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EFFECTS OF RADIATION ON OSCILLATORY MHD FLOW AND HEAT TRANSFER IN A POROUS MEDIUM PAST AN INFINITE VERTICAL MOVING HEATED POROUS PLATE

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ABSTRACT

This paper investigates the effects of radiation on oscillatory magnetohydrodynamic free-convection flow and heat transfer in a porous medium past an infinite vertically moving heated porous-perfectly-electrically-conducting plate with time-periodic surface temperature and variable suction in the presence of a horizontal magnetic field. The Cogley *et al* differential approximation is used to describe the radiative heat transfer in the limit of optically thin fluids. By imposing a sinusoidal time-dependent perturbation, the coupled nonlinear problem is solved for the velocity and temperature profiles. Also, expressions for the surface skin friction and heat transfer coefficient (Nusselt number) are presented. The results show that increase in radiation parameter leads to a decrease in the temperature profile. Also, simultaneous increases in radiation and magnetic field parameters are associated with decrease in the velocity profiles; while increases in the moving plate velocity, porosity and Grashof number lead to a rise in the velocity distribution. In addition, increase in the moving plate velocity and radiation parameters, as well as magnetic field parameter resulted in the increase in the absolute value of the skin-friction. Finally, increases in the radiation parameter lead to an increase in the surface heat parameter, *Nu*.

KEY WORDS: Radiation, Oscillatory MHD, Porous medium, Heat transfer, Skin-friction

Nomenclature

A	permeability parameter
B	suction velocity parameter
B_1	Planck's function
c_p	specific heat at constant pressure
Gr	Grashof number $= \nu g\beta(T'_w - T_\infty)/(U_0 v_0^2)$
g	acceleration due to gravity
H	surface heat transfer
K	permeability of the porous medium $= v_0^2 K'/\nu^2$
M	magnetic field parameter $= \nu\sigma H_0'^2/(\rho v_0^2)$
N	non-dimensional material parameter $= M + 1/k0$
Nu	Nusselt number $= H\,\mathrm{Re}_x$
q_y	radiative heat flux
R	radiation parameter $= 4\alpha^2\nu/(U_0 v_0^2)$
Re_x	Reynolds number
Pr	Prandtl number $= \nu\rho c_p/\kappa$
t	non-dimensional time $= v_0^2 t'/\nu$
U_0	scale of the free stream
u, v	non-dimensional velocities along and perpendicular to the plate, $(u', v')/U_0$
U	plate velocity $= u'_p/U_0$
v_0	scale of the suction velocity
x, y	non-dimensional distances along and perpendicular to the plate, $(x', y')v_0/\nu$

Greek symbols

α	positive parameter

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β coefficient of volumetric expansion
ε small parameter $(<<1)$
κ thermal conductivity
ρ fluid density
σ electrical conductivity
τ skin-friction coefficient
μ fluid viscosity
ν fluid kinematic viscosity
θ non-dimensional temperature $=(T'-T_\infty)/(T'_w-T_\infty)$
ω non-dimensional free stream frequency of oscillation

Subscripts

p plate
w wall
∞ free stream condition

INTRODUCTION

The study of magnetohydrodynamic flows through porous medium is useful in plasma physics, geothermal applications, magnetohydrodynamic generators, exploration and thermal recovery of oil, and in astrophysics (Soundalgekar and Takhar, 1977). In the area of space technology and in processes involving high temperature phenomena (as in ionosphere-plasma region, hypersonic flight, missile reentry, rocket combustion chambers, and power plants for interplanetary flight, gas-cooled nuclear reactors, and interstellar environment) the effects of radiation become significant (Ganessan and Loganathan, 2002). Researches on the flow of an electrically conducting incompressible viscous fluids with interaction of thermal radiation and free/forced convection have been carried out in the past (Takhar *et. al.*,1996; Bestman and Adjepong, 1988; Bestman, 1985; Alagoa *et. al.*, 1999). Takhar *et. al.* (1996). investigated the problem of a radiating gas past a semi-infinite vertical plate in an optically thin gas limit under the effects of buoyancy and magnetic field using similarity techniques. Alagoa *et. al.* (1999) studied the effects of radiation, free-convection and magnetic field on magnetohydrodynamic flow past parallel plates with time-dependent suction in a transparent medium. Israel-Cookey and Sigalo (2003) combined the effects of magnetic field, radiation free-convection and frequency to the study of a hydromagnetic fluid past a semi-infinite vertical porous plate with time-dependent suction in an optically thin environment when the plate is stationary. More recently, Israel-Cookey *et. al.* (2003) investigated the problem of unsteady MHD free-convection flow past an infinite heated vertical plate with time-dependent suction in a porous optically thin environment under the influence of viscous dissipation and radiation by imposing an oscillatory time-dependent perturbation.

In this present study, we investigate the problem of oscillatory MHD flow past an infinite moving heated porous vertical plate with time-dependent suction in the presence of a constant transverse magnetic field when the free stream velocity oscillates in time about a constant mean velocity. Also, we assume that both the suction velocity and the surface temperature of the plate oscillate periodically in time with the same frequency; while the working fluid is considered to be optically thin, absorbing/emitting but non scattering and perfectly electrically conducting. Finally, the porous heated plate is assumed to be non-reflecting, non-absorbing, ideally transparent and electrically con-ducting.

MATHEMATICAL FORMULATION

We consider two–dimensional, unsteady flow of an incompressible, electrically conducting fluid past an infinite moving heated porous vertical plate with pressure gradients and variable suction and permeability under the influence of a constant transverse magnetic field, H'_0. The induced magnetic field and magnetic Reynolds number are very small and hence are neglected . Also, we assume that there is no electric field on the account that the porous heated plate is perfectly conducting. The physical model and the coordinate system of the problem are shown in Fig.1.

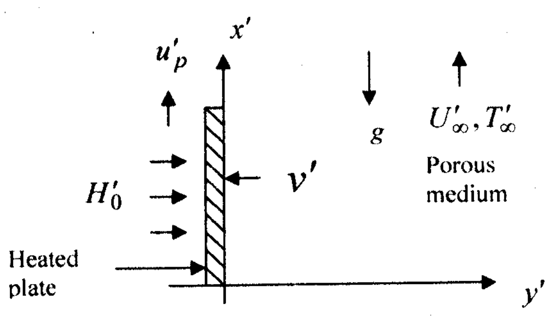

Fig.1: The schematic diagram of the physical model and Coordinate system

We take the x'-axis along the porous vertical plate in the upward direction and y'-axis normal to it. Initially the plate is maintained at wall temperature, T'_w which is high enough so that radiative heat is significant, while the fluid is sucked from the plate with time-dependent suction velocity v'. Under these conditions the governing equations for continuity, momentum and energy are

$$\frac{\partial v'}{\partial y'}=0 \tag{1}$$

$$\rho\left(\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}\right)=-\frac{\partial p'}{\partial x'}+\mu\frac{\partial^2 u'}{\partial y'^2}+\rho g\beta(T'-T_\infty)-\left(\frac{\mu}{K'}+\sigma H_0'^2\right)u' \tag{2}$$

$$c_p\rho\left(\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right)=\kappa\frac{\partial^2 T'}{\partial y'^2}-\frac{\partial q'_y}{\partial y'} \tag{3}$$

where u' and v' are the velocity components in the x' and y' directions respectively, t' is the time, ρ the density of the fluid, p' the pressure, μ the fluid viscosity, K' the permeability of the porous medium, g the acceleration due to gravity, σ the electrical conductivity, H'_0 the magnetic field, c_p the specific heat capacity, κ the thermal conductivity and q'_y the radiative heat flux.

Now, for applications in space science and at other high-operating temperature environments as one would have in interstellar medium, solar collectors, ionosphere (plasma region) and radiative equilibrium stratosphere, the absorption of radiation by the atmospheric gases is essentially non-gray (optically thin) to all infrared (of all) wavelengths (Pujol. 2002). The analysis of radiative heat flux is complicated and is usually represented by integro-differential equations. However, for optically thin limit, Cogley *et. al.* (1968) and Grief *et. al.* (1971) showed that for the fluid which does not absorb its own emitted radiation, but absorbs emitted radiation by the boundaries, the following relation holds:

$$\frac{\partial q'_y}{\partial y'}=4\alpha^2(T'-T_\infty) \tag{4a}$$

where

$$\alpha^2=\int_0^\infty \frac{\partial B_1}{\partial T'}d\lambda \tag{4b}$$

and B_1 is the Planck's function and λ the frequency of radiation. With Eq. (4) the energy equation (Eq.3) becomes

$$c_p\left(\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right)=\kappa\frac{\partial^2 T'}{\partial y'^2}-4\alpha^2(T'-T_\infty) \tag{5}$$

The appropriate boundary conditions are

$$u'=u'_p,\ T'=T_w+\varepsilon(T_w-T_\infty)e^{i\omega' t'} \quad \text{on } y'=0 \tag{6a}$$

$$u'\to U'_\infty=U_0(1+\varepsilon e^{i\omega' t'}),\ T'\to T_\infty \quad \text{as } y'\to\infty \tag{6b}$$

Further, from the continuity equation (Eq. 1), it is clear that the suction velocity is a function of time only and in the spirit of Pop and Soundalgekhar (1975), we assume

$$v'=-v_0(1+\varepsilon B e^{i\omega' t'}) \tag{7}$$

where $v_0>0$ is the mean suction velocity and $B>0,\ \varepsilon>0$ are small parameters such that $\varepsilon B<<1$. Now, outside the boundary layer the pressure gradient in the momentum equation (i.e. Eq. 2) yields

$$-\frac{1}{\rho}\frac{dp'}{dx'}=\frac{dU'_\infty}{dt'}+\frac{\nu}{K'}U'_\infty+\frac{\sigma H_0'^2}{\rho}U'_\infty \tag{8}$$

where $\nu=\dfrac{\mu}{\rho}$ is the kinematic viscosity and U'_∞ is the free stream velocity. Now, it is convenient to introduce the following non-dimensional variables

$$u' = U_0 u,\ v' = v_0 v,\ y' = \frac{\nu}{v_0} y,\ U'_\infty = U_0 U_\infty,\ u'_p = U_0 U,\ t' = \frac{\nu}{v_0^2} t,\ \omega' = \frac{v_0^2}{4\nu}\omega$$

$$\theta = \frac{(T' - T_\infty)}{T'_w - T_\infty},\ K = \frac{v_0^2}{\nu^2} K',\ \Pr = \frac{\nu\rho c_p}{\kappa},\ Gr = \frac{\nu g\beta(T'_w - T_\infty)}{U_0 v_0^2},\ R = \frac{4\alpha^2 \nu}{\rho c_p v_0^2}, \tag{9}$$

$$M = \frac{\nu\sigma H_0'^2}{\rho v_0^2}$$

In view of Eqs.(7)– 9) Eqs. (2), (5) and the boundary conditions (6) reduce to

$$\frac{\partial u}{\partial t} - (1 + \varepsilon B e^{i\omega t})\frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + M(U_\infty - u) + \frac{1}{K}(U_\infty - u) \tag{10}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon B e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - R\theta \tag{11}$$

subject to the boundary conditions

$$\begin{aligned} &u = U,\ \theta = 1 + \varepsilon e^{i\omega t} \quad \text{on } y = 0 \\ &u \to U_\infty = 1 + \varepsilon e^{i\omega t},\ \theta \to 0 \quad \text{as } y \to \infty \end{aligned} \tag{12}$$

The mathematical statement of the problem is now complete and embodies the solution to Eqs. (10)-(11) subject to **boundary** conditions (12).

METHOD OF SOLUTION

The problem as posed in Eqs. (10) and (11) subject to condition (12) are highly nonlinear coupled **partial differential** equations and hence are not easily amenable to analytical treatment. However, since $\varepsilon << 1$, we adopt **asymptotic expansion for** the flow velocity and temperature in the form

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) \tag{13}$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{i\omega t}\theta_1(y) \tag{14}$$

Substitution Eqs. (13) and (14) into Eqs. (10)–(12) and neglecting the coefficient of $O(\varepsilon^2)$ we obtain **the sequence of** approximations

$$u_0'' + u_0' - Nu_0 = -N - Gr\theta_0 \tag{15}$$

$$\theta_0'' + \Pr\theta_0' - R\Pr\theta_0 = 0 \tag{16}$$

subject to

$$\begin{aligned} &u_0 = U,\ \theta_0 = 1 \quad \text{on } y = 0 \\ &u_0 \to 1,\ \theta_0 \to 0 \quad \text{as } y \to \infty \end{aligned} \tag{17}$$

where $N = M + \frac{1}{K}$ for $O(1)$ equations, and

$$u_1'' + u_1' - (N + i\omega)u_1 = -(N + i\omega) - Bu_0' - Gr\theta_1 \tag{18}$$

$$\theta_1'' + \Pr\theta_1' - \Pr(R + i\omega)\theta_1 = -B\Pr\theta_0' \tag{19}$$

subject to the boundary conditions

$$\begin{aligned} &u_1 = 0,\ \theta_1 = 1 \ \text{ on } y = 0 \\ &u_1 \to 1,\ \theta_1 \to 0 \ \text{ as } y \to \infty \end{aligned} \tag{20}$$

for $O(\varepsilon)$ equations.

The solutions for the velocity and temperature profiles are obtained by first solving the $O(1)$-equations **and then** superimposing into the $O(\varepsilon)$-equations. They are given respectively

$$u(y,t) = 1 + (U - c_1 - 1)e^{-m_2 y} + c_1 e^{-m_1 y} + \varepsilon e^{i\omega t}\left(1 + c_3 e^{-m_4 y} + c_4 e^{-m_2 y} + c_5 e^{-m_1 y} + c_6 e^{-m_3 y}\right) \tag{21}$$

$$\theta(y,t) = e^{-m_1 y} + \varepsilon e^{i\omega t}\left((1 - c_2)e^{-m_3 y} + c_2 e^{-m_1 y}\right) \tag{22}$$

where

$$m_1 = \frac{1}{2}\left(\Pr + \sqrt{\Pr^2 + 4\Pr R}\right),$$

$$m_2 = \frac{1}{2}\left(1 + \sqrt{1 + 4N}\right),$$

$$m_3 = \frac{1}{2}\left(\Pr + \sqrt{\Pr^2 + 4\Pr(R + i\omega)}\right)$$

$$m_4 = \frac{1}{2}\left(1 + \sqrt{1 + 4(N + i\omega)}\right),$$

$$c_1 = \frac{-Gr}{m_1^2 - m_1 - N},$$

$$c_2 = \frac{m_1 B \Pr}{m_1^2 - \Pr m_1 - \Pr(R + i\omega)}$$

$$c_3 = -(1 + c_4 + c_5 + c_6),$$

$$c_4 = \frac{B m_2 (u_p - c_1 - 1)}{m_2^2 - m_2 - N - i\omega},$$

$$c_5 = -\frac{m_1 (Gr c_2 - B c_1)}{m_1^2 - m_1 - N - i\omega},$$

$$c_6 = \frac{Gr m_3 (1 - c_2)}{m_3^2 - m_3 - N - i\omega},$$

Now, with the velocity and temperature profiles given in Eqns. (21) and (22), respectively, we compute the skin friction (τ) and the surface heat transfer (*H*) are respectively given as:

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = (1 - U + c_1)m_2 - c_1 m_1 - \varepsilon e^{i\omega t}(c_3 m_4 + c_4 m_2 + c_5 m_1 + c_6 m_3) \qquad (23)$$

$$H = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = -m_1 - \varepsilon e^{i\omega t}\left(m_3(1 - c_2) + m_1 c_2\right) \qquad (24)$$

RESULTS AND DISCUSSION

In the previous sections, we have formulated and solved the problem of the effect of radiation on oscillatory MHD free-convection flow and heat transfer past an infinite vertical porous plate with time-periodic surface temperature and variable suction in the presence of a transverse magnetic field. By invoking the optically thin differential approximation for the radiative heat transfer and a sinusoidal time-dependent perturbation, we obtain the expressions for the flow velocity and temperature; as well as the expressions for the surface skin-friction and surface heat transfer.

In order to understand the physical situation of the problem, we have computed the numerical values of the velocity, temperature, surface skin-friction and surface heat transfer using the software *Mathematica*. The boundary condition $y \to \infty$ is approximated by $y \approx 4$ which is sufficiently large for the velocity to approach the relevant free-stream velocity. In addition, the Prandtl number, $\Pr = 0.71$ which corresponds to air and various values of the material parameters are used.

The temperature distributions against the boundary layer, y for different values of radiation parameter, R are presented in Fig. 2.

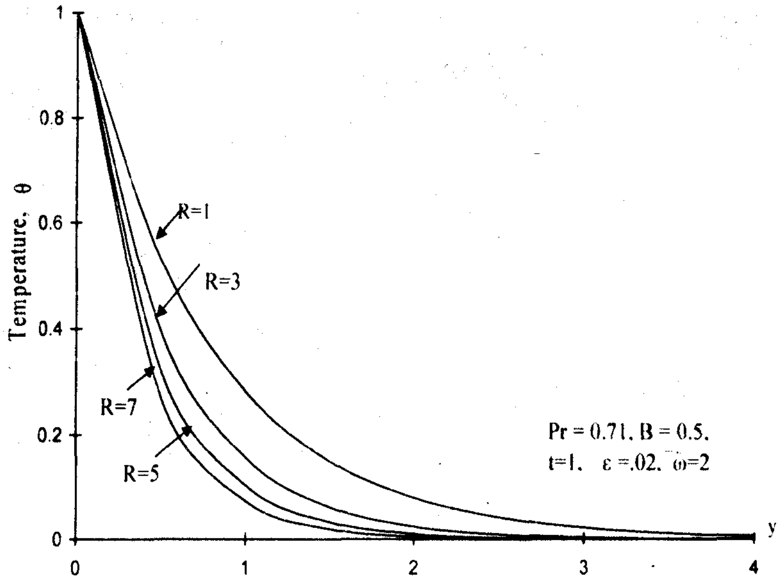

Fig. 2. Temperature profiles against the boundary layer, y for different material parameters

It is observed that the temperature of the plasma decays rapidly away from the plate with increase in radiation. Physically speaking, radiation has the tendency to reduce the temperature of the fluid. This result is in good qualitative agreement with the earlier results of Israel-Cookey *et. al.* (2003).

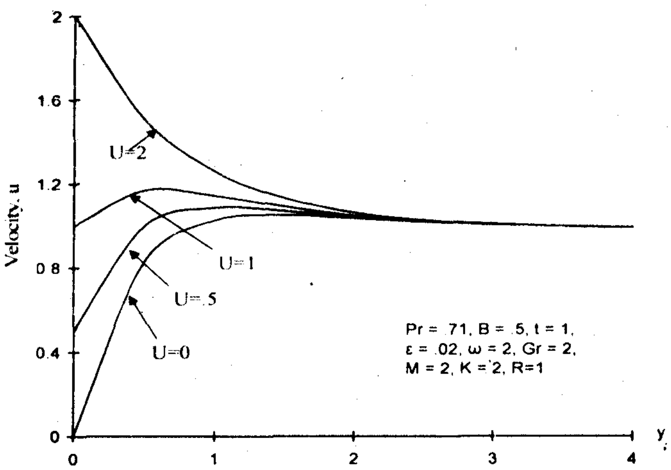

Fig. 3. Velocity profiles against the boundary layer, y for different values of plate velocity

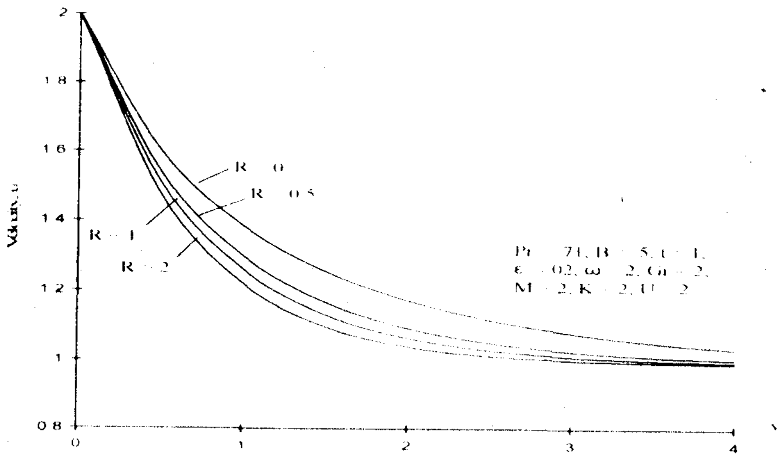

Fig. 4 Effect of radiation parameter, R on the velocity profiles against the boundary layer, y

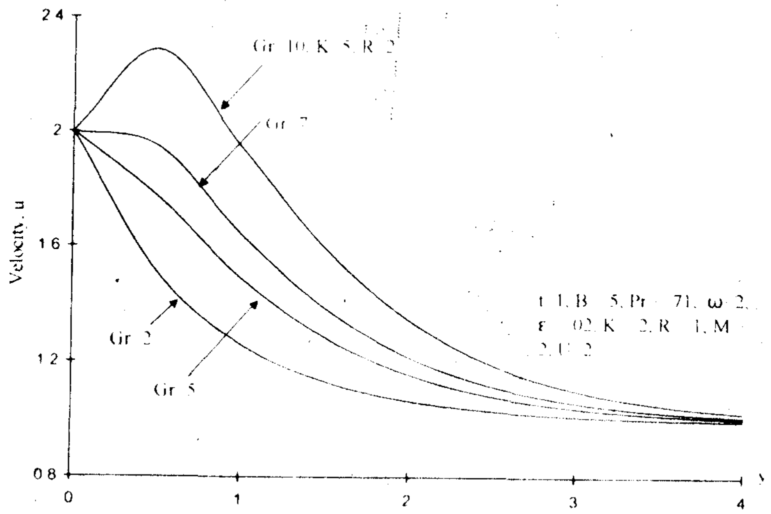

Fig.5 Effect of Grashoff number, Gr on the velocity profiles against the boundary layer, y

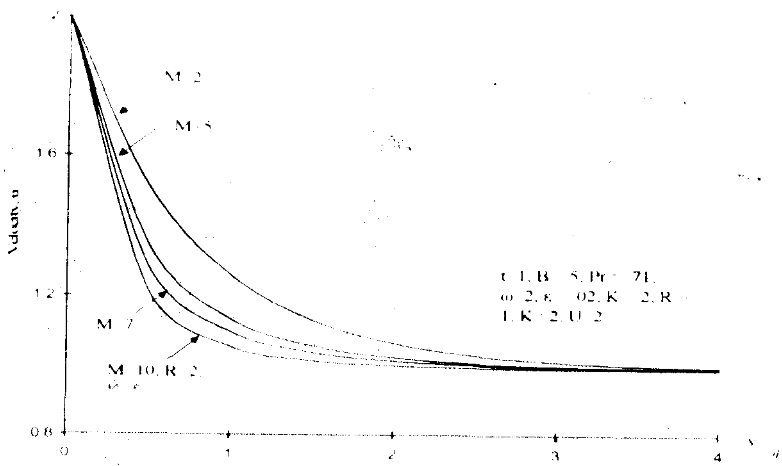

Fig. 6 Effect of magnetic field parameter, M on the velocity profiles against the boundary layer, y

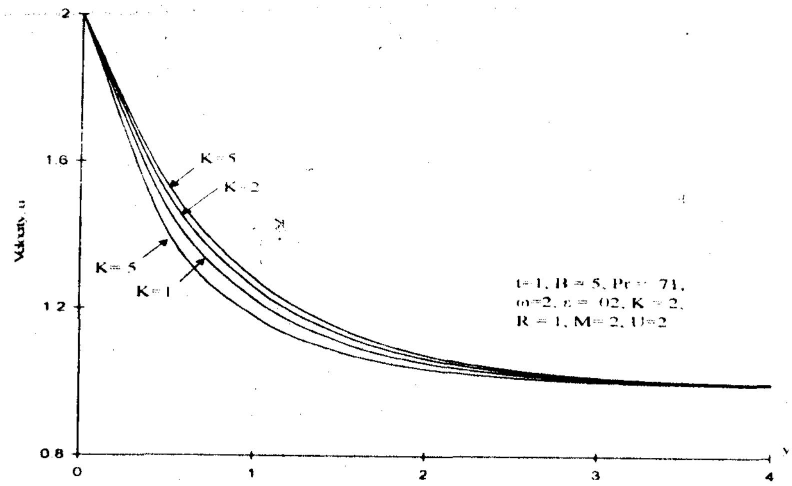

Fig 7 Effect of porosity parameter, K on the velocity profiles against the boundary layer, y

In Figs. 3-7, we presented some of the behaviour of the velocity distributions for various values of material parameters U, Gr, R, M and K against the boundary layer. It is observed from Fig. 3 that when the plate velocity lies between 0 and 2, the velocity rose steadily and converged close to the free stream velocity; while for $U \geq 2$, the velocity decayed steadily to the relevant free stream velocity at about $y = 2$. Also, increase in the plate velocity is associated with increase in the fluid velocity. These results are in good qualitative agreement with earlier results of Israel-Cookey *et. al.* (2003) and Kim (2001).
Figs. 4 and 5 demonstrates that while cooling the plate through convection currents separate increases in the material parameters *R* and *M* resulted in decrease in the velocity profiles. Also, Figs. 6 and 7, display the effects of porosity parameter, *K* and Grashof number, *Gr*, on the velocity profiles at $t = 1$, respectively. It is observed that increased values of *K* and *Gr* are associated in rise in the fluid velocity. Furthermore, simultaneous increases in the parameters *M*, *R*, and *K* resulted in decrease in fluid velocity (see Fig. 5); while simultaneous increases in *Gr*, *K* and *R* are associated with increase in fluid velocity. These results are in good qualitative agreement with earlier results of Israel-Cookey *et. al.* (2003) and Kim (2001)
In Table 1,

Table 1: Effect of radiation on the surface heat transfer, Nu for
$Pr = 0.71, B = 0.5, t = 1, \varepsilon = .2, \omega = 2$

R	Nu
1	1.057
3	1.641
5	5.050
7	2.370

We depict the values of the surface heat transfer, *Nu* for $Pr = 0.71, B = 0.5, t = 1, \omega = 2, \varepsilon = 0.2$, and various values of radiation parameter, *R*. It is observed that surface heat transfer increases with increasing radiation parameter up to R=5, and then decreases rapidly at R= 7.

Table 2: Surface skin-friction, $|\tau|$ for various values of material parameters

$t = 1, Pr = 0.71, B = .5, \omega = 2, \varepsilon = .2$.

Gr	*M*	*K*	*R*	*U*	$\|\tau\|$
2	2	2	0	2	1.29863
			.5		1.46941
			1		1.55909
			2		1.67668
2	2	2	1	2	1.55909
5					1.57095
7					1.59001
10					1.60588
2	2	2	1	2	1.55909
	5				2.50568
	7				2.98578
	10				3.57392
2	2	.5	1	2	2.07993
		1			1.74709
		2			1.55909
		5			1.43715
2	2	2	1	0	2.75445
				2	4.55909
				5	7.97351
				10	17.6437

Table 2 depicts the effects of various material parameters on the surface skin-friction, $|\tau|$ for $Pr = 0.71, B = 0.5, t = 1, \omega = 2, \varepsilon = 0.2$. It is observed that separate increases in the parameters *Gr, R, M, K* and *U* resulted in increase in $|\tau|$. Again, these results agree qualitatively with the earlier results of Kim (2001).

CONCLUSIONS

In conclusion therefore, the oscillatory MHD flow and heat transfer in a porous medium over an infinite heated moving vertical porous plate under the influence of radiation with time-dependent suction is affected by the material parameters. The conclusions are as follows:

Increase in radiation parameter led to decrease in the temperature and velocity profiles

Increase in radiation and magnetic parameters led to decrease in velocity distributions

Increase in velocity of the plate, Grashof number(convection current) and porosity parameter lead to increase in velocity distributions

Increase in radiation parameter is associated with increase in the rate of heat transfer

Increase in moving plate velocity, radiation and magnetic field parameters led to increase in the absolute value of the surface skin-friction; whereas increased porosity resulted in decrease of the absolute value of the skin-friction

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