upon relative humidity and air temperature. However, studies show that the dew point is significantly controlled by air

$$T_{dew\ point} = \frac{b \left(\left(\frac{aT}{b} + T \right) + \ln RH \right)}{a - \left(\left(\frac{aT}{b} + T \right) + \ln RH \right)} \tag{1}$$

Where $a = 17.27$, $b = 237.7$, $T_{dew\ point}$ is the dew point, T is the temperature and RH is the relative humidity.

2.2 Determination of embedding dimension and time delay

The time delay τ and embedding dimension d are two essential parameters needed to unfold the attractors in phase space: The autocorrelation method and mutual information approach are the two Standard approaches to estimate the optimal time delay τ (Fraser and Swinney, 1986). The autocorrelation method looks for linear independence of two variables while mutual information approach takes into account nonlinear correlations by measuring how far the pairs of random

$$I(\tau) = \sum_{X(i), X(i+\tau)} P(X(i), X(i+\tau)) \log_2 \left[\frac{P(X(i), X(i+\tau))}{P(X(i)) P(X(i+\tau))} \right] \tag{2}$$

Where i is the length of the time series, $P(X(i), X(i+\tau))$ denotes the joint probability density for the measurements $P(X(i))$ and $P(X(i+\tau))$, individual probabilities for the measurements of $X(i)$ and $X(i+\tau)$ are $P(X(i))$ and $P(X(i+\tau))$. The appropriate time delay τ is defined as the first minimum of the average mutual information $I(\tau)$. Then the values of $X(i)$ and $X(i+\tau)$ are independent enough of each other to be useful as coordinates in a time delay vector but not so independent as to have no connection with each other at all.

The most suitable choice for time delay (τ) is obtained from the first minimum in the AMI graph since this is the time when $X(i+\tau)$ adds maximum information to the knowledge we have from $X(i)$ (Shang *et al.*, 2005).

The false nearest neighbour (FNN) method to determine the minimal sufficient embedding dimension d was proposed by Kennel *et al* (1992). Each embedding dimension d of a set of time-series data is estimated

temperature rather than relative humidity (Lawrence, 2005).

The dew point temperatures are computed from air temperature and relative humidity using the mathematical relation given in Equation (1) (Lawrence, 2005).

variables of a time series data are dependent to each other (Renjini *et al.*, 2020). The method of average mutual information to determine the time delay was proposed by Fraser and Swinney (1986). This method gives vital information on how the measurements of the two state variables $X(i)$ and $X(t+\tau)$ are connected at time t and $t+\tau$, respectively, by presuming that the state $X(i)$ is known (Abarbanel, 1996). The average mutual information is computed by using equation (2):

from the percentage of false nearest neighbours (FNN). The point at which the first percentage of false nearest neighbours (FNN) drops to zero is chosen as suitable choice of the minimal sufficient embedding dimension d .

2.3 Lyapunov exponents

Lyapunov exponents determine a long-time average exponential rate of divergence or convergence of nearby trajectories in the phase space (Ott, 1993). If a system has at least one positive Lyapunov exponent, then it is assumed to be chaotic. Due to the sensitive dependence on initial conditions, nearby trajectories in phase space tends to diverge or converge making the system's evolution difficult to predict even after a few time steps. Many approaches have been proposed for computation of the maximal Lyapunov exponent out of which Rosenstein *et al.* (1993) is utilised. The maximum Lyapunov exponent is computed by using equation (3).

$$S(\Delta t) = \frac{1}{N} \sum_{t_0=1}^N \ln \left(\frac{1}{|U(S_{t_0})|} \sum_{S_t \in U(S_{t_0})} |S_{t_0+\Delta t} - S_{t+\Delta t}| \right) \tag{3}$$

Where S_{t_0} are embedding vectors or reference points, $U(S_{t_0})$ is the neighbourhood of S_{t_0} with diameter r . For a suitable diameter r and for all embedding dimensions $d > d_0$ which is the minimum dimension, if the exponential increase of the stretching factor $S(\Delta t)$ exhibits a flat region (a linear increase) signifying the saturation effect of exponential divergence, then its slope yields an estimation of the maximal Lyapunov exponent λ . A positive Lyapunov exponent λ shows exponential divergence of the nearby trajectories (an

unstable orbit) indicating chaotic behaviour of a system. When the orbits of dissipative or non-conservative systems are attracted to a stable fixed point or periodic orbit, then such a system indicates negative Lyapunov exponents while Zero Lyapunov exponents are characteristic of conservative systems for which the orbit is a neutral fixed point (Nagesh- Kumar and Dhanya, 2011).

Results and discussion

Air temperature, relative humidity and dew point temperature data from Nsukka station by the Centre for Atmospheric Research (CAR), measured in 5 minutes time steps have been analysed. These meteorological time series contain 8928 data points from 1st January till 31st December, 2012.

Table 1 shows the average mutual information (AMI) plot variations from January to December, 2012 with their first minimum corresponding to each value on the delay time axis, hence giving the optimal delay time, τ . The delay times obtained using the AMI method ranged from 58 to 77, 51 to 82 and 63 to 78 for air temperature, relative humidity and dew point temperature respectively (see Table 1).

Table 1: The average mutual information

Month	Air temperature	Relative humidity	Dew point temperature
Jan.	71	70	71
Feb.	71	79	71
Mar.	75	78	65
Apr.	77	76	75
May	73	69	77
Jun.	72	73	66
Jul.	73	74	78
Aug.	74	76	70
Sep.	74	70	75
Oct.	67	61	63
Nov.	70	51	76
Dec.	58	82	74

The embedding dimension (d) is calculated using the fraction of false nearest neighbours (FNN) depicted in figure 1, representing the 12 months. Figure 1 shows the variations of optimal values of the embedding dimension (d) for the 12 months at which the fraction of nearest neighbours drops to zero. The embedding dimension (d) were found to be in the range 4-5, 4-6 and 4-5 for air temperature, relative humidity and dew point temperature, respectively. For air temperature and dew

point temperature values, their dynamical transformation appears to be governed by a lower number of variables compared to relative humidity values. These values are variables that are necessary to correctly display the attractors of these meteorological parameters dynamics in the phase space. These variables give useful information about the dimensionality and complexity of the underlying dynamical behaviour of a system by giving rise to the variability of the time series.

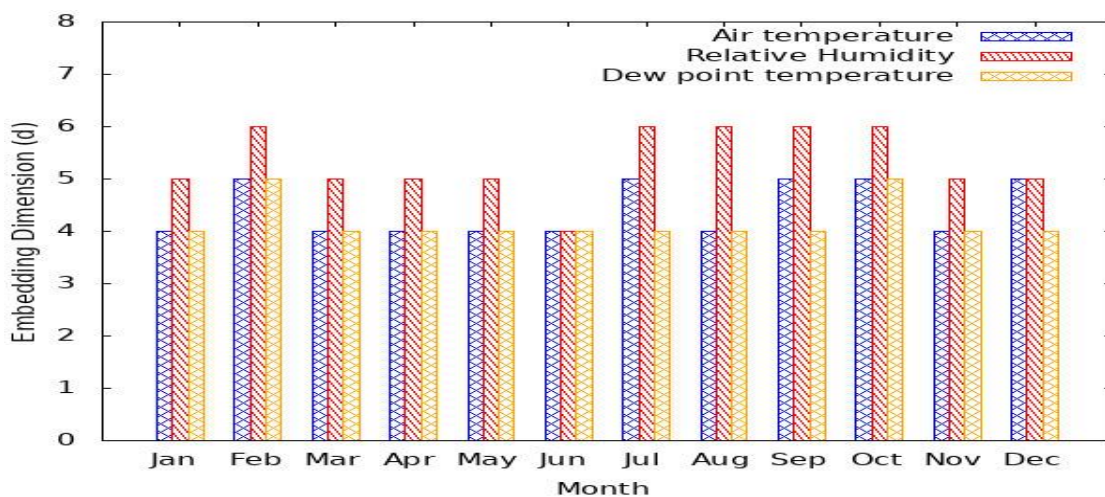


Figure 1: Variation of FNN for Air Temperature, Relative Humidity and Dew point temperature.

Table 2: Slope of stretching factor (maximum Lyapunov exponents)

Month	Air Temperature	Relative Humidity	Dew point temperature
Jan	0.5337	0.7721	0.8256
Feb	0.2956	0.1528	0.5763
Mar	-0.1484	0.484	0.2483
Apr	0.5798	-0.1912	0.5617
May	0.081	0.0668	0.1104
Jun	0.7599	0.3843	0.6498
Jul	0.7943	0.4121	0.7176
Aug	-0.2718	-0.1559	-0.3922
Sep	-0.4234	0.0432	-0.5403
Oct	0.2395	-0.0867	0.3366
Nov	-0.1048	0.1945	-0.1706
Dec	0.2108	0.1651	0.389

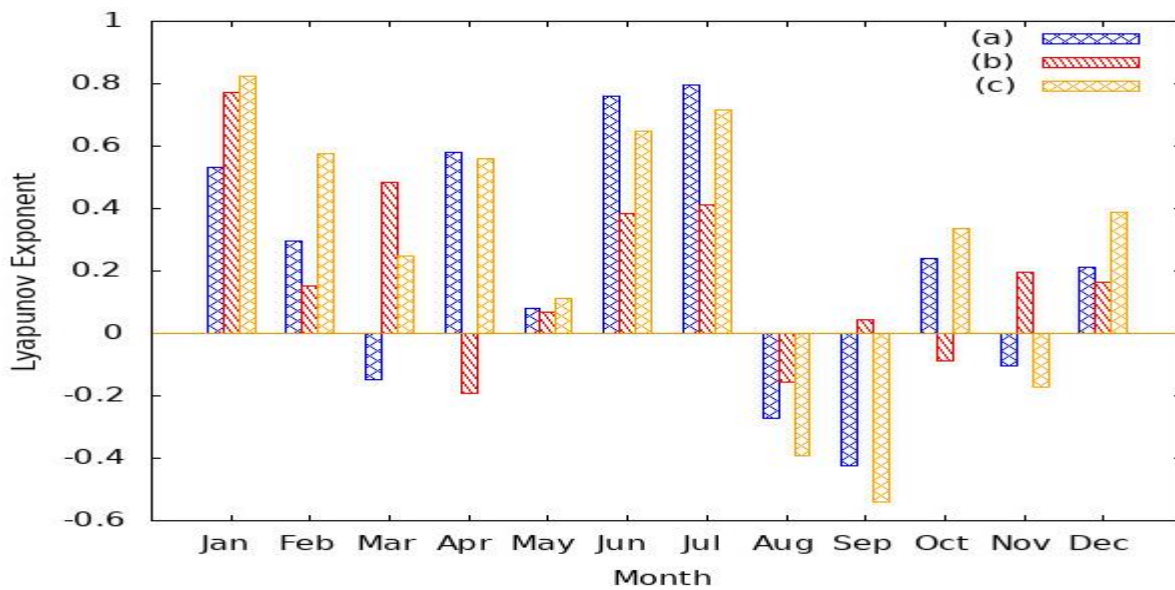


Figure 2: The global Lyapunov exponents for air temperature (a), relative humidity (b) and dew point temperature (c) time series.

The dynamic model needed to describe the data has 4-5 dimensions for air temperature, 4-6 for relative humidity and 4-5 for dew point temperature. This indicates the presence of more degrees of freedom in the system governing the relative humidity series than in that governing air temperature and dew point temperature series.

Table 2 shows the distribution of the negative and positive Lyapunov exponents of air temperature, relative humidity and dew point temperature for twelve months.

Figure 2 depicts the computed values of the Lyapunov exponents of air temperature, relative humidity and dew point temperature from January to December, 2012. The histogram (a) (Figure 2) shows the variation of maximum Lyapunov exponents of air temperature. The highest maximum Lyapunov exponents was observed in July, followed by June while the lowest was in May. Positive and negative Lyapunov exponents were observed in the air temperature time series. This indicates that there is transition for chaoticity (positive Lyapunov exponent) to periodicity (negative Lyapunov exponent) and thereafter to chaoticity (positive Lyapunov exponent). The chaotic regimes are evident in January, February, April, May, June, July, October and December, while periodic regimes are found in March, August and November.

Chaos (positive Lyapunov exponent) describes the behaviour of a system when its behaviour is rare (never completely repeated) (Abbaszadeh, *et al.*, 2020). The (approximate) period limit on accurate predictions of a chaotic system is a function of the largest Lyapunov exponent (Abarbanel, 1996): $\Delta t_{max} = \frac{1}{\lambda_{max}}$. Therefore,

the maximum length of prediction was observed in May with the value of 12 minutes ahead, followed by December value of 5 minutes, while the minimum value was in July where the region experiences maximum rainfall. However, the negative Lyapunov exponents indicate that the dynamical evolution gives property repeated at certain regular and periodic intervals, but without exact repetition (Sivakumar, 2017). As a result of this, high length of predictions are exhibited in the months of March, August and November.

The variation of Lyapunov exponents of relative humidity is displayed in the histogram (b) (Figure 2). The highest maximum Lyapunov exponents was observed in January, followed by March and while the lowest was in September. Positive and negative Lyapunov exponents were observed in the relative humidity time series. This indicates that there is transition from chaoticity (positive Lyapunov exponent) to periodicity (negative Lyapunov

exponent) and thereafter to chaoticity (positive Lyapunov exponent). The chaotic regimes are evident in the months of January, February, March, May, June, July, September, November and December, while periodic regimes are found in April, August and October. For the chaotic regime, the maximum length of prediction was observed in September with the value of 44 minutes ahead, followed by May which is 15 minutes ahead while March shows the minimum value of 2 minutes ahead. In periodic regime, high length of predictions are exhibited in April, August and October. The distribution of Lyapunov exponents of dew point temperature time series is displayed in the histogram (c) (Figure 2). The month of January has the highest maximum Lyapunov exponents, followed by July, while the lowest was in May. Positive and negative Lyapunov exponents were observed in the dew point temperature time series. This indicates that there exists periodicity inherent in the chaotic behaviour of dew point temperature time series causing transition from chaoticity (positive Lyapunov exponent) to periodicity (negative Lyapunov exponent) and thereafter to chaoticity (positive Lyapunov exponent). The chaotic regimes are observed in the months of January, February, March, April, May, June, July, October and December, while periodic regimes are found in August, September and November. For the chaotic regime, the maximum length of prediction was observed in May with the value of 9 minutes ahead, followed by March with the value of 4 minutes, while the minimum value was in January. A high length of predictions are exhibited in a periodic regime in the months of August, September and November. The transitions found in these meteorological parameters correspond to dynamical transitions caused by different changes in climate, which are confirmed by other studies using a nonlinear measure for transition detection (Malik, et al., 2012).

It is evident from the above discussion that the air temperature, relative humidity, and dew-point temperature fluctuations are affected by the local geographical topology and heavily influenced by the terrestrial climatic and atmospheric oscillations (Ray et al., 2019). The results, therefore, provide additional information about the climate transitions and for formulating weather prediction model.

CONCLUSION

Air temperature, relative humidity and dew point temperature data collected from Nsukka station by the Centre for Atmospheric Research (CAR), measured in 5 minutes time steps have been analysed using False Nearest Neighbour (FNN) and Lyapunov exponent methods. These meteorological time series contain 8928 data points from 1st January till 31st December, 2012. The Average mutual information was used to determine a nonlinear correlation time for the meteorological parameters. The dynamic model needed to describe the evolution of the observed data has 4 - 5 dimensions for air temperature, 4 - 6 for relative humidity and 4 - 5 for dew point temperature. The Lyapunov exponents were computed using average mutual information and false nearest neighbour to characterise the underlying dynamics of these meteorological parameters. Positive and negative Lyapunov exponents were observed in the air temperature, relative humidity and dew point temperature time series. This indicates that there exists

periodicity inherent in the chaotic behaviour of meteorological time series causing transition from chaoticity (positive Lyapunov exponent) to periodicity (negative Lyapunov exponent) and thereafter to chaoticity (positive Lyapunov exponent). The results, therefore, provide additional information about the climate transitions and also, for formulating a weather prediction model.

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