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A COMPARATIVE STUDY OF THE PERFORMANCES OF SOME ESTIMATORS OF LINEAR MODEL WITH FIXED AND STOCHASTIC REGRESSORS

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ABSTRACT

In linear regression model, regressors are assumed fixed in repeated sampling. This assumption is not always satisfied especially in business, economics and social sciences. Consequently in this paper, effort is made to compare the performances of some estimators of linear model with autocorrelated error terms when normally distributed regressors are fixed (non – stochastic) with when they are stochastic. The estimators are the ordinary least square (OLS) estimator and four feasible generalized least estimators which are Cochrane Orcutt (CORC), Hidreth – Lu (HILU), Maximum Likelihood (ML), Maximum Likelihood Grid (MLGD) estimator. These estimators are compared using the finite properties of estimators' criteria namely; sum of biases, sum of variances and sum of the mean squared error of the estimated parameter of the model at different levels of autocorrelation and sample size through Monte – Carlo studies.
Results show that at each level of autocorrelation the estimated value of the criteria with stochastic regressor are much lesser than that of the fixed regressor for all the estimators except CORC when the sample size is small (n=20) and the level of autocorrelation is very high $(\rho = 0.9)$. More comparatively, it is observed that the same estimator(s) that is more efficient with fixed regressors is also more efficient with stochastic regressors except when the sample size is large (n = 80) and the level of autocorrelation is either low $(\rho = 0.4)$ or high $(\rho = 0.8)$. At these instances, the CORC / HILU estimator is more efficient with fixed regressors while the ML / MLGD estimator is more efficient with stochastic regressors.

KEYWORDS: Fixed Regressors, Stochastic Regressors, Linear Model, Autocorrelated error, OLS estimator, Feasible GLS estimators.

INTRODUCTION

One of the basic assumptions that are made about the regressors in linear regression model is that they are fixed in repeated sampling. This assumption is not always satisfied especially in business, economics and social sciences. This is because their regressors are often generated by stochastic process beyond their control. For instance, consider regressing daily bathing suit sales by a departmental store on the mean daily temperature. Certainly, the departmental store can not control daily temperature, so it would not be meaningful to think of repeated samples when temperature levels are the same from sample to sample (Fomby et. al, 1984). Authors like Neter and Wasserman (1974), Maddala (2002) have given situations and instances where these assumptions may be not be tenable and have also discussed their consequences on the Ordinary Least Square (OLS) estimator when used to estimate the model parameters. Graybill (1961), Sampson (1974), Fomby et.al (1984) and many others emphasized that if regressors are stochastic and independent of the error terms; the OLS estimator is unbiased and has minimum variance even though it is not Best Linear Unbiased Estimator (BLUE).
When all the assumptions of the linear regression model hold except that the error terms are not homoscedastic (i.e. $E(UU^1) \neq \sigma^2 I_n$) but are heteroscedastic (i.e. $E(UU^1) = \sigma^2 \Omega$), the resulting model the Generalized Least Squares (GLS) Model. Aitken (1935) has shown that the GLS estimator $\hat{\beta}$ of β given as

$$\hat{\beta} = \left(X^1 \Omega^{-1} X\right)^{-1} X^1 \Omega^{-1} Y \qquad (1)$$

is efficient among the class of linear unbiased estimators of β with variance – covariance matrix of $\hat{\beta}$ given as

$$V\left(\hat{\beta}\right) = \sigma^2 \left(X^1 \Omega^{-1} X\right)^{-1} \qquad (2)$$

where Ω is assumed to be known. However, Ω is not always known, it is often estimated by $\hat{\Omega}$ to have what is known as Feasible GLS estimator. Many consistent estimates of $\hat{\Omega}$ can be obtained (Fomby et. al, 1984).
With first order autocorrelated error terms (AR (1)), among the Feasible GLS estimators in literature are the Cochrane and Orcutt estimator (1949), Hildreth and Lu estimator (1960), Prais – Winsten estimator (1954), Thornton estimator (1982), Durbin estimator (1960), Theil's estimator (1971), the Maximum Likelihood estimator and the Maximum Likelihood Grid estimator (Beach and Mackinnon, 1978). Some of these estimators have now been incorporated into White's SHAZAM program (White, 1978) and the new version of the time series processor (TSP, 2005).
Consequently, effort is made in this paper to compare the performances of some of these estimators of linear model when normally distributed regressors are fixed (non – stochastic) in repeated sampling with when they are stochastic.

LITERATURE REVIEW

The OLS estimator has been widely discussed to be unbiased but suffer efficiency in estimating the parameters of

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linear model in the presence of autocorrelation (Johnston, 1984; Chartterjee et.al, 2000; Maddala, 2002). To compensate for this lost of efficiency, Cochrane and Orcutt (1949) suggested a transformation of the regression model via the generalized least square (GLS) estimator. Chipman (1979), Kramer (1980), Kleiber (2001) and many others did observe that the efficiency of these estimators depends on the structure of the regressors that are used. Rao and Griliches (1969) did one of the earliest Monte Carlo studies on the performances of some of these estimators with autoregressive stochastic regressor. They observed that the OLS estimator is only more efficient than any of the GLS estimators considered when $|\rho| < 0.3$; and that the performances of the GLS estimators are not far apart. Park and Mitchell (1980) observed that when regressors are trended, the estimator that uses the P transformation (Paris – Winstern) is more efficient than the one that uses the Q transformation (Cochrane – Orcutt) and that the latter should even be avoided since it is less efficient than the OLS estimator.

More recently, Nwabueze (2005) examined the performance of some of these estimators with exponential independent variable. His result, among other things, show that the OLS estimator compares favorably with the Maximum Likelihood (ML) and Maximum Likelihood Grid (MLGD) estimators for small value of ρ but it appears to be superior to Cochrane – Orcutt (CORC) and the Hidreth and Lu (HILU) especially when ρ is large. Some other recent works that are done with different specification of regressors include that of Iyaniwura and Nwabuwze (2004a), Iyaniwura and Nwabuwze (2004b) and Olaomi and Iyaniwura (2006).

Consequently, this paper compares the performances of some of these estimators when normally distributed regressors are fixed in repeated sampling with when they are stochastic.

METHODOLOGY

Consider the GLS model with AR (1) of the form

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t \qquad (3)$$

where $u_t = \rho u_{t-1} + \varepsilon_t$

$$|\rho| < 1 \qquad t = 1, 2, ..., n \qquad \varepsilon_t \sim N\left(0, \sigma^2\right)$$

Its parameter estimations can be done using the OLS and the (feasible) GLS estimators. Thus, the performances of the OLS estimator and the following feasible GLS estimators are studied: CORC, HILU, ML and the MLGD estimators. Monte Carlo experiments were performed 120 times for three sample sizes $(n = 20, 40, 80)$ and four levels of autocorrelation ($\rho =$0.4, 0.8, 0.9, 0.99) with both fixed and stochastic regressors that are normally distributed. At a particular specification of n and ρ (a scenario), the first replication was obtained by generating $e_t \sim N(0,1)$ and hence u_t. Assuming the process start from infinite past and continue to operate, the initial value of U (i.e u_1) was thus drawn from a normal population with mean zero and variance $\frac{1}{1-\rho_1^2}$.

Hence

$$u_1 = \frac{\varepsilon_1}{\sqrt{1-\rho_1^2}} \qquad (4)$$

$$u_t = \rho_1 u_{t-1} + \varepsilon_t \quad t = 2, 3, \ldots, n \qquad (5)$$

Furthermore, $x_{1t} \sim N(0,1)$ and $x_{2t} \sim N(0,1)$ were generated. Hence, the values of y_t in equation (1) were also calculated by setting the true regression coefficients as $\beta_0 = \beta_1 = \beta_2 = 1$. This process continued until all replications in this scenario were obtained. Another scenario then started until all the scenarios were completed. The only difference in these procedures with stochastic regressors is that at each replication the $x_{1t} \sim N(0,1)$ and $x_{2t} \sim N(0,1)$ were newly generated.

Evaluation and comparison of estimators were examined using the finite sampling properties of estimators which include bias (B), and variance (Var) and the mean squared error (MSE) criteria. Mathematically, for any estimator $\hat{\beta}_i$ of β_i of model (3)

$$\bar{\hat{\beta}}_i = \frac{1}{120}\sum_{j=1}^{120}\hat{\beta}_{ij} \qquad (6)$$

$$B\left(\hat{\beta}_i\right) = \frac{1}{120}\sum_{j=1}^{120}\left(\hat{\beta}_{ij} - \beta_i\right) = \bar{\hat{\beta}}_i - \beta_i \qquad (7)$$

$$Var\left(\hat{\beta}_i\right) = \frac{1}{120}\sum_{j=1}^{120}\left(\hat{\beta}_{ij} - \bar{\hat{\beta}}_i\right)^2 \qquad (8)$$

$$MSE\left(\hat{\beta}_i\right) = \frac{1}{120}\sum_{j=1}^{120}\left(\hat{\beta}_{ij} - \beta_i\right)^2 = Var\left(\hat{\beta}_i\right) + \left[B\left(\hat{\beta}_i\right)\right]^2 \qquad (9)$$

for i = 0, 1, 2 and j= 1,2,…,120.

For each of the estimation methods, a computer program was written using TSP software to estimate all the model parameters and to evaluate the criteria. Often times, preference of estimators are based on bias (closest to zero), minimum variance and minimum (root) mean squared error. In this study, we utilized the criteria of sum of bias (SBIAS), sum of variance (SVAR), and the root mean squared error (SRMSE) of the estimated model parameters to compare the performances of the estimators. This approach has also been used by Iyaniwura and Nwabueze (2004a), Iyaniwura and Nwabueze (2004b), Nwabueze (2005), Olaomi and Iyaniwura (2006) and some others.

Consider an estimator $\hat{\beta}(.) = (\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$, then

SBIAS of $\hat{\beta}(.) = |BB0| + |BB1| + |BB2| \qquad (10)$

SVAR of $\hat{\beta}(.) = VARB0 + VARB1 + VARB2 \qquad (11)$

SRMSE of $\hat{\beta}(.) = RMSEB0 + RMSEB1 + RMSEB2 \qquad (12)$

The efficiency of the estimators was further examined using the sum of SRMSE. An estimator with the smallest SRMSE is most efficient whereas if two estimators are nearly equal in terms of their SRMSE, they are simply said to be more efficient.

SIMULATION RESULTS AND DISCUSSION

The summary of the performances of the estimators on the basis of the sum of the criteria is given in table 1, 2 and

3 in the appendix. Also, the summary of the most/ more efficient estimator(s) is shown in table 4 in the appendix. However, the estimated criteria of the model parameters for n = 20 for both fixed and stochastic regressors are given in table 5, 6,7,8,9 and 10 in the appendix while for all other sample sizes, see Ayinde (2006).

From the tables, it is observed that at each level of autocorrelation the estimated value of the criteria with stochastic regressor are much lesser than that of the fixed regressor for all the estimators except CORC when the sample size is small (n=20) where the CORC estimator has estimated value of the criteria greater with stochastic regressor than the fixed regressor especially when the level of autocorrelation is very high $(\rho = 0.9)$. Also, with both fixed and stochastic regressor, as ρ increases the estimated criteria of the estimators in all the sample sizes increase. Asymptotically, it is also observed that the estimated value of the criteria reduce at each level of autocorrelation.

Furthermore, in terms of the efficiency measured from table 1, 2 and 3 using the sum of root mean squared error of the estimated parameters, the summary of the results is shown in table 5 in the appendix.

From table 4, it can be seen that when the sample size is small (n =20) and the level of autocorrelation is both low $(\rho = 0.4)$ and high $(\rho = 0.8)$ the ML / MLGD estimator is more efficient; and that at the other levels of autocorrelation the HILU estimator is most efficient. When the sample size is moderate (n = 40), the results are essentially the same with when the sample size is small (n =20) except that the CORC estimator is now more efficient at high level of autocorrelation $(\rho = 0.8)$; and also it competes with the HILU estimator when autocorrelation is very high $(\rho = 0.9)$ under fixed regressors. Furthermore, when the sample size is large (n = 80) the results are the same with when the sample size is moderate (n = 40) except that when the autocorrelation level is high $(\rho = 0.8)$ under stochastic regressors, the ML / MLGD estimators are more efficient.; and under fixed regressors when autocorrelation is low $(\rho = 0.4)$ the CORC / HILU estimators are more efficient. Moreover, when the autocorrelation level is very high $(\rho = 0.9)$ under stochastic regressors the CORC estimator also competes with HILU estimator.

Comparatively from table 5, it is observed that the same estimator(s) that is more efficient with fixed regressors is also more efficient with stochastic regressors except when the sample size is large (n = 80) and the level of autocorrelation is either low $(\rho = 0.4)$ or high $(\rho = 0.8)$. At these instances, the CORC / HILU estimator is more efficient with fixed regressors while the ML / MLGD estimator is more efficient with stochastic regressors.

CONCLUSION

The performances of estimators of linear model in the presence of autocorrelated error terms with stochastic regressors are often much lesser than that of the fixed regressors on the basis of finite properties of estimator criteria. However, the estimators' performances in terms of their efficiency are much alike except when the sample size is large and the level of autocorrelation moderately high.

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APPENDIX:

TABLE 1: Sum of the estimated criteria of the model parameters when n = 20

ρ	Estimator	Sum of Biases		Sum of Variances		Sum of Root Mean Squared Error	
		Fixed	Stochastic	Fixed	Stochastic	Fixed	Stochastic
0.4	OLS	.238270	.038352	5.775682	.166334	2.984762	.656117
	CORC	.204020	.047266	5.798128	.175474	2.941968	.709747
	HILU	.205320	.050004	5.789748	.180270	2.940498	.724220
	ML	.234019	.039626	5.745688	.146264	2.924859	.630946
	MLGD	.233891	.037927	5.744684	.145209	2.924527	.627941
0.8	OLS	.266471	.056133	6.153009	.363389	3.263503	1.043938
	CORC	.039216	.125415	7.181041	.653176	3.157918	1.224348
	HILU	.186433	.073298	5.956764	.388445	2.917448	.993977
	ML	.216507	.027413	5.834747	.205884	2.881965	.770204
	MLGD	.218063	.041021	5.846870	.187885	2.885169	.734114
0.9	OLS	.281376	.067919	6.984565	.979242	3.502758	1.607896
	CORC	.124364	.839709	8.636493	78.423288	3.392219	9.314332
	HILU	.239270	.068723	5.998021	.451160	2.908556	1.029051
	ML	.226088	.047919	6.385660	.657157	2.973699	1.152051
	MLGD	.229164	.053341	6.396638	.661847	2.977209	1.155442
0.99	OLS	.283102	.080187	38.345911	31.889861	7.094184	6.389459
	CORC	.662101	.406078	43.485020	29.826651	7.067019	5.859107
	HILU	.266153	.036515	30.009107	24.180186	5.913894	5.301591
	ML	.252761	.074358	36.939508	30.899853	6.499316	5.932051
	MLGD	.249847	.072162	36.939690	30.989355	6.500296	5.940578

TABLE 2: Sum of the estimated criteria of the model parameters when n = 40

ρ	Estimator	Sum of Biases		Sum of Variances		Sum of Root Mean Squared Error	
		Fixed	Stochastic	Fixed	Stochastic	Fixed	Stochastic
0.4	OLS	.247033	.019700	5.667157	.068791	2.776989	.411387
	CORC	.240610	.024383	5.639120	.053530	2.721794	.379146
	HILU	.240863	.024651	5.639213	.052998	2.721970	.377509
	ML	.239337	.019273	5.646078	.051251	2.720489	.356836
	MLGD	.239800	.018713	5.645847	.050760	2.719910	.355102
0.8	OLS	.279389	.045788	5.821598	.142195	2.999537	.642281
	CORC	.225009	.027036	5.651256	.053457	2.675785	.400956
	HILU	.225662	.030867	5.650916	.061062	2.675680	.425629
	ML	.243775	.021199	5.697144	.060774	2.686009	.424054
	MLGD	.245195	.025081	5.700142	.064486	2.687561	.435030
0.9	OLS	.343057	.086436	6.099065	.354921	3.175278	1.010561
	CORC	.226712	.027416	5.757145	.134072	2.686770	.566352
	HILU	.224773	.027344	5.741233	.107847	2.682936	.522869
	ML	.276233	.050594	5.921211	.241752	2.724562	.705002
	MLGD	.276014	.049323	5.912873	.231940	2.722778	.694261
0.99	OLS	.760960	.456681	29.513130	22.885019	6.301095	5.419174
	CORC	.511734	.370717	159.821391	16.674900	12.926464	4.326374
	HILU	.501084	.248016	22.810143	16.397971	5.073257	4.283857
	ML	.671973	.423473	29.416089	22.913213	5.735637	5.032298
	MLGD	.665383	.418786	29.144429	22.647244	5.709859	5.004314

TABLE 3: Sum of the estimated criteria of the model parameters when n = 80

ρ	Estimator	Sum of Biases		Sum of Variances		Sum of Root Mean Squared Error	
		Fixed	Stochastic	Fixed	Stochastic	Fixed	Stochastic
0.4	OLS	.250802	.014099	5.636964	.035007	2.684490	.286915
	CORC	.252363	.017475	5.606360	.025552	2.644359	.254305
	HILU	.252816	.018032	5.608606	.025365	2.645570	.253706
	ML	.252489	.015358	5.627886	.024661	2.649754	.242854
	MLGD	.252547	.016199	5.628093	.024715	2.649549	.243370
0.8	OLS	.263009	.037144	5.678403	.085684	2.812667	.475116
	CORC	.240028	.020998	5.576679	.025315	2.611712	.276711
	HILU	.236382	.024039	5.583061	.026128	2.613064	.281463
	ML	.247864	.013629	5.668202	.024962	2.632163	.274328
	MLGD	.246783	.012750	5.663375	.025273	2.631302	.276023
0.9	OLS	.265469	.055593	5.730489	.187614	2.916687	.752543
	CORC	.234145	.028050	5.570998	.061649	2.602313	.392062
	HILU	.215455	.043050	5.581487	.062544	2.603192	.395977
	ML	.259666	.028731	5.800086	.096059	2.652877	.460007
	MLGD	.254529	.025007	5.782619	.090554	2.648944	.450261
0.99	OLS	.565490	.414638	27.642830	21.321966	5.981096	5.290383
	CORC	.419035	.201564	16.338757	10.815012	4.289645	3.459889
	HILU	.328511	.270800	7.995106	8.112876	3.070842	3.025455
	ML	.545399	.368202	25.739784	19.552746	5.328961	4.602012
	MLGD	.515371	.346511	26.498670	20.258198	5.400255	4.679430

TABLE 4: Summary of the more / most efficient estimator(s)

Sample size (n)	ρ	Regressors	
		Fixed	Stochastic
20	0.4	ML/MLGD	ML/MLGD
	0.8	ML/MLGD	ML/MLGD
	0.9	HILU	HILU
	0.99	HILU	HILU
40	0.4	ML/MLGD	ML/MLGD
	0.8	CORC/HILU	CORC
	0.9	CORC/HILU	HILU
	0.99	HILU	HILU
80	0.4	CORC/HILU	ML/MLGD
	0.8	CORC/HILU	ML/MLGD
	0.9	CORC/HILU	CORC/HILU
	0.99	HILU	HILU

TABLE 5: Bias of β with fixed regressor when n = 20 and R = 120.

ρ	Estimator	BB0	BB1	BB2	SBIAS
0.4	OLS	.218460	-.006482	-.013328	.238270
	CORC	.182840	-.017640	.003540	.204020
	HILU	.184910	-.017215	.003195	.205320
	ML	.212750	-.019451	-.001818	.234019
	MLGD	.213070	-.018807	-.002014	.233891
0.8	OLS	.232580	-.011302	-.022589	.266471
	CORC	.038880	.000311	-.000026	.039216
	HILU	.176440	.006256	.003738	.186433
	ML	.208870	.005978	-.001659	.216507
	MLGD	.209900	.006465	-.001698	.218063
0.9	OLS	.249260	-.006339	-.025777	.281376
	CORC	.118030	.001118	.005217	.124364
	HILU	.232320	.000479	.006471	.239270
	ML	.222830	.001941	-.001317	.226088
	MLGD	.225710	.002104	-.001351	.229164
0.99	OLS	.251330	.007021	-.024751	.283102
	CORC	.644360	.013240	.004501	.662101
	HILU	.254380	.011488	-.000285	.266153
	ML	.238320	.011103	-.003338	.252761
	MLGD	.235750	.011575	-.002522	.249847

TABLE 6: Variance of β with fixed regressor when n = 20 and R = 120.

ρ	Estimator	VB0	VB1	VB2	SVAR
0.4	OLS	5.589455	.097461	.088765	5.775682
	CORC	5.641770	.080830	.075528	5.798128
	HILU	5.633298	.080601	.075849	5.789748
	ML	5.594737	.078916	.072035	5.745688
	MLGD	5.593811	.078683	.072190	5.744684
0.8	OLS	5.793627	.211262	.148120	6.153009
	CORC	7.055148	.058578	.067315	7.181041
	HILU	5.833769	.056623	.066372	5.956764
	ML	5.718863	.056167	.059716	5.834747
	MLGD	5.730682	.056453	.059735	5.846870
0.9	OLS	6.545189	.268890	.170486	6.984565
	CORC	8.525999	.051646	.058849	8.636493
	HILU	5.886957	.052009	.059055	5.998021
	ML	6.280947	.051563	.053150	6.385660
	MLGD	6.291375	.051390	.053873	6.396638
0.99	OLS	37.905503	.273501	.166907	38.345911
	CORC	43.384100	.044421	.056499	43.485020
	HILU	29.912821	.043909	.052377	30.009107
	ML	36.849624	.043389	.046496	36.939508
	MLGD	36.849352	.043932	.046406	36.939690

TABLE 7: Root mean squared of β with fixed regressor when n = 20 and R = 120.

ρ	Estimator	RMB0	RMB1	RMB2	SRMSE
0.4	OLS	2.374275	.312255	.298233	2.984762
	CORC	2.382268	.284853	.274847	2.941968
	HILU	2.380649	.284424	.275425	2.940498
	ML	2.374868	.281592	.268399	2.924859
	MLGD	2.374702	.281135	.268689	2.924527
0.8	OLS	2.418206	.459772	.385526	3.263503
	CORC	2.656437	.242029	.259451	3.157918
	HILU	2.421756	.238038	.257655	2.917448
	ML	2.400519	.237072	.244375	2.881965
	MLGD	2.403069	.237687	.244414	2.885169
0.9	OLS	2.570471	.518585	.413703	3.502758
	CORC	2.922316	.227260	.242644	3.392219
	HILU	2.437402	.228055	.243099	2.908556
	ML	2.516068	.227084	.230547	2.973699
	MLGD	2.518396	.226702	.232110	2.977209
0.99	OLS	6.161872	.523020	.409292	7.094184
	CORC	6.618104	.211178	.237737	7.067019
	HILU	5.475174	.209859	.228860	5.913894
	ML	6.075065	.208595	.215655	6.499316
	MLGD	6.074943	.209919	.215434	6.500296

TABLE 8: Bias of β with stochastic regressors when n = 20 and R =120

ρ	Estimator	BB0	BB1	BB2	SBIAS
0.4	OLS	.007344	-.029209	.001799	.038352
	CORC	-.014375	-.018075	-.014816	.047266
	HILU	-.017108	-.018296	-.014600	.050004
	ML	.008449	-.024703	-.006475	.039626
	MLGD	.007297	-.024030	-.006600	.037927
0.8	OLS	.020933	-.018791	-.016409	.056133
	CORC	-.093311	-.010060	-.022044	.125415
	HILU	-.031898	-.008305	-.033095	.073298
	ML	.004288	-.012049	-.011076	.027413
	MLGD	.009819	-.011101	-.020101	.041021
0.9	OLS	.038194	-.004922	-.024803	.067919
	CORC	.806880	-.003197	-.029632	.839709
	HILU	.035737	-.003658	-.029328	.068723
	ML	.023303	-.005196	-.019420	.047919
	MLGD	.028056	-.005956	-.019329	.053341
0.99	OLS	.055476	.004100	-.020611	.080187
	CORC	.377050	-.003457	-.025571	.406078
	HILU	.006200	-.004365	-.025951	.036515
	ML	.049108	-.003146	-.022104	.074358
		.047197	-.002862	-.022103	.072162

TABLE 9: Variance of β with stochastic regressors when n = 20 and R =120

ρ	Estimator	VB0	VB1	VB2	SVAR
0.4	OLS	.008582	.076754	.080999	.166334
	CORC	.026956	.064870	.083647	.175474
	HILU	.031960	.065082	.083228	.180270
	ML	.012941	.059106	.074218	.146264
	MLGD	.012628	.058566	.074015	.145209
0.8	OLS	.104922	.119527	.138941	.363389
	CORC	.531943	.044476	.076757	.653176
	HILU	.281693	.043200	.063553	.388445
	ML	.107592	.040263	.058029	.205884
	MLGD	.102914	.039077	.045895	.187885
0.9	OLS	.667371	.145056	.166815	.979242
	CORC	78.332465	.038960	.051864	78.423288
	HILU	.359883	.039142	.052136	.451160
	ML	.581877	.035551	.039729	.657157
	MLGD	.586503	.035784	.039560	.661847
0.99	OLS	31.594642	.135223	.159995	31.889861
	CORC	29.750583	.036910	.039158	29.826651
	HILU	24.104022	.037053	.039112	24.180186
	ML	30.828288	.034149	.037415	30.899853
	MLGD	30.917592	.034183	.037579	30.989355

TABLE 10: Root mean squared error of β with stochastic regressors when n = 20 and R =120

ρ	Estimator	RMB0	RMB1	RMB2	SRMSE
0.4	OLS	.092928	.278580	.284609	.656117
	CORC	.164812	.255337	.289598	.709747
	HILU	.179591	.255767	.288862	.724220
	ML	.114070	.244369	.272507	.630946
	MLGD	.112610	.243193	.272138	.627941
0.8	OLS	.324592	.346237	.373109	1.043938
	CORC	.735289	.211133	.277926	1.224348
	HILU	.531705	.208012	.254260	.993977
	ML	.328040	.201017	.241147	.770204
	MLGD	.320952	.197990	.215172	.734114
0.9	OLS	.817820	.380894	.409182	1.607896
	CORC	8.887267	.197408	.229656	9.314332
	HILU	.600966	.197876	.230209	1.029051
	ML	.763164	.188621	.200265	1.152051
	MLGD	.766348	.189259	.199835	1.155442
0.99	OLS	5.621185	.367750	.400525	6.389459
	CORC	5.467426	.192151	.199529	5.859107
	HILU	4.909589	.192541	.199462	5.301591
	ML	5.552540	.184822	.194689	5.932051
	MLGD	5.560559	.184908	.195110	5.940578