GLOBAL JOURNAL OF PURE AND APPLIED SCIENCES VOL. 13, NO. 4, 2007: 571 - 577

 1SSN 1118-0579

A VIBRATION ANALYSIS OF NON-MINDLIN RECTANGLAR PLATES TRAVERSED BY UNIFORMLY DISTRIBUTED MOVING LINE MASSES

J.A. GBADEYAN, M.S. DADA AND A.S. IDOWU

(Received 29 September 2006; Revision Accepted 30 May 2007)

ABSTRACT

The dynamic analysis of non-Mindlin plates traversed by uniformly distributed moving masses has been developed. The familiar governing equation is transformed into new series of ordinary coupled differential equations that apply to plates with various boundary conditions. The numerical solutions for simply supported edges are given. The results show that magnitude of the distribution of the load vary directly as the deflection for the moving forces and inversely as the deflection for the moving mass problems. Furthermore, the response of structures due to moving masses differs significantly with moving forces. Also, the results compared excellently well with the previous ones involving the limiting case when the load's interval (ε) reduces to zero.

KEY WORDS: Non-Mindlin Plates, Uniformly Distributed Moving Line Mass, coupled differential equations, inertial effects and Central difference Scheme

1. INTRODUCTION

Many researchers have studied the transverse vibrations of rectangular plates of uniform thickness. The literature is found in the review articles of Leissa (1969). In addition, Singh and Chakraverty (1994) investigated the flexural vibration of skew plates using boundary characteristic orthogonal polynomials in two variables. J.A. Gbadeyan and S.T. Oni (1995) studied the dynamic behaviour of beams and rectangular plates under moving loads. The effects of centripetal and Coriolis forces on the dynamic response of light bridges under moving load was considered by G.T. Michaltsos and A.N. Kounadis (2001). Moreover, G.T. Michaltsos (2002) focused his attention on the dynamic behaviour of a single-span beam subjected to loads with variable speeds. He found that the effect of a variable speed is significant for the deflections of the bridges. These problems are concerned with concentrated moving masses.

The model becomes closer to the physical reality when the moving masses are evaluated taking the length of the distributed load into consideration. This is the focus of this present paper. The moving point masses become a special case of the present model. As a matter of fact, the form is obtained, as the length of the distributed load tends to zero. E. Esmailzadeh and M. Ghorahi (1995) studied the vibration analysis of beams traversed by uniform partially distributed moving masses. Results from their work indicated that the inertial effect of the moving mass is important in the dynamic behaviours of beams.

The main aim of this present paper is to extend the study to plates by considering the vibration analysis of non-mindlin rectangular plates traversed by uniformly distributed moving masses.

2. THE GOVERNING EQUATION

According to the classical theory of thin isotropic elastic plate, the lateral deflection w (x,y,t) at time t of a rectangle plates satisfies the partial differential equation

$$D\left\{\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial^2 y} + \frac{\partial x^4 w}{\partial x^4}\right\} + \rho h \frac{\partial^2 w}{\partial^2 t} = P(x,y,t) \qquad \ldots\ 2.01$$

where

$$D = \frac{1}{12} E h^2 \left(1 - v^2\right)^{-1}$$

is the flexural rigidity of the plate,ρ is the density of the plate per unit volume, E and v are young's modulus and Poisson ratio respectively for the material of the plate and h is the plate thickness.

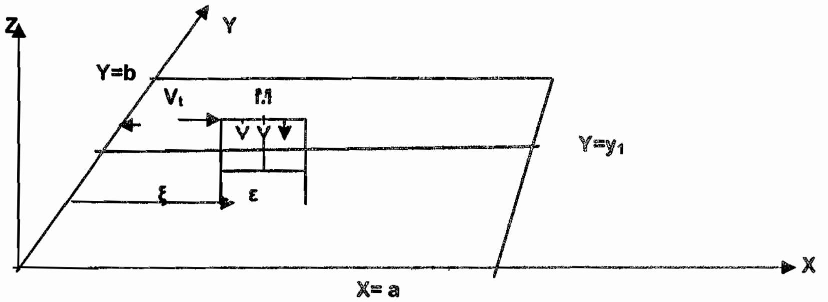

Figure 1

The time varying force p(x,y,t) per unit length can be expressed as

J.A. Gbadeyan, Department of Mathematics, University of Ilorin, Ilorin, Kwara State, Nigeria
M.S. Dada, Department of Mathematics, University of Ilorin, Ilorin, Kwara State, Nigeria
A.S. Idowu, Department of Mathematics, University of Ilorin, Ilorin, Kwara State, Nigeria

$$P(x,y,t) = \frac{1}{\varepsilon}\left[-M_l g - M_l \frac{\partial^2 w}{\partial t^2}\right]$$

$$\left[H\left(x - \xi + \varepsilon/2\right) - H\left(x - \xi - \varepsilon/2\right)\right]\delta\left(y - y_1\right) \quad \ldots \quad 2.02$$

where H is the Heaviside unit function, M_1 is the mass of the load and $\delta(y - y_1)$ is the Dirac delta function. The symbol ξ is defined as $\xi = vt + \varepsilon$.

The expression for the concentrated force can be obtained by taking the limit of the time vary force p(x,y,t) as ε tend to zero since

$$\delta\left(x, -\xi\right) = \lim_{\varepsilon \to 0}\left[\frac{1}{\varepsilon}\left\{H\left(x, -\xi + \frac{\varepsilon}{2}\right) - H\left(x, -\xi - \frac{\varepsilon}{2}\right)\right\}\right] \quad \ldots \quad 2.03$$

The constant force exerted by the mass on the plate is defined to be negative if the force acting on the plate is pointing downward in the z-direction. The load is assumed to be in contact with the plate during the course of the motion. It should be noted that the first term in the bracket at the right hand side of the equation (2.02) represents the constant gravitation force, while the second term represents the inertia effect of the moving mass.

The traversed displacement w(x,y) and applied force p(x,y) at time t is defined respectively by summation-mode method as

$$w\left(x,y,t\right) = \sum_{q=1}^{M}\sum_{p=1}^{N} U_{pq}(t)\, w_p(x) w_q(y) \quad \ldots \quad 2.04$$

$$p\left(x,y,t\right) = \sum_{q=1}^{M}\sum_{p=1}^{N} Q_{pq}(t)\, w_p(x) w_q(y) \quad \ldots \quad 2.05$$

where $U_{pq}(t)$ and $Q_{pq}(t)$ unknown functions of time t. $W_p(x)$ and $W_q(y)$ are the normalized deflection curves for the p^{th} modes respectively of the vibration of the beam.

Substitute equation (2.04) and (2.05) into equation (2.02) and multiply the result by $W_n(x)$ $W_m(y)$ gives

$$\frac{1}{\varepsilon}\left[-M_l g w_n(x)\, w_m(y) - M_l w_n(x)\, w_m(y) \sum_{q=1}^{M}\sum_{p=1}^{N} U^{11} pq(t)\, w_p\ (x) w_q(y)\right]$$

$$\left[H\left(x - \xi + \varepsilon/2\right) - H\left(x - \xi + \varepsilon/2\right)\right]\delta(y - y_1)$$

$$= w_n\ (x)\ w_m\ (y) \sum_{q=1}^{M}\sum_{p=1}^{N} Q''_{pq}\ (t)\ w_p\ (x)\, w_q(y) \quad \ldots \quad 2.06$$

where $U''_{pq}(t)$ denote the second derivative of the function with respect to time t

Taking the double integrals of both sides of equation (2.06) with respect to x and y results in

$$\frac{-M_l g}{\varepsilon}\int_0^a w_n(x)\ Bdx \int_0^b w_m\ (y)\ \delta(y - y_1)\ d\ -$$

$$\frac{-M_l}{\varepsilon}\sum_{q=1}^{M}\sum_{p=1}^{N} U^{11}\ pq\ (t) \int_0^a w_n(x)\ w_p(x)\ Bdx \int_0^b w_m(y) w_q(y)\ \delta(y - y_1)\ dy\ =$$

$$\sum_{q=1}^{M}\sum_{p=1}^{N} Q\ pq\ (t) \int_0^a w_p(x)\ w_n(x)\ dx \int_0^b w_m(y) w_q(y)\ dy\ = \quad \ldots \quad 2.07$$

where $B = H\left(x - \xi + \varepsilon/2\right) - H\left(x - \xi - \varepsilon/2\right)$

Recall the delta integral properties:

$$\int_{x_1}^{x_2} w(x)\,\delta(x - x_0)\,dx = w(x_0) \qquad \dots \quad 2.08$$

$$\int_{x_1}^{x_2} \delta(x - x_0)\,dx = H(x - x_0) \qquad \dots \quad 2.09$$

Provided ... $x_1 < x_0 < x_2$

Integrating the first integral in equation (2.07) and using equations (2.08) and (2.09), we obtained

$$\frac{1}{\varepsilon}\int_0^a w_n(x)\,B\,dx = \frac{1}{\varepsilon}\left[\int_0^a w_n\left(\xi + \varepsilon/2\right) - w_n\left(\xi - \varepsilon/2\right)\right] d\xi \qquad \dots \quad 2.10$$

We now use taylor series expansion for the integrand function in the right hand side of equation (2.10) and carrying out integration.

$$\frac{1}{\varepsilon}\int_0^a \left[w_n\left(\xi + \varepsilon/2\right) - w_n\left(\xi + \varepsilon/2\right)\right] d\xi =$$

$$w_n(\xi) + \left(\varepsilon/2\right)^2 \left(1/3!\right) w_n''(\xi) + \left(\varepsilon/2\right)^4 \left(1/5!\right) w_n^{iv}(\xi) +$$

$$\cong w_n(\xi) + \left(\varepsilon/2\right)^2 \left(1/3!\right) w_n''(\xi) \qquad (2.11)$$

Similarly,

$$\frac{1}{\varepsilon}\int_0^a w_p(x)\,B\,dx \cong w_p(\xi)w_n(\xi) + \left(\varepsilon/2\right)^2 \left(1/3!\right)\left[w_p(\xi)\,w_n(\xi)\right]'' =$$

$$w_p(\xi)w_n(\xi) + \frac{\varepsilon^2}{24}\left[w_n(\xi)w_p''(\xi) + 2w_n'(\xi)w_p'(\xi) + w_n''(\xi)w_p(\xi)\right] \qquad \dots \quad (2.12)$$

The two integrals in right hand side of equation (2.07) can be normalized to a constant. The normalized constant is unity in this paper. Hence, equation (2..07) implies

$$Q_{pq}(t) = -M_i g w_m(y_1)\left[w_n(\xi) + \varepsilon^2/24\; w_n''(\xi)\right] -$$

$$-M_i \sum_{q=1}^{M}\sum_{p=1}^{N} U_{pq}^{ii}(t)\,w_q(y_1)w_m(y_1)$$

$$\left[w_p(\xi)\,w_n(\xi) + \frac{\varepsilon}{24}\left\{w_p(\xi)\,w_p''(\xi) + 2w_p'(\xi)w_n'(\xi) + w_n''(\xi)\,w_p(\xi)\right\}\right] \qquad \dots \quad (2.13)$$

The substitution of equation (2.13) into equation (2.05) and the resultant equation couple with equation (2.04) into (2.01) yields

$$\sum_{p=1}^{N}\sum_{q=1}^{M} U_{pq}(t)\left[\left\{w_q(y)w_p^{iv}(x) + 2w_q''(y)w_p''(x) + w_q^{iv}(y)w_p(x)\right\} D + \mu U_{pq}^{ii}(t)w_p(x)w_q(y)\right]$$

$$= \sum_{p=1}^{M}\sum_{q=1}^{M} w_p(x)\,w_p(y)[-M_i g w_m(y_1)\left\{w_n(\xi) + \varepsilon^2/24\; w_n''(\xi)\right\} - M_i \sum_{p=1}^{N}\sum_{q=1}^{M} w_q(y_1)w_m(y_1)U_{pq}(t)$$

$$\left\{w_p(\xi)w_p(\xi) + \frac{\varepsilon^2}{24}\left\{w_n(\xi)_p''(\xi) + 2w_n'(\xi)w_p'(\xi) + w_n''(\xi)w_p(\xi)\right\}\right\}] \qquad (2.14)$$

where $\mu = h\rho$

NUMERICAL WORK AND DISCUSSION

The numerical analysis for a simply supported rectangular plate of L_x by L_y is carried out for the following values $E = 2.07 \times 10^8$ kgm^{-2}, $M/\rho h L_x L_y = 0.25$, $v = 5$m^{-1}, $\nu = 0.2$, $\varepsilon = 0.01$m, 0.1m and 1m.

The limit as ε tends to zero in this analysis gives the results for the concentrated moving forces/masses. The deflection of the plate converges as ε approaches zero.

The deflection of the plates due to moving masses is greater than the deflection due to moving forces both for moving point (concentrated) problems and uniformly distributed moving mass problems. It's result for concentrated moving forces/masses is in agreement with [5] as indicated in tables 1 & 2. However, the gap between the deflection of the deflection of the distributed moving mass problems and the concentrated moving mass problems is wider than that of moving forces (See figure 2, table 1 and 2). This shows the significance of taking the inertial effect of the load into account in considering the vibration analysis of distributed moving masses. Neglecting the inertial effect of the load is grossly misleading.

The study reveals that the degrees of the distribution of the moving load vary directly as the deflection of the plate for the moving force problems. On the other hand, the degree of the distribution of the moving load vary inversely as the deflection for the moving mass problems (See tables 1 and 2).

The critical speed for moving masses and moving forces is 11.5ms^{-1} and 22.5ms^{-1} respectively.

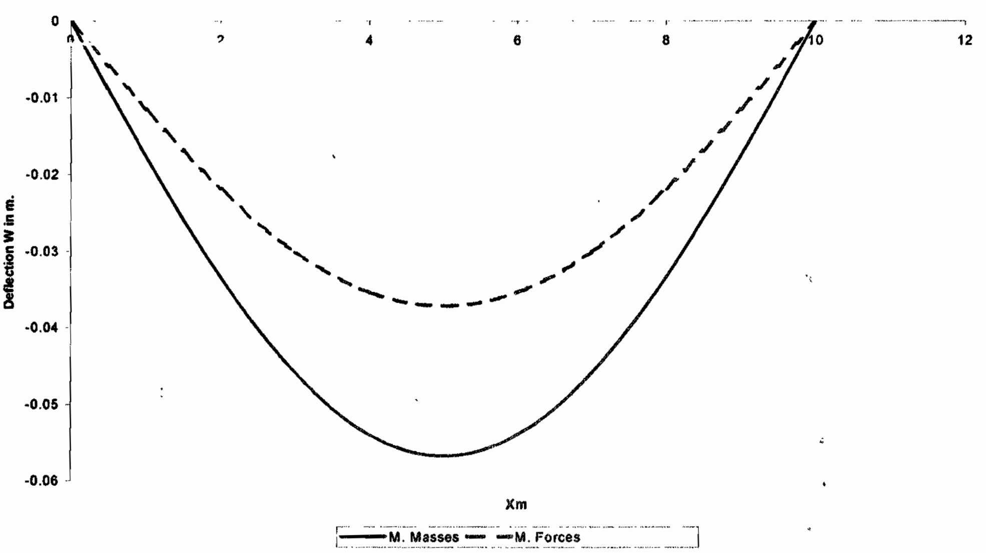

Figure 2: The deflection along y = 4m for E = 1.0m at t = 0.6s

Table 1: Displacement (W) due to moving forces along y = 4m at t = 0, 6s.

X(m)	ε = 1.0(m)	ε = 0.1 (m)	ε = 0.01 (m)
0	0	0	0
1	-0.01144	-0.00972	-0.00967
2	-0.02176	-0.01849	-0.01840
3	-0.02996	-0.02545	-0.02533
4	-0.03522	-0.02992	-0.02977
5	-0.03703	-0.03146	-0.03130
6	-0.03522	-0.02992	-0.02977
7	-0.02996	-0.02545	-0.02532
8	-0.02176	-0.01849	-0.01840
9	-0.01144	-0.00972	-0.00967
10	0	0	0

Table 2: Displacement (W) due to moving masses along y = 4m at 0.6s

X(m)	ε = 1.0(m)	ε = 0.1 (m)	ε = 0.01 (m)
0	0	0	0
1	-0.01750	-0.14152	-0.19445
2	-0.03320	-0.26919	-0.36985
3	-0.04582	-0.37087	-0.50908
4	-0.05307	-0.43556	-0.59846
5	-0.050[illegible]	-0.45798	-0.62926
6	-0.05307	-0.43556	-0.59846
7	-0.04582	-0.37051	-0.50908
8	-0.03320	-0.26919	-0.36986
9	-0.01750	-0.14152	-0.19445
10	0	0	0

CONCLUSION

The analytical – numerical method for analyzing the vibration of rectangular plates with various boundary conditions has been presented. The moving mass problems, due to the inertial effect of the load, result in coupled differential equations [8]. However, the problem results to ordinary differential equations when the inertial effect of the distributed load is ignored. These ordinary differential equations represent the moving force problems. Central difference method is used for the differential equations. It is simple enough to be carried out on most personal computers. The results show the importance of taking the load distribution and the inertial effect of the loads into consideration in designing structures like bridges, railways and others.

REFERENCES

Dada, M.S., 2002. The Vibration analysis of elastic plates under uniform partially distributed moving loads. Unpublished PhD Thesis, Department of mathematics, University of Ilorin, Ilorin Nigeria.

Esmailzadeh, E. and Ghorashi, M., 1995. Journal of sound and vibration. 184 (1) 9–17 Vibration analysis of beams traversed by uniform partially distributed moving masses.

Gbadeyan, J. A. and Oni, S.T., 1995. Journal of sound and vibration 182(5) 677 – 695 Dynamic behaviour of beams and rectangular plates under moving loads.

Leissa, A.W., 1969. Vibration of plates (NSA SP 160) WashinOgton, D.C. U.S. Government Printing Office.

Leissa, A.W., 1987.The shock and vibration Digest 19, 10. Recent studies in plate vibration 1981 - 1985 Part II; complicating effects.

Michalisos, G.T. and Kounadis, A. N., 2001. Journal of vibration and control 7: 315-326. The effects of Centripetal and Coriolios Forces on the dynamic response of light bridges under moving load.

Michalisos, G.T., 2002. Journal of vibration and control 258(2) 359-372. Dynamic behaviour of a single- span beam subjected to moving loads with variable speeds.

Singh, B. and Chakraverty, S., 1994. Journal of sound and vibration. 173, 157 - 178. Flexural vibration of skew plates using boundary characteristic orthogonal polynomials in two variables.