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HEAT TRANSFER IN MAGNETOHYDRODYNAMIC (MHD) COUETTE FLOW OF A TWO-COMPONENT PLASMA WITH VARIABLE WALL TEMPERATURE

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ABSTRACT

In this paper we study the field structure and the heat transfer at the walls of two-component plasma. The flow is induced by two horizontal walls moving relative to each other along their common axis in the presence of a uniformly applied transverse magnetic field and the analysis made under the following assumptions: (i) The flow is viscous and incompressible (ii) The flow is fully developed. (iii) The temperature varies linearly along the wall. (iv) The temperature difference between the walls is not large enough to cause free convection current to flow. Exact solutions for the velocities and temperatures for the ionized and neutral particles and the induced magnetic field are derived. These together with the heat transfer are discussed quantitatively

KEYWORDS: Couette flow, two-component plasma, field structure, heat transfer, Hartmann number, Magneto hydrodynamic, Nusselt number

INTRODUCTION

Studies of MHD flow of electrically conducting viscous fluids within boundaries have important practical applications in the design of MHD generators, cross field accelerators, shock tubes and pumps. Chang and Yen (1962) analyzed the effect of conducting walls on such flows and found that the viscous drag and the mass flow rate decrease while the magnetic drag increase with the sum of wall conductance ratio. An extension to this was done by Rayleigh and Snyder who considered heat transfer. One of the initiators of MHD couette flow studies was Lehnert (1952) and later Yen and Chang (1964) studied the effect of wall electrical conductance and came up with the result that there was distortion of the velocity profile. Recently the problem of heat transfer in MHD couette flow as affected by wall electrical conductance with temperature varying linearly along the walls and in the presence of a constant heat source was studied by Seshagiri Rao (1979). In all these studies as is usual in classical magneto-hydrodynamics and plasma physics, fully ionized plasma is assumed. This simplifies most of the difficult problems but deviates from real life situations. The more realistic situation is the partially ionized plasma with attendant neutral-ionized particles interaction. Such studies have been undertaken by Chhjlani and Vaghela (1987) who studied the gravitational stability of magnetized self gravitating two-component plasma of finite conductivity in a porous medium. Also Bestman (1989) studied the thermal stability of radiating two-component plasma in a porous medium. Warmate and Bestman (1996) also studied the transient flow of fully developed two-component plasma in a magnetized cylinder.

In this paper we intend to extend the work of Seshagiri Rao (1979) to two-component plasma by studying the field structure and then the heat transfer.

The following procedure has been adopted, first the formulation of the problem is done with derivation of the resulting governing equations and solutions for the velocity and induced magnetic field. Using these solutions the expression for temperatures are obtained. Finally the results are presented with a quantitative discussion of the field structure and heat transfer at the walls for the ionized and neutral species of the plasma.

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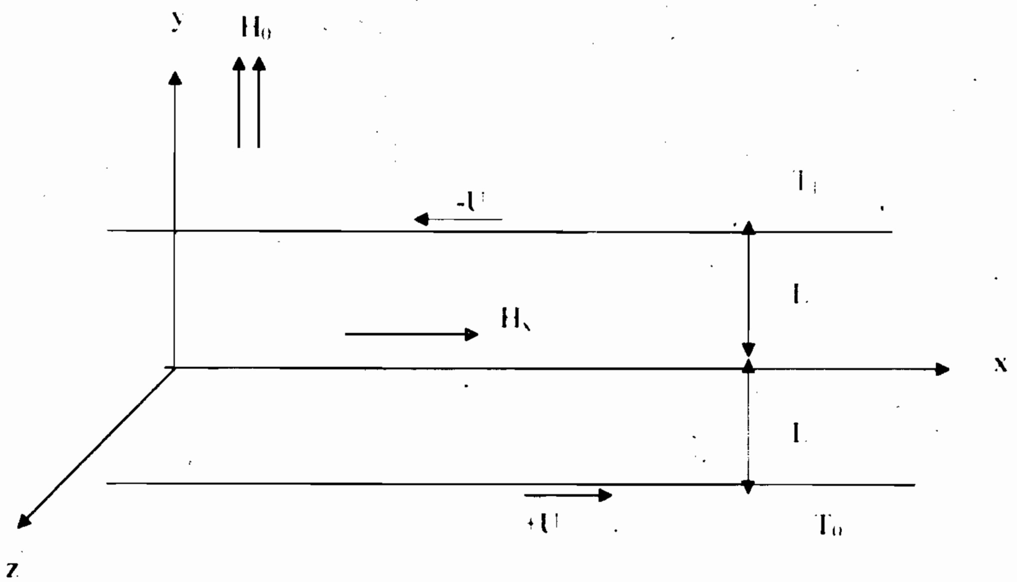

Fig. 1: Coordinate system of parallel plate couette flow

FORMULATION

We consider the flow of two-component plasma (ionized and neutral species) with the ionized specie electrically conducting The flow is viscous, incompressible and bounded by two horizontal parallel plates moving relative to each other along their common axis with velocities $\mp U_0$ The plates are maintained at temperatures T_1 and T_0 ($T_1 > T_0$) and we employ a cartesian coordinate system (x, y', z') with one plate situated at y = L while the other is at y = -L

Along the x and z directions they are infinite and a uniform magnetic field is applied along y which is in the vertical upward direction.

Assuming a steady and fully developed flow along x, all the physical variables will depend on y except the temperature Such a flow is governed by the following equations

Continuity

$$\nabla \bullet \vec{V}' = 0 \qquad (1)$$

Modified Navier- Stokes equation

$$\rho_{i,n}(\vec{V}'_{i,n} \bullet \nabla)\vec{V}'_{i,n} = \mu_{i,n}\nabla^2 \vec{V}'_{i,n} + \mu_m \vec{J}_{i,n} \times \vec{H}' + F(\pm \vec{V}'_{i,n}) \qquad (2)$$

Energy equation

$$\rho_{i,n} C_p (\vec{V}'_{i,n} \bullet \nabla) T_{i,n} = k_{i,n}\nabla^2 T_{i,n} + \mu_{i,n}\left(\frac{\partial U'_{i,n}}{\partial y}\right)^2 + \frac{J_{i,n}^2}{\sigma} + Q_{i,n} \qquad (3)$$

where $J_n = 0$, the subscripts and later superscripts i, n are ionized and neutral components respectively. $\frac{J^2}{\sigma}$ is the Ohmic dissipation. $F(\pm \vec{V}'_{i,n})$ is the collision frictional force term.

The electromagnetic field equations take the form:

$$\nabla \times \vec{H}' = \vec{J} \qquad (4)$$

$$\nabla \bullet \vec{B}' = 0, \quad \nabla \bullet \vec{D} = 0 \qquad (5)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}'}{\partial t} = 0 \qquad (6)$$

$$\vec{B}' = \mu_m \vec{H}', \quad \vec{D} = \varepsilon \vec{E} \qquad (7)$$

and the modified Ohms law $\vec{J} = \sigma\{\vec{E} + \mu_m[\vec{V}' \times \vec{H}']\}$ (8)

The equation of interaction between flow velocity and magnetic field is obtained from Eqs. (4), (6) and (8) as

$$\nabla \times \left(\nabla \times \vec{H}'\right) = \mu_m \sigma \nabla \times \left(\vec{V}' \times \vec{H}'\right) \tag{9}$$

From Eqs. (5) and (7) we get H_y = constant and this we write as H_0, the applied magnetic field.

Consequently we set

$$\vec{V}_{i,n}' = U'\left(y'\right)\hat{i} \ , \quad \vec{H}' = H_x'\left(y'\right)\hat{i} + H_0\hat{j} \tag{10}$$

where H_x is the induced magnetic field and $\hat{i}$, $\hat{j}$ are respectively unit vectors along x and y directions. It can be deduced that all variables are functions of y only, except T which has an additional dependence on x. Splitting Eqs. (2), (3) and (9) into ionized particle and neutral specie equations and writing them in component form gives the dimensional equations:

$$\mu_m H_0 \frac{dH_x'}{dy'} + \mu_i \frac{d^2U_i'}{dy'^2} + f_i\rho_i\beta\left(U_n' - U_i'\right) = 0 \tag{11}$$

$$\mu_n \frac{d^2U_n'}{dy'^2} - f_i\rho_i\beta\left(\cdots\right) \tag{12}$$

$$\frac{\cdots}{dy'^2} + \mu_m \sigma H_0 \frac{dU_i'}{dy'} = 0 \tag{13}$$

$$\rho_i C_p U_i' \frac{\partial T_i'}{\partial x'} = k_i\left(\frac{\partial^2 T_i'}{\partial x'^2} + \frac{\partial^2 T_i'}{\partial y'^2}\right) + \mu_i\left(\frac{dU_i'}{dy'}\right)^2 + \frac{1}{\sigma}\left(\frac{dH_x'}{dy'}\right)^2 + Q \tag{14}$$

$$\rho_n C_p U_n' \frac{\partial T_n'}{\partial x'} = k_n\left(\frac{\partial^2 T_n'}{\partial x'^2} + \frac{\partial^2 T_n'}{\partial y'^2}\right) + \mu_n\left(\frac{dU_n'}{dy'}\right)^2 + Q \tag{15}$$

where the collision frictional force which is the coupling factor between the ionized and neutral species is defined as $\vec{F}\left(\vec{V}'\right) = f_i\rho_i\beta\left(\vec{V}_n' - \vec{V}_i'\right)$ (Chhajlani and Vaghela, 1987)

To facilitate analysis it is expedient to introduce the non-dimensional variables:

$$x, y = \frac{x', y'}{L} \ ; \quad U_{i,n} = \frac{U_{i,n}'}{U_0} \ ; \quad H_x = \frac{H_x'}{H_0}$$

Substituting this into Eqs. (11), (12) and (13) we get

$$\frac{d^2U_i}{dy^2} + R_M N \frac{dH_x}{dy} + \sigma_i^2 \beta\left(U_n - U_i\right) = 0 \tag{16}$$

$$\frac{d^2U_n}{dy^2} - \frac{\sigma_n^2}{\beta}\left(U_n - U_i\right) = 0 \tag{17}$$

$$\frac{d^2H_x}{dy^2} + M_A R_M \frac{dU_i}{dy} = 0 \tag{18}$$

where we have introduced the following parameters defined as follows:

$$R_M = \frac{A_0 L}{\nu_i} \ , \quad A_0 = \sqrt{\left(\frac{\mu_m H_0^2}{\rho_i}\right)} \ , \quad \nu_i = \frac{\mu_i}{\rho_i} \ , \quad N = \frac{A_0}{U_0} \ , \quad \sigma_{i,n}^2 = \frac{f_{i,n}L^2}{\nu_{i,n}} \ , \quad M_A = \frac{\nu_i}{\lambda}$$

$$\lambda = \frac{1}{\mu_m \sigma}$$

The three equations (16), (17) and (18) are subject to the following boundary conditions:

(i) Hydrodynamic $\qquad U_{i,n} = \mp 1$ on $y = \pm 1$

(ii) Magneto-hydrodynamic (Seshagiri Rao, 1979)

$$\frac{dH_x}{dy} + \frac{1}{\phi_u} H_x = 0 \text{ on } y=1$$

$$\frac{dH_x}{dy} - \frac{1}{\phi_l} H_x = 0 \text{ on } y=-1$$

subscripts u and l stand for upper and lower plates and $\phi_{u,l} = \frac{\sigma_{u,l}}{\sigma}$

We now solve the equations simultaneously and after integration, substitution and reconciliation of the constants we get the expressions

$$U_l = A_1 Cosh(m_1 y) + A_2 Sinh(m_1 y) + A_3 Cosh(m_2 y) + A_4 Sinh(m_2 y) + A_5 \quad (19)$$

$$U_u = B_1 A_1 Cosh(m_1 y) + B_1 A_2 Sinh(m_1 y) + B_2 A_3 Cosh(m_2 y) + B_2 A_4 Sinh(m_2 y) + A_5 \quad (20)$$

$$H_x = -M_A R_M \left(\frac{A_1 Sinh(m_1 y)}{m_1} + \frac{A_2 Cosh(m_1 y)}{m_1} + \frac{A_3 Sinh(m_2 y)}{m_2} + \frac{A_4 Cosh(m_2 y)}{m_2} \right) + A_6 \quad (21)$$

$A_1 : A_2 : A_3 ; A_4 : A_5 : A_6$ are arbitrary constants which are determined from the boundary conditions as a 6X6 determinant.

FLUID TEMPERATURE UNDER VARIABLE WALL TEMPERATURE

We assume that the temperature varies linearly along the wall with a constant heat source We therefore set

$$T(x, y) = T(y) + x\Gamma \quad (22)$$

and write

$$T_{l,u}(x, -L) = T_{l,u}(-L) + x\Gamma = T_0 \quad (23)$$

$$T_u(x, L) = T_u(L) + x\Gamma = T_1 \quad (24)$$

$T_1 > T_0$

Differentiating Eq (22) twice and substituting in Eq (14) and (15) we get

$$\rho_l C_p U_l \Gamma = k_l \frac{d^2 \bar{T}_l}{dy^2} + \mu_l \left(\frac{dU_l}{dy}\right)^2 + \frac{1}{\sigma}\left(\frac{dH_x}{dy}\right)^2 + Q \quad (25)$$

$$\rho_u C_p U_u \Gamma = k_u \frac{d^2 \bar{T}_u}{dy^2} + \mu_u \left(\frac{dU_u}{dy}\right)^2 + Q \quad (26)$$

In addition to the non-dimensional variables we include these two

$$\theta(y) = \frac{T_{l,u} - T_0}{T_1 - T_0} \qquad \Gamma = \frac{\Gamma L}{T_1 - T_0}$$

Eqs. (25) and (26) yield the following dimensionless equations after substituting the above dimensionless variables

$$\frac{d^2\theta_l}{dy^2} + \beta_1^l \left(\frac{dU_l}{dy}\right)^2 + \beta_2^l \left(\frac{dH_x}{dy}\right)^2 + \beta_3^l - \beta_4^l U_l = 0 \quad (27)$$

$$\frac{d^2\theta_u}{dy^2} + \beta_1^u \left(\frac{dU_u}{dy}\right)^2 + \beta_3^u - \beta_4^u U_u = 0 \quad (28)$$

with the additional parameters defined as follows

$$\beta_1^{l,u} = \frac{\mu_{l,u} U_0^2}{k_{l,u}(T_1 - T_0)} = Pr^{l,u} Ec^{l,u}, \qquad \beta_2^l = \frac{H_0^2}{\sigma k_l (T_1 - T_0)} = \frac{R_H Pr_l Ec}{M}$$

$$\beta_3^{l,u} = \frac{QL^2}{k_{l,u}(T_1 - T_0)} \qquad \beta_4^{l,u} = \frac{\rho_{l,u} C_p U_0 L\Gamma}{k_{l,u}} = Pr \; Re \; \Gamma$$

where $Pr^{l,u} = \frac{\mu_{l,u} C_p}{k_{l,u}}$; $Ec^{l,u} = \frac{U_0^2}{C_p (T_1 - T_0)}$; $R_H = \frac{\mu_m H}{\rho_l U_0}$; $M = HL\sqrt{\frac{\mu_m \sigma}{\nu}}$

Substituting for $\left(\frac{dU_t}{dy}\right)^2$ and $\left(\frac{dH_x}{dy}\right)^2$ in Eq. (27) and integrating twice we get an equation for θ_t with two arbitrary constants subject to the boundary conditions:

$$\theta_t = 0 \text{ on } y = -1$$
$$\theta_t = 1 \text{ on } y = 1$$

This gives the final expression for θ_t as

$$\theta_t = \frac{1}{2}(1+y) + \beta_4' \begin{bmatrix} \frac{A_1}{m_1^2}\{Cosh(m_1 y) - Cosh(m_1)\} + \frac{A_2}{m_1^2}\{Sinh(m_1 y) - ySinh(m_1)\} + \\ \frac{A_3}{m_2^2}\{Cosh(m_2 y) - Cosh(m_2)\} + \frac{A_4}{m_2^2}\{Sinh(m_2 y) - ySinh(m_2)\} + \frac{A_5}{2}(y^2 - 1) \end{bmatrix} -$$

$$-\frac{\beta_3'}{2}(y^2-1) -$$

$$(\beta_2' M_4 M^2) \begin{bmatrix} \frac{A_1^2}{2}\left\{\frac{Cosh(2m_1 y)}{4m_1^2} + \frac{y^2}{2} - \frac{Cosh(2m_1)}{4m_1^2} - \frac{1}{2}\right\} + \frac{A_2^2}{2}\left\{\frac{Cosh(2m_1 y)}{4m_1^2} - \frac{y^2}{2} - \frac{Cosh(2m_1)}{4m_1^2} + \frac{1}{2}\right\} + \\ \frac{A_3^2}{2}\left\{\frac{Cosh(2m_2 y)}{4m_2^2} + \frac{y^2}{2} - \frac{Cosh(2m_2)}{4m_2^2} - \frac{1}{2}\right\} + \frac{A_4^2}{2}\left\{\frac{Cosh(2m_2 y)}{4m_2^2} - \frac{y^2}{2} - \frac{Cosh(2m_2)}{4m_2^2} + \frac{1}{2}\right\} \end{bmatrix}$$

$$-\beta_2' M_4 M^2 \begin{bmatrix} \frac{A_1 A_2}{4m_1^2}\{Sinh(2m_1 y) - ySinh(m_1)\} + \\ A_1 A_3\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2} + \frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2} - \frac{Cosh(m_1+m_2)}{(m_1+m_2)^2} - \frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\} + \\ A_1 A_4\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2} - \frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2} - \frac{ySinh(m_1+m_2)}{(m_1+m_2)^2} + \frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\} + \\ A_2 A_3\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2} + \frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2} - \frac{ySinh(m_1+m_2)}{(m_1+m_2)^2} - \frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\} + \\ A_2 A_4\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2} - \frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2} - \frac{Cosh(m_1+m_2)}{(m_1+m_2)^2} + \frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\} + \\ \frac{A_3 A_4}{4m_2^2}\{Sinh(2m_2 y) - ySinh(2m_2)\} \end{bmatrix}$$

$$
-\beta_1'\left[\begin{array}{l}
\frac{A_1^2 m_1^2}{2}\left\{\frac{Cosh(2m_1y)}{4m_1^2}-\frac{y^2}{2}-\frac{Cosh(2m_1)}{4m_1^2}+\frac{1}{2}\right\}+\frac{A_2^2 m_1^2}{2}\left\{\frac{Cosh(2m_1y)}{4m_1^2}+\frac{y^2}{2}-\frac{Cosh(2m_1)}{4m_1^2}-\frac{1}{2}\right\}+\\
\frac{A_3^2 m_2^2}{2}\left\{\frac{Cosh(2m_2y)}{4m_2^2}-\frac{y^2}{2}-\frac{Cosh(2m_2)}{4m_2^2}+\frac{1}{2}\right\}+\frac{A_4^2 m_2^2}{2}\left\{\frac{Cosh(2m_2y)}{4m_2^2}+\frac{y^2}{2}-\frac{Cosh(2m_2)}{4m_2^2}-\frac{1}{2}\right\}+\\
\frac{A_1A_2}{4}\{Sinh(2m_1y)-ySinh(2m_1)\}+A_1A_3m_1m_2\left\{\begin{array}{l}\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\\ -\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}+\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\end{array}\right\}+\\
A_1A_4m_1m_2\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}-\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\
A_2A_3m_1m_2\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}+\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\
A_2A_4m_1m_2\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}-\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\
\frac{A_3A_4}{4}\{Sinh(2m_2y)-ySinh(2m_2)\}
\end{array}\right]
$$

In similar vein we deduce θ_n

$$
\theta_n=\frac{1}{2}(1+y)+\beta_4''\left[\begin{array}{l}\frac{B_1A_1}{m_1^2}\{Cosh(m_1y)-Cosh(m_1)\}+\frac{B_1A_2}{m_1^2}\{Sinh(m_1y)-ySinh(m_1)\}+\\ +\frac{B_2A_3}{m_2^2}\{Cosh(m_2y)-Cosh(m_2)\}+\frac{B_2A_4}{m^2}\{Sinh(m_2y)-ySinh(m_2)\}+\frac{A_5}{2}(y^2-1)\end{array}\right]-
$$

$$
-\frac{\beta_3''}{2}(y^2-1)-\beta_1''\left[\begin{array}{l}
\frac{B_1^2A_1^2m_1^2}{2}\left\{\frac{Cosh(2m_1y)}{4m_1^2}-\frac{y^2}{2}-\frac{Cosh(2m_1)}{4m_1^2}+\frac{1}{2}\right\}+\frac{B_1^2A_2^2m_1^2}{2}\left\{\begin{array}{l}\frac{Cosh(2m_1y)}{4m_1^2}+\frac{y^2}{2}-\\ \frac{Cosh(2m_1)}{4m_1^2}-\frac{1}{2}\end{array}\right\}+\\
+\frac{B_2^2A_3^2m_2^2}{2}\left\{\frac{Cosh(2m_2y)}{4m_2^2}-\frac{y^2}{2}-\frac{Cosh(2m_2)}{4m_2^2}+\frac{1}{2}\right\}+\frac{B_2^2A_4^2m_2^2}{2}\left\{\begin{array}{l}\frac{Cosh(2m_2y)}{4m_2^2}+\frac{y^2}{2}\\ \frac{Cosh(2m_2)}{4m_2^2}\quad\frac{1}{2}\end{array}\right\}\\
+\frac{B_1^2A_1A_2}{4}\{Sinh(2m_1y)-ySinh(2m_1)\}
\end{array}\right]
$$

$$-\beta_1''\left[\begin{array}{l} B_1B_2A_1A_3m_1m_2\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}+\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\}+ \\ B_1B_2A_1A_4m_1m_2\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}-\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+ \\ B_1B_2A_2A_3m_1m_2\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}+\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+ \\ B_1B_2A_2A_4m_1m_2\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}-\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\} \end{array}\right]$$

$$-\beta_1''\left[\frac{B_2A_3A_4}{4}\{Sinh(2m_2y)-ySinh(2m_2)\}\right]$$

RESULTS AND DISCUSSION

In the previous two sections we have formulated and solved exactly for the velocity, temperature and induced magnetic field for parallel plate couette flow of a two-component plasma A primary observation is the separation of the ionized specie from the neutral.

Fig. 2 shows the field structure of the ionized and neutral species To have a feel of the varying parameters used, numerical results are presented with the constants as follows:

N=2.0, $Pr' = Pr'' = 0.71$, R_M=3.0, $Re' = Re'' = 1.0$, R_H=2.0, M=3.0, $\Gamma = M_A = \sigma_i^2 = \sigma_n^2 = 1.0$,

$Ec' = Ec'' = 0.1$, $\beta_3' = \beta_3'' = 2.0$, β = 1.0 (50% ionization),

$\phi_u = \phi_l = 1.0$ (upper and lower walls are conducting)

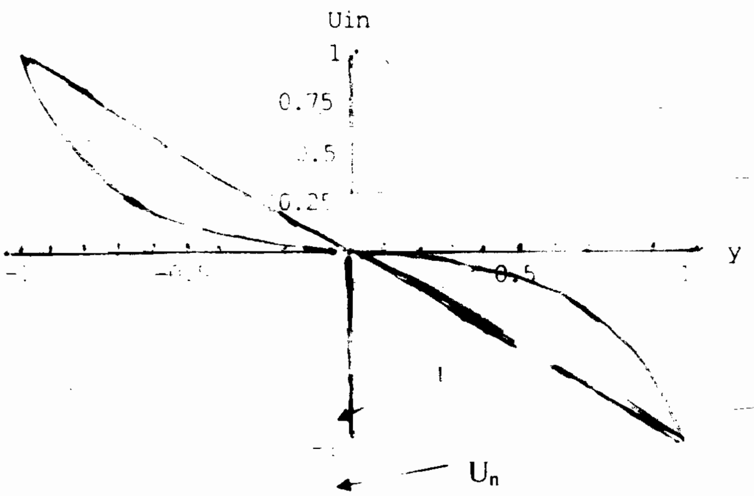

Fig. 2a: Velocity profile in MHD Couette flow of a two-component plasma

Fig. 2(a) shows the velocity profile This agrees perfectly well with the result of Shih-I Pai (1961) where when the Hartmann number R_n which is M in our problem equals zero correspond to the neutral particles. Also the boundary condition is

$U = \mp 1$ on $y = \mp 1$ which rotates our result 180° clockwise

At the centre of the channel $U_i = U_n = 0$ meaning the ionized and neutral particles are stationary. Moving up or down the channel the particles separate with the neutral specie speed higher than the ionized This is also in good agreement with the result of Alabraba et al (2002)

Fig. 2(b) depicts the induced magnetic field which is almost constant and in the $-i$ direction.

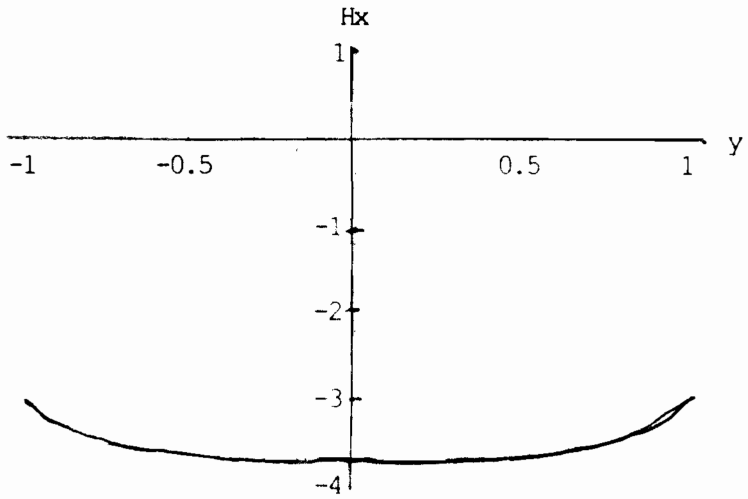

Fig. 2b: Induced magnetic field profile in MHD Couette flow of a two-component plasma.

Fig. 2 (c) shows the temperature profile with θ_i always higher than θ_n except at the walls. At a distance about a quarter up the channel $\theta_n = 1$ i.e. the temperature of the hotter wall and $\theta_i = 2$ Just beyond half the channel θ_i peaks at 2.6 and θ_n at 1.6.

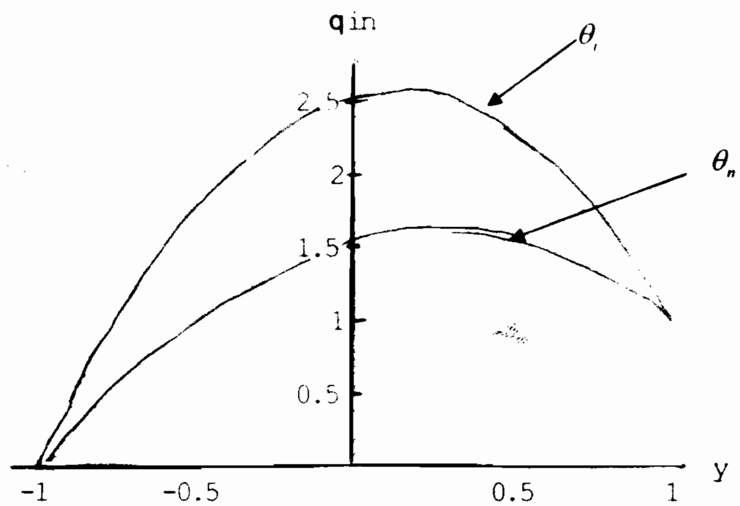

Fig. 2c : Temperature profile in MHD Couette flow of a two-component plasma

HEAT TRANSFER

The heat transfer in terms of the Nusselt number at the walls for the ionized and neutral particles is defined as:

$$Nu^{i,n} \pm 1 = \frac{\mp L}{T_1 - T_0}\left(\frac{\partial \bar{T}'}{\partial y'}\right)_{y'=\pm l} \quad \text{(Seshagiri Rao, 1979)}$$

In Fig. 3 we have plotted the Nusselt number Nu' against the Hartmann number M with $R_{xt} = \beta_3' = 2.0$ for the ionized particles at the upper and lower walls.

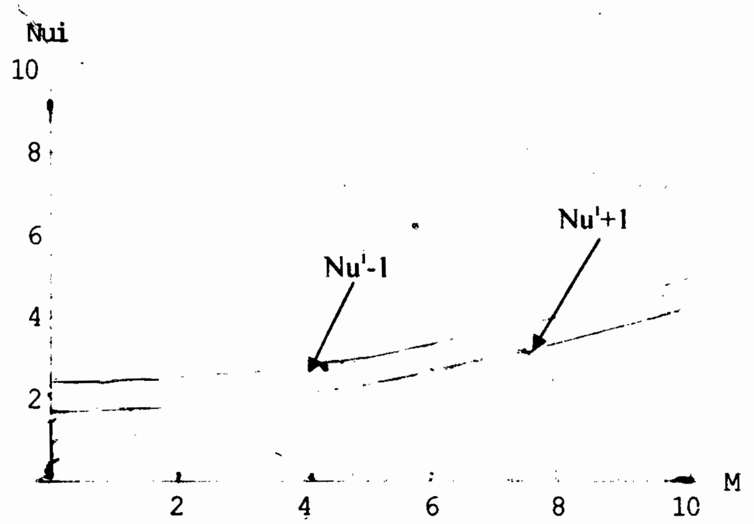

Fig. 3a: Nusselt number versus M for R_M = 2.0, β_3' = 2.0

Fig. 3 (a) shows the case when both walls are either conducting or non conducting. Nu' increases slowly at both walls but with that at the lower wall higher.

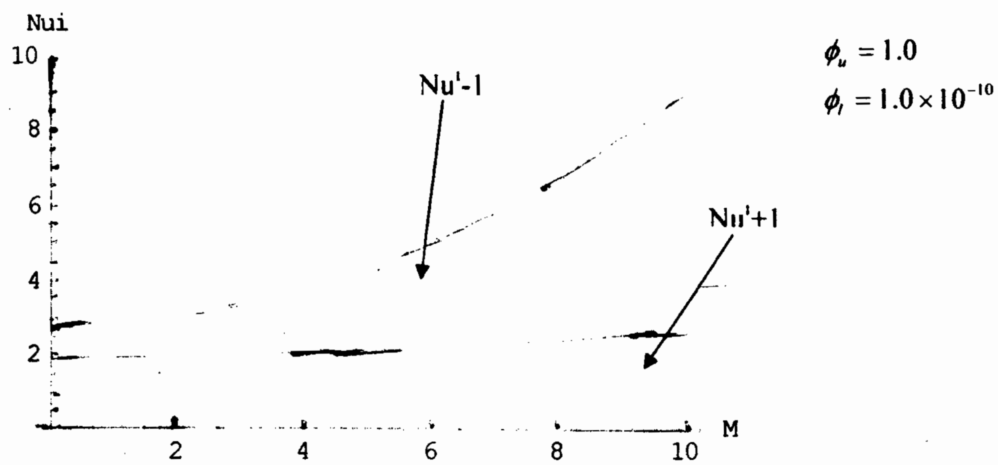

Fig. 3b: Nusselt number versus M for R_M = 2.0, β_3' = 2.0

Fig. 3 (b) shows when the upper wall is conducting and lower wall is non conducting. We notice that Nu' increases sharply at the lower wall but steadily at the upper wall

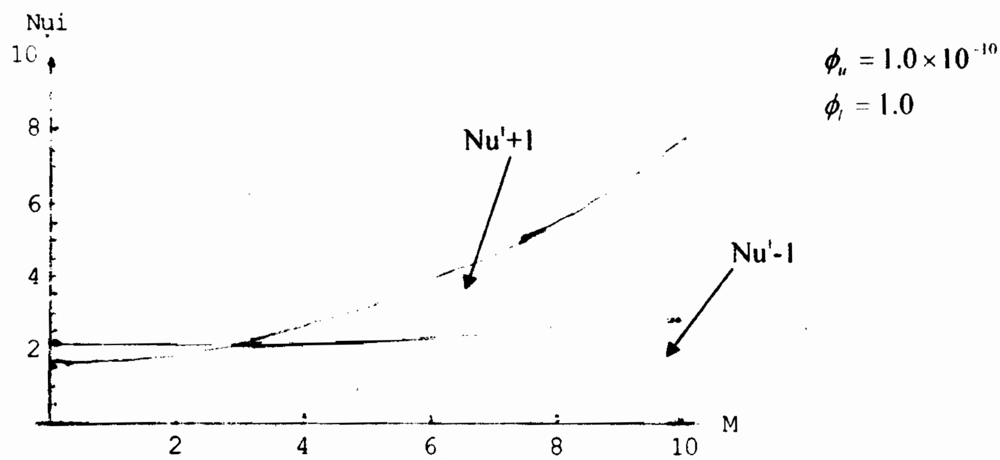

Fig. 3c : Nusselt number versus M for R_M = 2.0, β_3' = 2.0

Fig 3 (c) is the case when the conductivity of the walls are reversed when compared with that in Fig. 3 (b). Nu' increases sharply at the upper wall and steadily at the lower wall.
The results in Fig. 3 (b) and 3 (c) shows that the Nusselt number increases sharply at the non conducting wall and steadily at the conducting wall when the walls are such that one is conducting and the other non conducting.

Finally Fig. (4) shows the Nusselt number $Nu^{i,n}$ for both the ionized and neutral particles plotted against the heat source parameter $\beta_3^{i,n}$ with R_M = 2.0 and M = 3.0

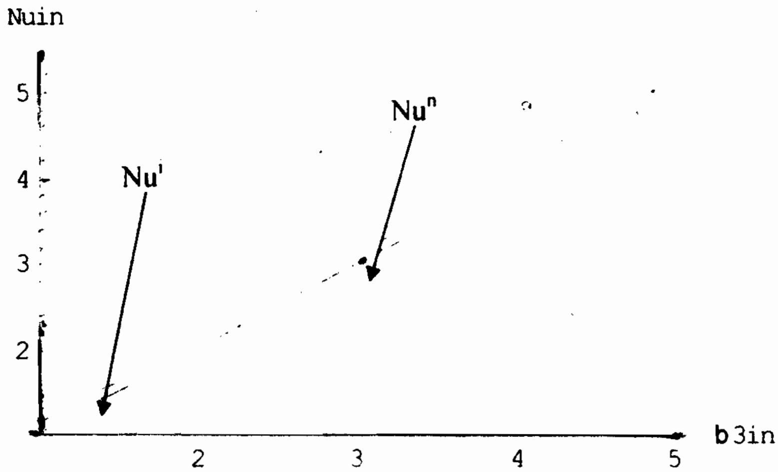

Fig. 4a: Nusselt number versus $\beta_3^{i,n}$ at the upper wall for R_M =2.0, M = 3.0

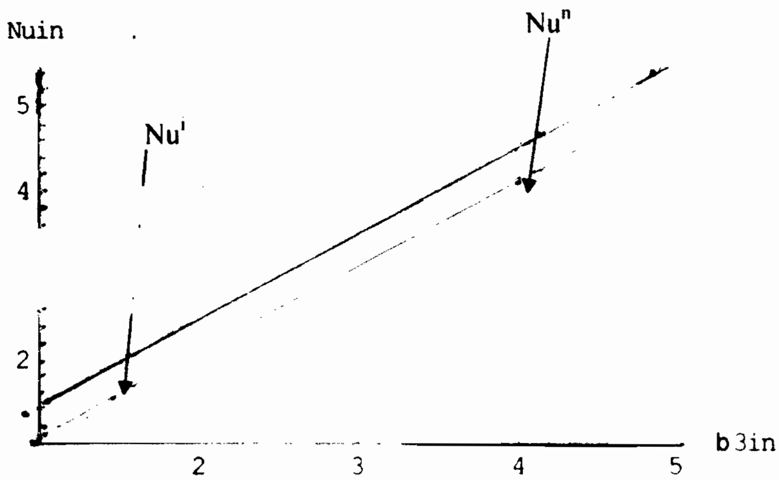

Fig. 4b: Nusselt number versus $\beta_3^{i,n}$ at the lower wall for R_M = 2.0, M = 3.0

Fig. 4(a) shows a steady increase in $Nu^{i,n}$ at the upper wall with the value of the ionized just higher, while Fig 4(b) which is at the lower wall shows similar increase but with a bigger difference.
This suggests that the rate of heat transfer from the upper wall to the particles is not significantly different for the ionized and neutral while the rate of transfer from the particles to the lower wall is significantly different and higher for the ionized than the neutrals.

CONCLUSIONS

In conclusion we deduce the following in the problem of heat transfer in MHD couette flow of two-component plasma with variable wall temperature:

- There is a clear seperation of the neutral species from the ionized species with the former having a higher speed. (This speed difference is due to collision effects)
- There is an almost constant induced magnetic field in the reverse direction of $\hat{i}$.
- The temperature of the ionized is always higher than the neutral except at the walls (This is due to joule heating on the ionized specie)

- The Nusselt number increases slowly with Hartmann number at the walls when both walls are conducting or not, but when one of the wall is conducting and the other non conducting, it increases sharply at the non conducting wall and steadily at the other wall.

APPENDIX

$$a = \left(\sigma_i^2 \beta + \frac{\sigma_n^2}{\beta} + R_M^2 M_A N \right) \qquad b = \frac{R_M^2 M_A \sigma_n^2 N}{\beta}$$

$$m_1 = \sqrt{\frac{a + \sqrt{(a^2 - 4b)}}{2}} \qquad m_2 = \sqrt{\frac{a - \sqrt{(a^2 - 4b)}}{2}}$$

$$B_1 = 1 - \frac{m_1^2}{\sigma_i^2 \beta} + \frac{R_M^2 M_A N}{\sigma_i^2 \beta} \qquad B_2 = 1 - \frac{m_2^2}{\sigma_i^2 \beta} + \frac{R_M^2 M_A N}{\sigma_i^2 \beta}$$

u' dimensional velocity component
(x', y', z') dimensional cartesian coordinates
k thermal conductivity
D dimensional electric displacement
C_p specific heat at constant pressure
Ec Eckert number
H_0 constant transverse magnetic field
R_H Magnetic pressure number
T_0 constant temperature at the lower wall

Pr Prandtl number
Q volumetric heat source
M Hartmann number
f collision frequency
R_M Magnetic Reynolds number
A_0 Alfven speed
N Ratio of Alfven to wall velocity
M_A dimensionless parameter
T_1 constant temperature at the upper wall

Greek symbols

σ electrical conductivity of ionized particles
ε electric permittivity
Γ dimensionless temperature gradient
$\sigma_{u,l}$ electrical conductance of plate's
ν kinematic viscosity
ρ hydrodynamic density
λ magnetic diffusivity

μ coefficient of viscosity
μ_m magnetic permeability
$\sigma_{i,n}^2$ Wormesley frequency type parameter
$\phi_{u,l}$ electrical conductance ratio
β degree of ionization
β_3 heat source parameter

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