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ANALYSIS OF RETURN ON INVESTMENT IN SOME NIGER-DELTA OIL FIELD PROJECTS: USING THE VARIANCE-GAMMA PROCESS

CHISARA PEACE OGBOGBO

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ABSTRACT

Oil field development projects face market risks, largely because the parameter of key importance, the oil price, fluctuates rapidly over time. The decision to invest or not in an oil field project is therefore very challenging, because information concerning the field is often scarce. Neither the future production, nor sales prices are known with certainly. Constraints on production level also exist, in addition to OPEC quota, which is used in this work as proxy for other production limitations. The price process that best describes the fluctuations in oil price is expected to yield better analysis with respect to expected returns (viability of the project). The Variance Gamma process which is a subordinated Lévy process effectively captures jumps in a process. Thus, Variance Gamma process and a mean reverting process are considered in the analysis. The expected revenue was found to be higher, when the variance Gamma process is used as the model for oil price. This indicates that the variance Gamma process performs better than the Ornstein-Uhlenbeck process as a model for oil price. It therefore provides a good basis for price forecasting, and optimality of investment decision.

KEYWORDS: Variance Gamma Process, Mean Reverting Process, Parameters, Niger-Delta Oil Field Projects, Expected Return, OPEC constraint.

INTRODUCTION

This paper discusses findings that emerged while seeking an answer to a different question. The emphasis here is not exactly, on optimality of decision, but on analysis of revenue accruing (to an investor) with respect to the process describing fluctuations in oil price. Traditional tools such as net present value (NPV), discount factor (DCF), had been used generally in analysis involving expected returns on investment, to provide information about (viability of) a project. In these static models, calculations are done with fixed oil price premise. This can grossly over estimate projected gains or severely undermine a project's viability. The case study here is some Niger Delta fields. The oil price data is calibrated to obtain parameters to fit the process. A computational attempt is made at obtaining maximum return, using two different price processes: - a mean reverting process, and a subordinated Levy processes, the Variance Gamma, (VG) process. This is done in the face of Organization of petroleum exporting countries, OPEC constraints, which is a proxy for other production limitations. This is because production is not without an upper bound. Analysis is done with respect to various parameters of these processes.

Mean reverting processes are processes with the tendency to move away from their initial positions but return to a long -term mean (price) in the long run. The simplest mean reverting process is the Ornstein and Uhlenbeck (O-U) process. The process is stationary, Gaussian, and Markovian. The O-U process though a Gaussian process is different from the Weiner process by the drift term. If the current value of the process is less than the (long-term) mean, the drift will be positive; if the current value of the process is greater than the (long-term) mean, the drift will be negative. In other words, the mean acts as an equilibrium level for the process. The Weiner process has constant drift.

Mean reverting processes such as the O-U process are used in finance to model interest rates, currency exchange rates as well as commodities prices. This recommends the O-U model for the study of crude oil price, (which is characterized by fluctuations) to determine whether the process would capture the type of movement in crude oil price. Black Scholes Model is a Gaussian model used for pricing options. The main shortcoming of the Black Scholes model is the assumption that the volatility in the price of the underlying security is constant. This is responsible for the great incentive to produce other models which can describe the movement in crude oil price. The O-U process is therefore considered, to ascertain if mean reversion is a characteristic exhibited by the crude oil price series. On the other hand, the Variance gamma process is an infinite activity Lévy process, which can model small and frequent large jumps. As a subordinated Levy process, its ability to capture jumps informs its use in the analysis to investigate the presence or not of jumps in crude oil price series. The result of the analysis done in this work is expected to highlight the price process describing crude oil price.

The aim of this work is to determine which process leads

Chisara Peace Ogbogbo, Department of Mathematics University of Ghana, Legon. Accra, Ghana.

to better performance for a field project given constraint on production. Against this backdrop, we use a model which computes expected return on the project for each process, given OPEC production constraint. The price process that best describes the fluctuations in oil price is expected to yield better analysis with respect to expected returns (viability of the project). We compare results from analysis of returns on investment in oil field, using O-U process and VG process.

Data used is data for 13 crude types and fields from some Niger Delta oil field projects. The results are obtained computationally using PYTHON programming language. Parameter estimation for the O-U and VG processes are done using MLE. Codes for these computations are lengthy and can be obtained from author, through the given contact. The data was obtained from Department of Petroleum Resources (DPR) Victoria Island Lagos, and the Crude Oil Marketing Division (COMD). It was confirmed at National Petroleum Investment Management Services (NAPIMS) Ikoyi Lagos. For the period under study, January 2005 - December 2009, two sets of data were collected. Data on price of various crude types, and data on production for 13 fields. The data available are monthly data and not daily data as may be quoted on NYMEX and IPE. (Commodities and futures market). The prices are considered as spot prices and are used by the Federal Government of Nigeria to collect royalty from oil companies drilling on these fields.

First sieving of the comprehensive data procured had to be done, to match crude types with fields from where they are drilled. The Nigerian crude basket is made up of different crude types from various fields. Some price and production columns were blank probably due to intermittent crisis in the Niger Delta during part of the period. Care had to be taken to choose fields and crude types corresponding to prices, that had meaningful data.

The rest of the paper is presented as follows: The second section gives a brief review of work done in oil price modeling. Section 3 presents some important concepts with respect to the processes, calibration of data is also done here. Section 4 presents the model and analysis of results obtained. Conclusion is drawn.

2. Brief Review of Work on Oil Price Modeling and Useful Information about OPEC Quota

Crude oil price futures have been studied by Krichene (2006), (2008). He modeled oil futures price returns as a Lévy process, in particular as a Variance Gamma process. He assumed that oil futures price returns, followed a Lévy process with a variance-gamma (VG) distribution. He defined log price return as $x_t = \Delta \log S_t = \mu + X_t - X_{t-1}$

Where X_t ,is a variance Gamma process, and crude oil price S_t is modeled as

$$S_t = S_0 \exp\left[\mu t + X_t\right]$$

He used data on daily oil futures prices January 2, 2002-July 7, 2006, to estimate the parameters of the VG process using Empirical characteristic function, ECF. He obtained the following parameters Drift μ, Skewness α, Volatility σ, Variance of VG, ν. Going by his finding, crude oil price exhibited a high drift coefficient (μ), also exhibited high volatility (σ), frequent and large jumps (determined by ν) and skewness (controlled by α).

OPEC sets up oil production quota to pursue stability and harmony in the petroleum market, for the benefit of both oil producers and consumers. Production regulation is simply one possible response of OPEC to market conditions: if demand grows, or some producers are producing less oil, OPEC can increase its oil production in order to prevent a sudden rise in prices. It might also reduce its oil production, in response to market conditions.

Table 1: Nigeria: OPEC Quota

2005	2006	2007	2008	2009
2.4 mbpd	3.30 mbpd	2.2 mbpd	1.3 mbpd	1.67 mbpd

Source: www.oilandenergytrends.com/ger/ger_opec oil and energy trends: A monthly publication of International Energy Statistics and Analysis.

3 Preliminary Concepts

3.1 Mean Reverting process

Mean reversion is the tendency for the Brownian motion to move away from its initial position but return to a mean (price) in the long run or after some time. The simplest mean reverting process is the Ornstein-Uhlenbeck process given in equation (1) below

$$dP_t = \eta(P_t - a)dt + \sigma dW_t \qquad (1)$$

Where η is speed of reversion. P_t is oil price, t is time, σ is variance parameter, a is level the price tends to revert to (the mean), dW_t is increment of a Wiener process.

Note: Smaller η implies that price drifts away from its mean, (η is non-negative). Also $\eta = 1$ implies oil price reverts surely to the long run mean α. This process is driven by a Brownian motion. The main parameters of the O-U process are the mean α and the volatility σ.

To obtain the price process, P_t the stochastic differential equation in (1) is solved using Itô's lemma and for given initial conditions. i.e.

$$dP_t = \eta(P_t - a)dt + \sigma dW_t P_0 = P(0) \qquad (2)$$

to obtain

$$P_t = P_0 e^t + a(1 - e^t) + \int_0^t e^{t-s}\, \sigma dW_s \qquad (3)$$

Due to the autoregressive quality, increments or jumps are not independent. The farther away the process has moved from the mean, the stronger the tendency for a movement towards the mean. Thus, the speed of reversion is proportional to the distance between the current position and the equilibrium level. So, the variance grows at first but then stabilizes.

3.2 The Variance Gamma (VG) process

A variance-gamma (VG) process is defined as a Brownian motion, BM with drift $\mu$and volatility σ i.e. $\mu t + \sigma B_t$, where $B_t$is an ordinary Brownian motion, time-changed by a gamma process.

Let $G = \{G_t, t \geq 0\}$be a gamma process with mean $\alpha > 0$ and variance $b > 0$.

Let $B = \{B_t, t \geq 0\}$, denote a Brownian motion, and let $\sigma > 0$ and $\mu \in \mathbb{R}$; then the VG process,

$X^{(VG)} = \left\{X_t^{(VG)}, t \geq 0\right\}$ $\sigma > 0, v > 0$ and μ can be defined as

$$X_t^{(VG)} = \mu G_t + \sigma B_{G_t}$$

Thus, the VG process is obtained on evaluating Brownian motion at a time given by the gamma process. It is also called the Laplace motion, and is a pure jump process. When Brownian motion is time changed by a Gamma process, the Variance Gamma (VG) process is obtained. The VG process has three parameters, namely volatility of the BM, σ, variance of the gamma time change, ν, and drift in the Brownian motion, μ. More generally, for VG process defined as

$X_x^{VG} = \theta G_t + \sigma B_{G_t}$

There are four parameters: - drift of BM, μ ,volatility of BM, σ, volatility of time change, ν, (variance of VG) and drift of VG, θ.

3.3 Calibration of data for the parameters of the oil process

Calibration of data was done using maximum likelihood estimation process, MLE. Two processes considered are the mean reverting, Ornstein-uhlenbeck (O-U) , process and a subordinated Lévy process, Variance Gamma (VG). Parameter values are obtained for each of the two processes using R statistical package.

Table 2: OU PARAMETERS

TYPE	Mean α	Vol σ
BL	71.49338901	21.23606388
FB	70.38885929	23.58284355
QL	53.4152558	32.52533274
BB	71.82665219	21.89534891
ESC	70.31012184	23.69099342
PL	72.20670171	22.07915967
ANTAN	69.44279201	22.05442617
YOHO	68.68628743	25.12259855
AME	68.56037041	25.15704196
UKPO	60.04738667	35.10172905
ABO	60.83765706	28.97048242
IME	59.66053667	35.01734336
OKONO	57.24444831	27.25513794

Table 3: Parameters of the VG-model

Crude Type	Location parameter μ	Speed Parameter θ	The asymmetry parameter σ	The Shape Parameter ν
BL	9.0076	3.1837	56.5214	0.1374
FB	71.320	49..178	-34.817	1.833
QL	5.2540	8.8676	64.2039	0.1184
BB	181.5583	7.0123	-132.0270	0.1184
ESC	121.4919	22.5143	-53.7653	0.1184
PL	-22.8995	4.6734	93.8197	0.1184
ANTAN	179.0202	9.0396	-133.0744	0.1184
YOHO	44.2191	10.9447	27.8540	0.4389
AME	139.4743	23.7300	-133.8352	0.4821
UKPO	73.340	53.023	-35.221	2.288
ABO	-42.4688	5.1721	102.5921	0.1189
IME	59.640	19.445	10.007	1.448
OKONO	-64.0324	5.7250	116.6728	0.1184

3.3.1 On the parameters

Drift of the Brownian Motion, θ, accounts for the skewness. The variance of time change, ν, accounts for jumps in the process. It is known as the shape parameter because it relates to kurtosis or peakedness of the process. The spread parameter (which accounts for volatility) are generally single digit for the VG. Lower values of skewness parameter implies there are more of higher prices (than the average). High values of skewness parameter imply there are more of lower prices (than average). There are some crude types whose distributions are left skewed. The others are right skewed. The first three crude types with the highest values of (left) skewness eventually yielded highest V_t values.

3.3.2 Comparing Parameters for Return Series and Price Series

Parameters obtained for price data here, are far higher than values obtained in krichene's work. In this work actual prices are used because actual price P_t is used in the model formulation. Most financial studies involve returns instead of price for two reasons. For the average investor return on an asset is a complete and scale free summary of investment opportunity. Return series are easier to handle than price series because the former have more attractive statistical properties. Despite the above reasons for preference for returns, prices are used, because there are several varying definitions of asset return. Moreover, an investor in a field project may be more concerned about actual price of crude oil.

Remark:
Lévy processes are preferred for modeling crude oil price largely to make up for limitations of the Black-Scholes model. They are more versatile than the Gaussian driven processes. Lévy processes are proposed to make up for the short-fall of the Black-Scholes model, by capturing the notion of stochastic volatility and to model small and frequent jumps.

3.4 Empirical Crude oil spot price data as a Lévy process

To show that a price series is a Lévy process, basically it is shown that the price increments (difference series) are stationary and independent. The augmented Dickey-Fuller (ADF) test is a test for stationarity. The Kolmogorov -Smirnov normality test together with Durbin-Watson autocorrelation test establish independence. The crude oil spot price process has been shown to be Lévy process. [6] Crude oil spot price data for some Niger Delta fields was used.

4 Model and Analysis of Results

4.1 The model

It is assumed in the model that trading is done only in the spot market, and the aggregate OPEC quota as used in the model is interpreted to filter down in the micro sense to each field. Also, the expected return on the investment is assumed to depend largely on production and price. The expected value problem is given below:
Let $V_t(P_t, n_t, r_t)$ be the expected return on the project.

$$V_t(P_t, n_t, r_t) = E\left[\int_T^{\tau} P_t e^{r(\tau - T)} n_t dt\right]$$

We want the maximum $V_t(P_t, n_t, r_t)$ subject to constraints on n_t
Thus,

$$V_t = \max_{n_t, P_t} E\left[\int_T^{\tau} P_t e^{r(\tau-T)} n_t dt\right]$$

$$n_t \leq n_{max}$$

Equation (4) defines the expected return on the investment. Suppose initial time is
$T = 0$, then,

$$V_t = \max_{n_t, P_t} E\left[\int_0^{\tau} P_t e^{r\tau} n_t dt\right]$$

P_t is oil price at time t, and is modeled as a stochastic process. $r$is interest rate, n_t is number of barrels produced and sold at time t which is regulated. (A barrel of oil is 42 gallons or 159 litres). τ is the break through time. T is time when production started. We set $T = 0$. $e^{r\tau}$ is the discount factor.

Given information on price P_t, $n_t$is a bounded quantity and is related to price P_t

$$n_t = \prod(P_t)$$

n_t is measurable with respect toP_t, n_t is adapted to P_t, $n_t$is considered a filtration generated by P_t. The relationship between n_t and $P_t$is defined by

$$n_t = \frac{CP_t^{\alpha}}{P_t^{\alpha}+1}, \quad C = n_{max} \tag{5}$$

Equation (5) defines the relationship between $n_t$and $P_t$with respect to OPEC quota. C is maximum quantity that can be produced. α is the bound regulator which controls the speed at which the upper bound is attained. The actual bound on n_t i.e. $n_t \leq n_{max}$ is determined by some technical constraints along with the OPEC regulatory constraints. We assume optimality for other factors.

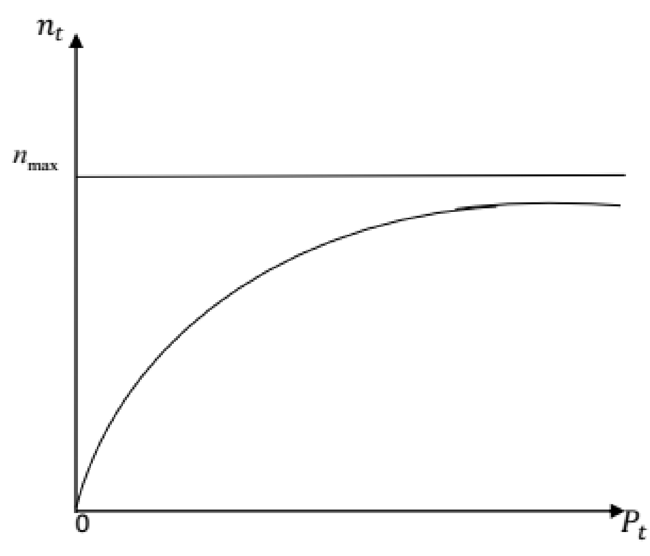

Figure 1: Illustration of Bound on quantity produced

Table 4: Lending I-rate and Deposit I- rate Nigeria 2005-2009

Year	2005	2006	2007	2008	2009
Deposit Rate	10.53	9.74	10.29	11.29	13.3
Lending Rate	17.95	16.9	16.94	15.48	18.36

Source: Lending I-rate and Deposit I-rate Nigeria 2005-2009 http://www.tradingeconomics.com/nigeria/lending-rate-precent-wb-data.html and CBN Bulletins, 2006, 2007, 2008, 2009. Average of deposit rate and lending rate is used in the model.

4.2 The Computational program

The objective: is to obtain optimal expected return on investment for each price process, O-U and VG. The price processes are generated using their various parameters as displayed on Tables [2] [3]. Computations for optimal expected return, (on investment) were done using a program written with PYTHON. The start position for the simulation is price for each crude type as at January 2005. An alpha value of $0.5$was selected for the program because the system achieved equilibrium better for alpha equal to 0.5. (from initial simulation results). For $n_{max} = 5$ million, a time period of 60 months the following was done. Each time, the process $P_t$is generated for each process, using the estimated parameters on Tables [2] [3].The OPEC constraint was taken into consideration, i.e. for each set of values generated, $n_t = \frac{CP_t^\alpha}{P_t^\alpha+1}$ was obtained, n_t, was applied on each P_t generated process. Several integral values for these products were obtained. The mean of these set of integral values, is the expected value V_t.

4.3 Expected Returns V_t for each process

The values of $V_t$obtained from the program for each process for $\alpha = 0.5$ are displayed on the following tables. [5] and [6]

FIELD	CRUDE TYPE	V_t(USD)
Inda	BL	23,437,045.15
Ughelli East	FB	24,602,995.74
USari	QL	22,853,370.99
Jisike	BB	23,213,462.11
Meren	ESC	22,686,595.70
Funiwa	PL	24,430,838.24
Noron	Antan	24,682,512.15
Yoho	Yoho	25,188,623.38
Amenam	Ame	23,432,838.51
Ukpokiti	Ukpo	25,188,179.40
Abo	Abo	23,252,820.80
Ngo	Ima	25,242,415.60
Okono/Okpoho	Okono	21,809,242.31

Table 5: V_tVALUES FOR O-U PROCESS $\alpha = 0.5$
Lowest V_tValue $= 21,809,242.31$
Highest V_tValue $= 25242,415.60$(Ima)
Lowest V_tValue $= 21,809,242.31$(Okono)

Table 6: V_tVALUES FOR VG PROCESS $\alpha = 0.5$

FIELD	CRUDE TYPE	V_t (USD)
Inda	BL	570,823,546.47
Ughelli East	FB	193,334,997.71
Usari	QL	690,651,072.43
Jisike	BB	1,416,029,327.64
Meren	ESC	591,572,829.02
Funiwa	PL	964,004,252.92
Noron	Antan	1,423,038,808.46
Yoho	Yoho	295,404,198.65
Amenam	Ame	1,506,0999,994.00
Ukpokiti	Ukpo	328,576,402.93
Abo	Abo	1,027,559,313.32
Ngo	Ima	160,498,726.65
Okono/Okpoho	Okono	1,221,572,908.25

4.4 Analysis

Highest V_t value $= 1,506,099,994.00$ (Ame): Lowest V_t value $= 160,498,726.65$ (Ima)

4.4 Analysis of Results

4.4.1 Ornstein-Uhlenbeck Process

The O-U process has only two parameters, Mean and Volatility. There is not much difference in volatility values for various crude types. This accounts for the small difference in $V_t$values for various fields. The O-U process yields less values of V_t, when compared with VG processes. Volatility is the only parameter accounted for in the price movement. Not very good results are expected with the O-U process. The O-U process usually generates both positive and negative values (oil prices) over time. This can be seen on graph in Fig 2. Therefore, considering the price process as a mean reverting (O-U) process will lead to Sub-optimal results on investments and incorrect calculation of expected returns.

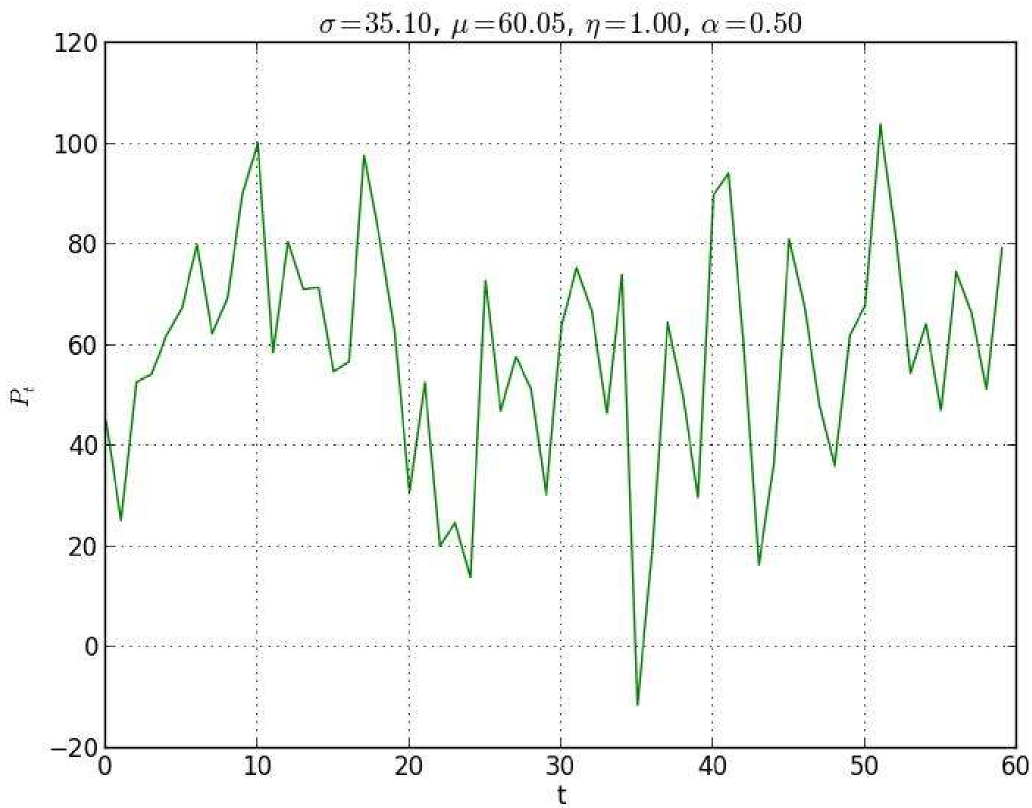

Figure 2: UKPO: O-U PRICE-TIME GRAPH

4.4.2 The Variance Gamma Process

V_t values, for the VG process are generally higher than $V_t$values for the O-U process. For the VG process, comparatively low values of volatility, together with high drift (of BM) values, yielded higher V_t values. The trend is broken only when volatility and drift (of BM) are very low and very high respectively. Fields with higher vvalues yielded less V_t,values. Crude types with most skewed (left) price distribution yielded the highest $V_t$value. This is in consonance with krichene's work. Findings from his work , shows that VG process is left skewed.

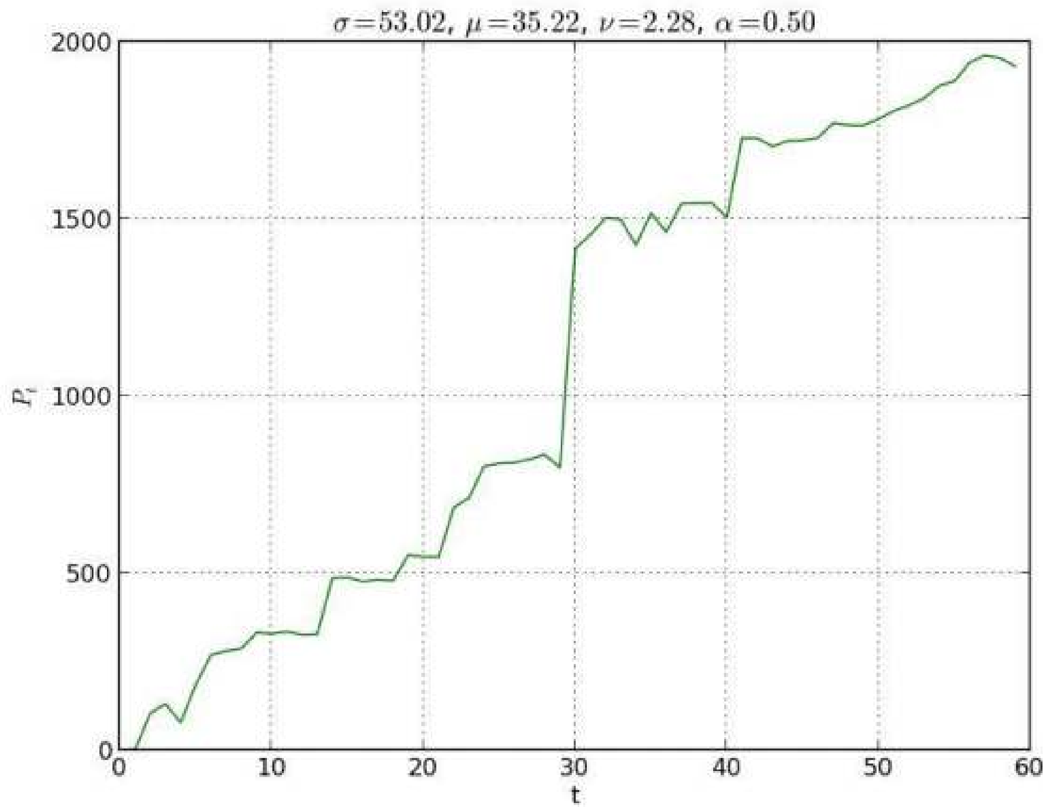

Figure 3: UKPO: VG PRICE-TIME GRAPH

CONCLUSION

From the model (which includes production constraints), expected revenue are higher for the subordinated Levy process than for the Ornstein Uhlenbeck process. This is an indication of better performance for the Variance Gamma process as the process describing the crude oil price. The contribution of drift and volatility parameter values in the analysis is important in terms of explaining fluctuations and jumps in the price process. Despite the fact that gaps in the data may be responsible for the type of parameters obtained for some crude types. The VG process is considered a better price model for the data. The missing data could be accounted for by crises that were prevalent in the Niger Delta region during the period under study.

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REMARK:

Appendix includes:- CRUDE OIL PRICES 2005 - 2009 , R- CODE FOR PARAMETER ESTIMATION FOR VG AND O-U PROCESSES, PYHON CODE IMPLEMENTED FOR VG PROCESS,AND PYTHON CODE IMPLEMENTED FOR O-U PROCESS.

Author can be contacted for data and codes.

REFERENCES

Adelman M.A., "Crude oil supply curve". International Association for Energy Economics Vol 7, issue 4-October 1986. (Revision of MIT Laboratory Working paper). *www.iaee.org/documents/vol-7(4)*

Coding Algorithm for Stochastic Processes using PYTHON.pyprocess.70percentfatfree.com

Daniel T. Gillespie “Exact numerical simulation of the Ornstein-Uhlenbeck process and its integral" Phys. Rev. E 54, 2084 Published 1 August 1996.

David Scott and Christine Yan Dong, "Package Variance Gamma version, 0.3 1" Http//www.r-project.org. April 13. 2012.

Krichene Noureddine, “Recent Dynamics of Crude Oil Prices” IMF Working Paper. WP/06/299. http://wwww.imf.org. 2006

Noureddine Krichene. Crude Oil Prices: Trends and Forecasts. IMF working paper. WP/08/133. *https://www.imf.org/external/pubs/ft/wp/2008/wp08133. 2008*

Ogbogbo Chisara P. “The crude oil spot price process is a Lévy process" Journal of the Nigerian

Association of Mathematical Physics. Vol 42. NO 1. 2017

Roger J-B. Wells and Ignacio Rios," Modeling and Estimating Commodity Prices: Copper Prices. *https://www.math.ucdavis.edu/rjb-* /mypage/Mathematics. 2012.

Ruey S. Tsay" Analysis of Financial Time Series" 2nd Edition. Wiley and Sons Inc. 2005

Schwartz E. S. "The Stochastic behavior of commodity Prices: Implications for valuation and Hedging". The Journal of Finance 52 (3) pp. 923-973. 1997.

William Smith. "On the Simulation and Estimation of the Mean-Reverting Ornstein Uhlenbeck Process: Especially as Applied to Commodities Markets and Modeling." Commodity Models, 2010. *http://commodity* models.les. word press. com/2010/02/estimating-the- parameters-of-a-mean-reverting-ornstein-uhlenbeck-process1.pdf. Checked December 2016.