

INFLUENCE OF RADIATION ON UNSTEADY MHD FREE-CONVECTION FLOW OF POLAR FLUIDS PAST A CONTINUOUSLY MOVING HEATED VERTICAL PLATE IN A POROUS MEDIUM

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ABSTRACT

This paper investigates the influence of radiation on the unsteady MHD flow of polar fluids past a heated continuously moving vertical plate in a porous medium with time-dependent suction, in the presence of a horizontal magnetic field. The unsteadiness in the flow is characterized by an oscillatory perturbation in the flow of order $\varepsilon \exp[i\omega t]$, where ε is a small positive parameter and ω is the free stream frequency of the system. Analytical solutions are obtained using an asymptotic expansion for small ε , and adopting Rosseland optically thick differential approximation for the radiative heat flux in the energy equation. The effects of various material parameters on the temperature, linear velocity and angular velocity distributions across the boundary layer are presented in graphical form. The results show that increases in radiation and the free stream frequency lead to a decrease in the temperature. Also, increased cooling of the plate ($Gr > 0$), and porosity parameter lead to a rise in the velocity profile; while increases in radiation, and magnetic field are associated with decrease in the velocity. In addition, the effects of material parameters show a lower velocity value for the viscosity ratio, $\beta_r = 0$ when compared with $\beta_r \leq 0.5$; whereas the velocity decreases with increasing values of $\beta_r (> 0.5)$. Also, increased magnetic field and cooling of the plate lead to a decrease in the angular velocity; while increase in radiation shows a slight growth in the angular velocity. Finally, the results show that the temperature, velocity and angular velocity profiles grew with time.

KEYWORDS: Rosseland approximation, polar fluids, porous medium, oscillatory perturbation, continuously moving plate

Nomenclature

- A suction velocity parameter
 c_p specific heat at constant pressure
 Gr Grashof number = $\nu g \beta (T_w^* - T_\infty^*) / (U_0 v_0^2)$
 g acceleration due to gravity
 K permeability of the porous medium = $v_0^2 K' / \nu^2$
 M magnetic field parameter = $\nu \sigma H_0^2 / (\rho \nu_0^2)$
 N non-dimensional material parameter = $M + 1/K$
 q_z^* radiative heat flux
 R radiation parameter = $k^* \kappa / (4\sigma^* T_\infty^*)$
 Pr Prandtl number = $\nu \rho c_p / \kappa$
 t non-dimensional time = $v_0^2 t' / \nu$
 U_0 scale of the free stream
 u, v non-dimensional velocities along and perpendicular to the plate, $(u', v') / U_0$
 u_p plate velocity = u'_p / U_0
 v_0 scale of the suction velocity
 x, z non-dimensional distances along and perpendicular to the plate, $(x', y') v_0 / \nu$
 Greek symbols
 β coefficient of volumetric expansion

β_r	Viscosity ratio = ν_r / ν
ε	small parameter ($\ll 1$)
κ	thermal conductivity
γ	spin-gradient viscosity
η	constant depending on β_r
ρ	fluid density
σ_c	electrical conductivity
σ_r	angular velocity
μ	fluid viscosity
ν	fluid kinematic viscosity
ν_r	kinematic rotational viscosity
θ	non-dimensional temperature = $(T' - T_\infty) / (T_w' - T_\infty)$
ω	non-dimensional free stream frequency of oscillation

Subscripts

p	plate
r	rotation
w	wall
∞	free stream condition

INTRODUCTION

The theory of micropolar fluids could be used to explain the flow behaviour of non-Newtonian fluids such as colloidal fluids and fluids with suspensions, for example. This theory first formulated by Eringen (1966) deals with a class of viscous fluids consisting of assemblage of microstructures with rigid and spherical or randomly oriented particles. It has received a lot of interest by many researchers and excellent reviews on the properties and phenomena may be found in literature (Ariman *et al.* 1973, 1974; Aero *et al.* 1965; Shenoy and Mashekar, 1982; Elbashbeshy and Bazid, 2000). In particular, Elbashbeshy and Bazid (2000) considered the problem of heat transfer over a continuously moving plate embedded in a non-Darcian porous medium using numerical methods and show that the exponential parameters in the velocity and heat flux functions influence the heat transfer coefficients.

Ahmadi (1976) considered the effect of the gyration vector normal to the xy - plane and micro-inertia on the boundary layer of a micropolar fluid past a stationary semi-infinite plate, while Raptis (1988) investigated the problem of the same nature past a continuously moving plate in the presence of radiation.

Recently, attention have been focused on the flow and heat transfer of electrically conducting viscous polar fluids past a semi-infinite stationary or moving porous plate in the presence of magnetic field as this could find useful applications in geophysics, oil, reservoir engineering, MHD generators, astrophysics and plasma physics (Soundalgekhkar and Takhar, 1977; Kim, 2001). Kim (2001) investigated an unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting polar fluid in a porous medium past a semi-infinite vertical porous moving plate in the presence of a transverse magnetic field with suction.

Most of the previous studies of the same problem neglected the effects of radiation. However, in high temperature regions such area as the opaque galaxy, the effects of radiation cannot be ignored. This present study is an attempt to complement the work of Kim (2001) by investigating the influence of radiation on the problem of unsteady magnetohydrodynamic flow of polar fluids past a heated continuously moving vertical plate in a porous medium with oscillating time-dependent suction in an optically thick environment. This is an attempt at widening the applicability of the problem of this nature.

MATHEMATICAL FORMULATION

We consider an unsteady two-dimensional flow of an incompressible viscous electrically conducting polar fluid past a continuously moving heated porous plate in a porous medium in the presence of a horizontal magnetic field and radiative heat transfer. Assuming that the magnetic Reynolds number is very small, the induced magnetic field can be neglected. Also, the porous medium is considered to consist of an assemblage of small identical spherical particles so that the medium is that of an optical thickness fixed in space.

The x^* -axis is chosen along the vertical porous plate and the z^* -axis normal to it. At time $t^* = 0$, the temperature of the plate, T^* which is high enough to initiate radiative heat, is described as an oscillatory function of t^* . The plate moves in the vertical upward direction with velocity U_p^* , while a constant magnetic field of strength H_0^* is applied in the z^* -direction.

Under these assumptions and Boussinesq approximation, and following Kim [10] the governing equations for continuity, linear momentum, angular momentum, and energy are:

$$\frac{\partial v^*}{\partial z^*} = 0 \quad (1)$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial z^*} = (v + v_r) \frac{\partial^2 u^*}{\partial z^{*2}} + \frac{dU_\infty^*}{dt^*} - \left(\frac{\nu}{K^*} + \frac{\mu^2 \sigma_c H_0^{*2}}{\rho} \right) (u^* - U_\infty^*) + g\beta(T^* - T_\infty) + 2v_r \frac{\partial \sigma_r^*}{\partial z^*} \tag{2}$$

$$\rho \rho_1 \left(\frac{\partial \sigma_r^*}{\partial t^*} + v^* \frac{\partial \sigma_r^*}{\partial z^*} \right) = \gamma \frac{\partial^2 \sigma_r^*}{\partial z^{*2}} \tag{3}$$

$$\rho c_p \left(\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial z^*} \right) = \kappa \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{\partial q_z^*}{\partial z^*} \tag{4}$$

subject to the following boundary conditions

$$u^* = U_p^*, T^* = T_w + \varepsilon(T_w - T_\infty)e^{i\omega t^*}, \frac{\partial \sigma_r^*}{\partial z^*} + \frac{\partial^2 u^*}{\partial z^{*2}} = 0 \quad \text{on } z^* = 0 \tag{5a}$$

$$u^* \rightarrow U_0(1 + \varepsilon e^{i\omega t^*}), T^* \rightarrow T_\infty, \sigma_r^* \rightarrow 0 \quad \text{as } z^* \rightarrow \infty \tag{5b}$$

Since the fluid is considered to be grey, absorbing-emitting and radiating in a non-scattering medium, the radiative heat flux, q_z^* , in the energy equation (4) can be approximated by Rosseland optically thick approximation in the form (Pomraning, 1973; Sparrow and Cess, 1978)

$$q_z^* = -\frac{4\sigma^*}{3k^*} \frac{\partial T^{*4}}{\partial z^*} \tag{6}$$

where σ^* is the Stefan-Boltzmann constant and k^* the mean absorption coefficient. Now, assuming that the temperature differences within the flow environment are sufficiently small, then T^{*4} may be expressed as a linear function of the temperature. By employing Taylor's series expansion about the reference temperature, T_∞ and neglecting higher order terms, we obtain

$$T^{*4} \cong 4T_\infty^3 T^* - 3T_\infty^4 \tag{7}$$

Further, from the continuity equation (1), the suction velocity is normal to the plate, and so we assume it to be a function of time, t^* only:

$$v^* = -v_0(1 + \varepsilon A e^{i\omega t^*}) \tag{8}$$

such that $\varepsilon A \ll 1$, where A is a small positive parameter and v_0 is a dimensional suction velocity.

It is now convenient to define the following non-dimensional variables: $u = \frac{u^*}{U_0}, v = \frac{v^*}{v_0}, z = \frac{v_0 z^*}{\nu}$

$$U_\infty = \frac{U_\infty^*}{U_0}, u_p = \frac{u_p^*}{U_0}, t = \frac{v_0^2 t^*}{\nu}, \sigma_r = \frac{\nu \sigma_r^*}{v_0 U_0}, \omega = \frac{\nu \omega^*}{v_0^2}, \theta = \frac{(T^* - T_\infty)}{(T_w - T_\infty)}, K = \frac{v_0^2 K^*}{\nu^2}, \rho_1 = \frac{v_0^2 \rho_1^*}{\nu^2},$$

$$Pr = \frac{\nu \rho c_p}{\kappa}, \eta = \frac{\mu \rho_1^*}{\gamma} = \frac{2}{2 + \beta_r}, \beta_r = \frac{v_r}{\nu}, M = \frac{\sigma_c H_0^{*2}}{\rho \nu^2}, Gr = \frac{\nu \beta g (T_w - T_\infty)}{U_0 v_0^2}, R = \frac{k^* \kappa}{4\sigma^* T_\infty^3} \tag{9}$$

In view of eqs. (6) – (9), the governing eqs. (2) – (4) in non-dimensional form become

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial u}{\partial z} = \frac{dU_\infty}{dt} + (1 + \beta_r) \frac{\partial^2 u}{\partial z^2} + Gr\theta - N(u - U_\infty) + 2\beta_r \frac{\partial \sigma_r}{\partial z} \tag{10}$$

$$\frac{\partial \sigma_r}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \sigma_r}{\partial z} = \frac{1}{\eta} \frac{\partial^2 \sigma_r}{\partial z^2} \tag{11}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial z} = \frac{1}{\text{Pr}} \left(1 + \frac{4}{3R} \right) \frac{\partial^2 \theta}{\partial z^2} \quad (12)$$

where $N = \frac{1}{K} + M$ and the non-dimensional boundary conditions are:

$$u = u_p, \quad \theta = 1 + \varepsilon e^{i\omega t}, \quad \frac{\partial \sigma_r}{\partial z} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \text{on } z = 0 \quad (13a)$$

$$u \rightarrow U_\infty = 1 + \varepsilon e^{i\omega t}, \quad \theta \rightarrow 0, \quad \sigma_r \rightarrow 0 \quad \text{as } z \rightarrow \infty \quad (13b)$$

The mathematical statement of the problem is now complete and embodies the solution of Eqs. (10)-(12) subject to the boundary conditions (13).

METHOD OF SOLUTION

The problem as posed above in Eqs (10)-(12) subject to boundary conditions (13) are highly coupled equations, and generally require numerical integration. However, in the neighbourhood of the plate and since ε is a small positive parameter, we assume a regular perturbation for the flow variables of the form

$$u(z, t) = u_0(z) + \varepsilon e^{i\omega t} u_1(z) + O(\varepsilon^2) \quad (14a)$$

$$\sigma_r(z, t) = \sigma_0(z) + \varepsilon e^{i\omega t} \sigma_1(z) + O(\varepsilon^2) \quad (14b)$$

$$\theta(z, t) = \theta_0(z) + \varepsilon e^{i\omega t} \theta_1(z) + O(\varepsilon^2) \quad (14c)$$

Substituting Eqs. (14) into Eqs. (10)-(12) and the boundary conditions (13), neglecting the coefficients of $O(\varepsilon^2)$, we obtain the sequence of approximations

$$(1 + \beta_r) u_0'' + u_0' - N u_0 = -N - Gr \theta - 2\beta_r \sigma_0' \quad (15)$$

$$\sigma_0'' + \eta \sigma_0' = 0 \quad (16)$$

$$(4 + 3R) \theta_0'' + 3R \text{Pr} \theta_0' = 0 \quad (17)$$

subject to the conditions

$$u_0 = u_p, \quad \sigma_0' + u_0'' = 0, \quad \theta_0 = 1 \quad \text{on } z = 0 \quad (18a)$$

$$u_0 \rightarrow 1, \quad \sigma_0 \rightarrow 0, \quad \theta_0 \rightarrow 0 \quad \text{as } z \rightarrow \infty \quad (18b)$$

for $O(1)$ equations, and

$$(1 + \beta_r) u_1'' + u_1' - (N + i\omega) u_1 = -A u_0' - (N + i\omega) - 2\beta_r \sigma_1' - Gr \theta_1 \quad (19)$$

$$\sigma_1'' + \eta \sigma_1' - i\omega \eta \sigma_1 = -A \eta \sigma_0' \quad (20)$$

$$(4 + 3R) \theta_1'' + 3R \text{Pr} \theta_1' - 3i\omega \text{Pr} R \theta_1 = -3A \text{Pr} R \theta_0' \quad (21)$$

subject to the conditions

$$u_1 = 0, \quad \sigma_1' + u_1'', \quad \theta_1 = 0 \quad \text{on } z = 0 \quad (22a)$$

$$u_1 \rightarrow 0, \quad \sigma_1 \rightarrow 0, \quad \theta_1 \rightarrow 0 \quad \text{as } z \rightarrow \infty \quad (22b)$$

for $O(1)$ equations. In the above equations the primes denote differentiation with respect to z

Solving the $O(1)$ equations and substituting into the $O(\varepsilon)$ and using Eq. (14), we obtain linear velocity, angular velocity and temperature distributions of the flow respectively as:

$$u(z,t) = 1 + d_1 e^{-m_1 z} + d_2 e^{-\Gamma_1 z} + d_3 e^{-\eta z} + \varepsilon e^{i\omega t} \left(1 + \alpha_6 e^{-m_4 z} + \alpha_1 e^{-m_3 z} + \alpha_2 e^{-\Gamma_1 z} + \alpha_3 e^{-\eta z} + \alpha_4 e^{-m_2 z} + \alpha_5 e^{-m_1 z} \right) \quad (23)$$

$$\sigma_r(z,t) = c_1 e^{-\eta z} + \varepsilon e^{i\omega t} \left(c_2 e^{-m_3 z} - i \frac{A\eta}{\omega} c_1 e^{-\eta z} \right) \quad (24)$$

$$\theta(z,t) = e^{-\Gamma_1 z} + \varepsilon e^{i\omega t} \left(c_2 e^{-m_3 z} + i \frac{A\Gamma_1}{\omega} \left(e^{-\Gamma_1 z} - e^{-m_3 z} \right) \right) \quad (25)$$

where

$$m_1 = \frac{1}{2(1 + \beta_r)} \left(1 + \sqrt{1 + 4N(1 + \beta_r)} \right)$$

$$m_2 = \frac{\Gamma_1}{2} \left(1 + \sqrt{1 + \frac{4i\omega}{\Gamma_1}} \right)$$

$$m_3 = \frac{\eta}{2} \left(1 + \sqrt{1 + \frac{4i\omega}{\eta}} \right)$$

$$m_4 = \frac{1}{2(1 + \beta_r)} \left(1 + \sqrt{1 + 4(N + i\omega)(1 + \beta_r)} \right)$$

$$\Gamma_1 = \frac{3PrR}{4 + 3R}$$

$$d_1 = u_p - 1 + \frac{Gr}{(1 + \beta_r)\Gamma_1^2 - \Gamma_1 - N} - \frac{2\beta_r \eta c_1}{(1 + \beta_r)\eta^2 - \eta - N}$$

$$d_2 = \frac{Gr}{(1 + \beta_r)\Gamma_1^2 - \Gamma_1 - N}$$

$$d_3 = \frac{2\beta_r \eta c_1}{(1 + \beta_r)\eta^2 - \eta - N}$$

$$\alpha_1 = \frac{Am_1 d_1}{(1 + \beta_r)m_1^2 - m_1 - N - i\omega}$$

$$\alpha_2 = \frac{A\Gamma_1(d_2\omega - iGr)}{\omega((1 + \beta_r)\Gamma_1^2 - \Gamma_1 - N - i\omega)}$$

$$\alpha_3 = \frac{A\eta(d_3\omega + 2\beta_r \eta c_1)}{\omega((1 + \beta_r)\eta^2 - \eta - N - i\omega)}$$

$$\alpha_4 = \frac{2\beta_r c_2 m_3}{(1 + \beta_r)m_3^2 - m_3 - N - i\omega}$$

$$\alpha_5 = \frac{Gr(iA\Gamma_1 - \omega)}{\omega((1 + \beta_r)m_2^2 - m_2 - N - i\omega)}$$

$$\alpha_6 = -1 - \sum_{i=1}^5 \alpha_i$$

$$c_1 = \frac{(1 + \beta_r)\eta - 1 - \frac{N}{\eta}}{(1 + \beta_r)\Gamma_1^2 - \Gamma_1 - N} \times \frac{(u_p - 1)((1 + \beta_r)\Gamma_1^2 - \Gamma_1 - N)m_1^2 + Gr(m_1^2 - \Gamma_1^2)}{(1 - \beta_r)\eta^2 - \eta - N + 2\beta_r m_1^2}$$

$$c_2 = \frac{(1 + \beta_r)m_3 - 1 - \frac{(N + i\omega)}{m_3}}{(1 - \beta_r)m_3^2 - m_3 - N - i\omega + 2\beta_r m_4^2} \times \left(m_1^2 \alpha_1 + \Gamma_1^2 \alpha_2 + \eta^2 \alpha_3 + m_2^2 \alpha_5 - m_4^2 (1 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_5) - \frac{iA\eta^2 c_1}{\omega} \right)$$

RESULTS AND DISCUSSION

In the previous sections, we have formulated and solved the problem of the influence of radiation on the unsteady MHD free convection flow of polar fluids past a continuously moving heated vertical plate in a porous medium with time-dependent suction. By using an oscillatory perturbation series technique and adopting Rosseland optically thick differential approximation for the radiative heat transfer in the energy equation, analytical expressions for the velocity, angular velocity, and temperature profiles are obtained. These results are given in equations (23), (24) and (25) respectively. In the computations that follow, the software "mathematica" is employed, while the Prandtl number $Pr = 0.71$ which corresponds to air and various values of the material parameters are used. In addition, the boundary condition $z \rightarrow \infty$ is approximated by $z_{max} = 12$ (for the temperature and velocity profiles) and $z_{max} = 6$ (for the angular velocity) which are sufficiently large enough for the profiles to approach the relevant free stream profiles.

Figs. 1 – 3 show the temperature profiles for various material parameters and varying radiation, free stream frequency of oscillation and time, respectively. The results revealed that the temperature profiles within the fluid decayed away from the moving plate. In addition separate increases in radiation, frequency and time are associated with decrease in the temperature profile. These results are in good qualitative agreement with earlier results of Raptis (1988).

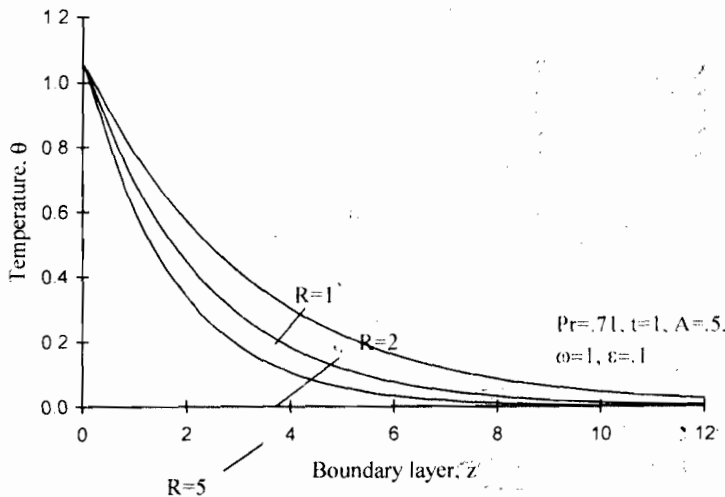


Fig. 1. Temperature profile against the boundary layer, z for different values of radiation parameter.

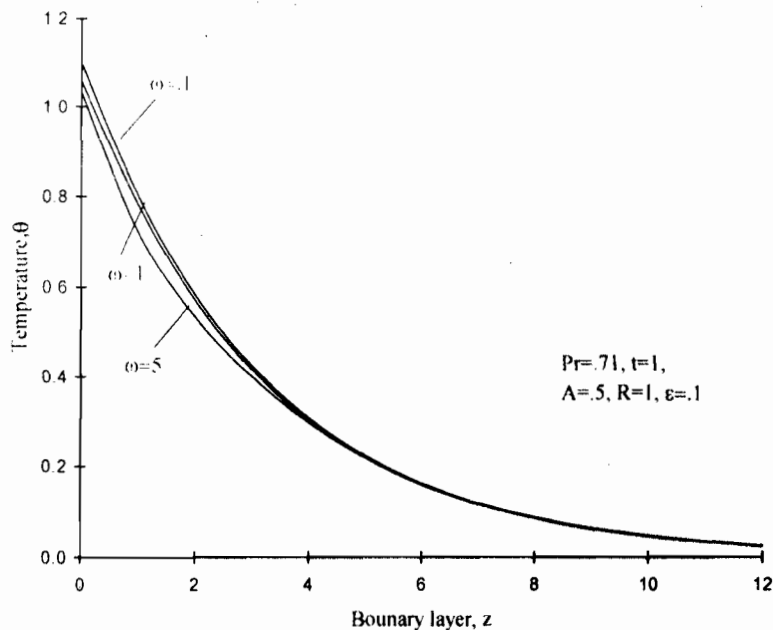


Fig. 2. Temperature profile against the boundary layer, z for different values of free stream frequency of oscillation.

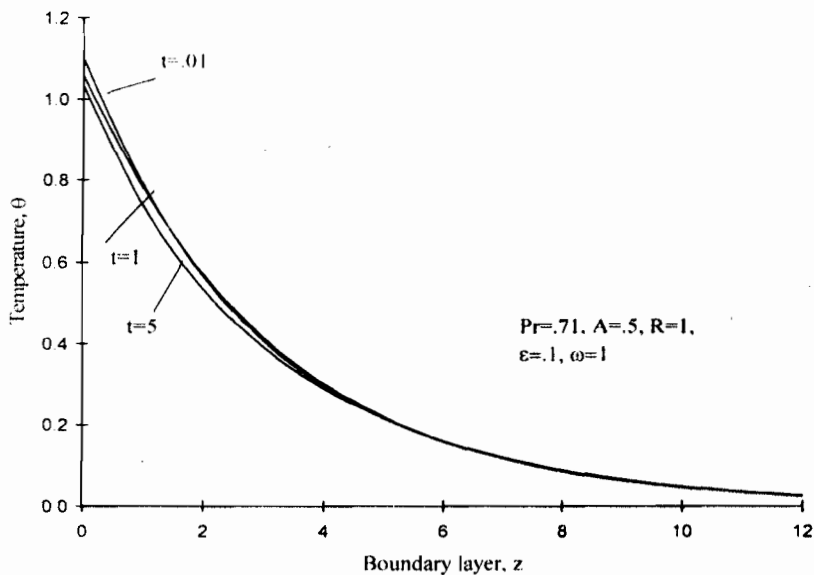


Fig.3. Temperature profile against the boundary layer, z at different times.

In Figs. 4 – 8, we show the behaviour of the velocity of the flow for different material parameters. It is observed that the velocity grew steadily and then converges to the relevant free stream velocity away from the moving plate. It is obvious that the velocity profile decreases with increase in radiation and magnetic field (see Figs. 4 – 5).

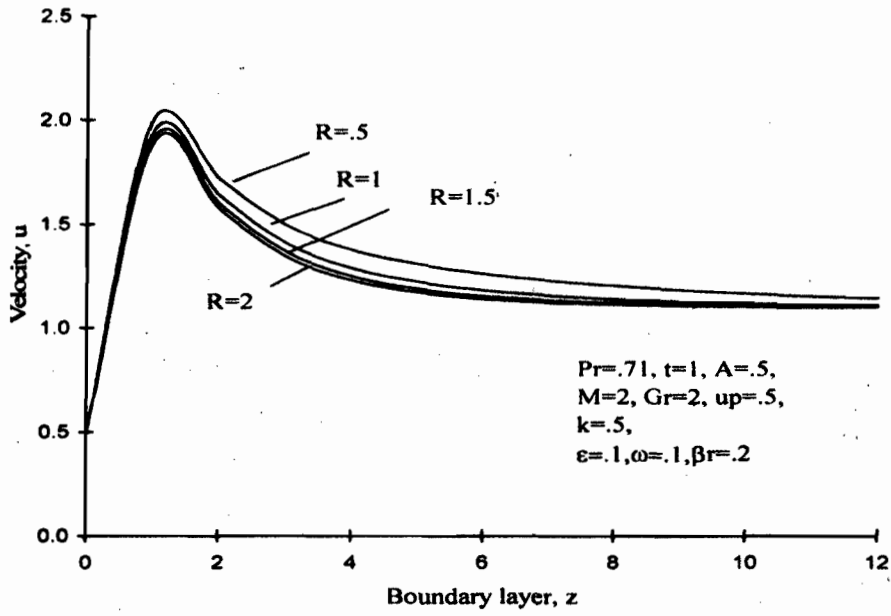


Fig4. Velocity Profile against the boundary layer, z for different values of radiation Parameters.

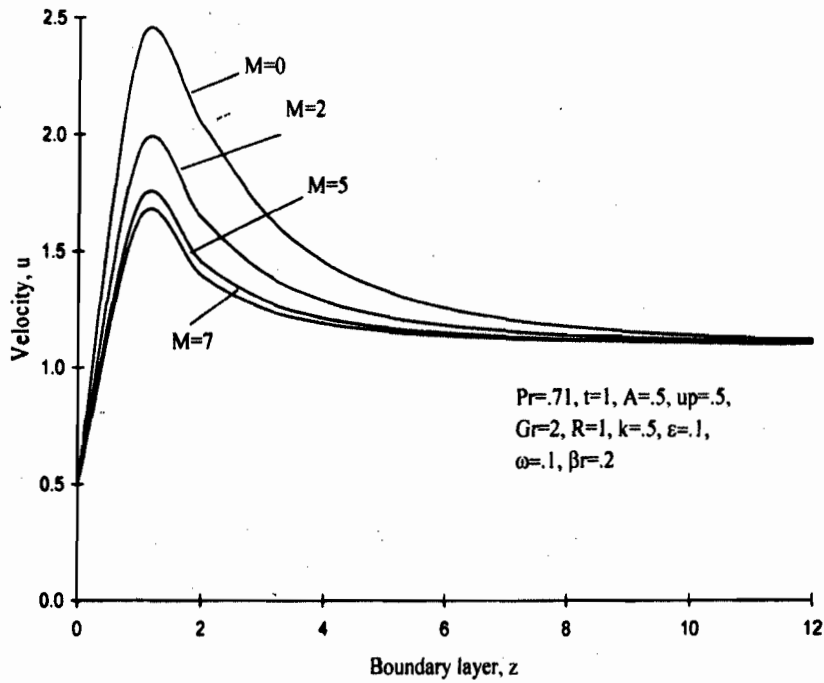


Fig.5. Velocity profile against the boundary layer, z for different values of magnetic field parameter.

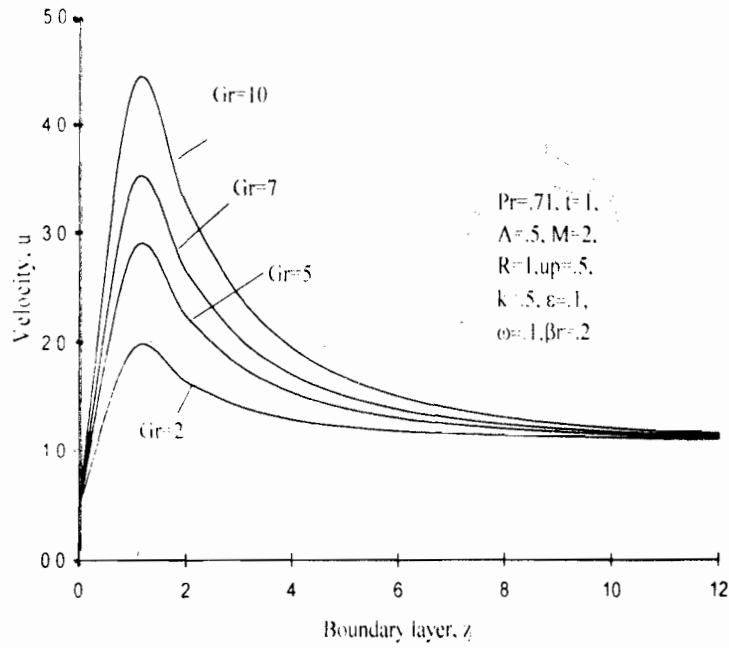


Fig. 6. Velocity profile against the boundary layer, z for different values of free convection parameter

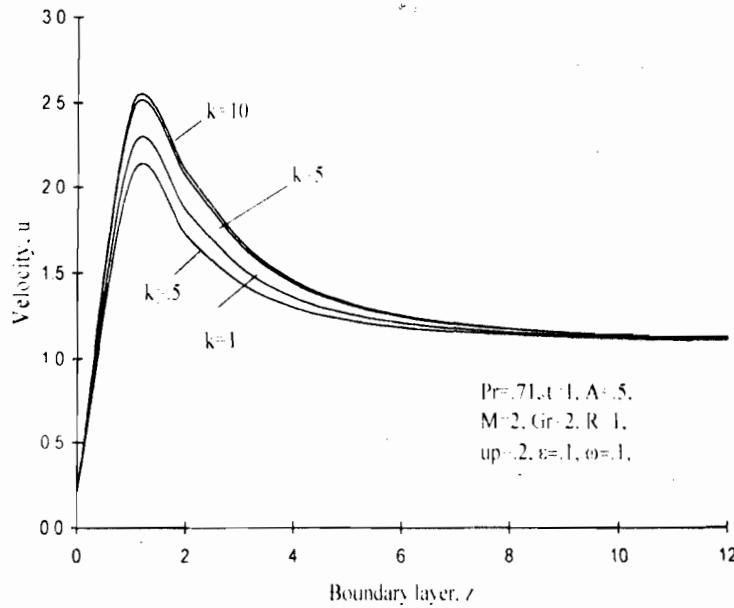


Fig. 7. Velocity profile against the boundary layer, z for different values of porosity parameter

We observed from Figs. 6 – 7 that greater cooling of the plate ($Gr > 0$) and increased porosity of the medium resulted in increase in the velocity. Also, the peak values of the velocity rose rapidly close to the moving plate. In the absence of radiation, these results are in good qualitative agreement with the earlier results of Kim (2001). In addition, the velocity profile grew with time (see Fig. 8).

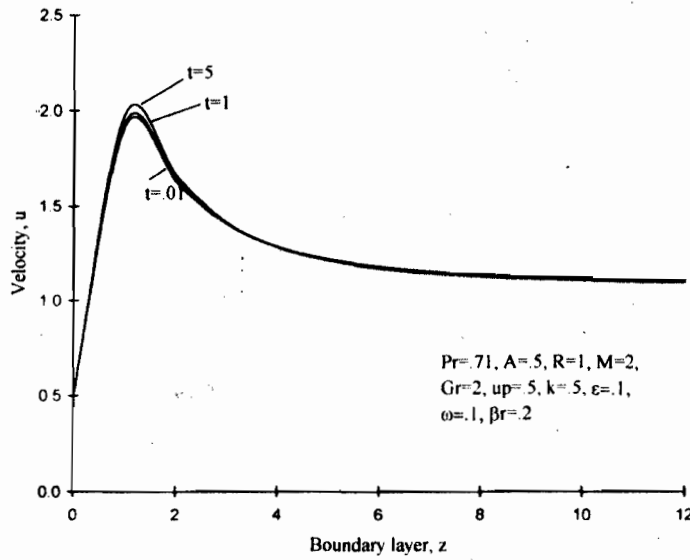


Fig. 8. Velocity profile against the boundary layer, z at different times.

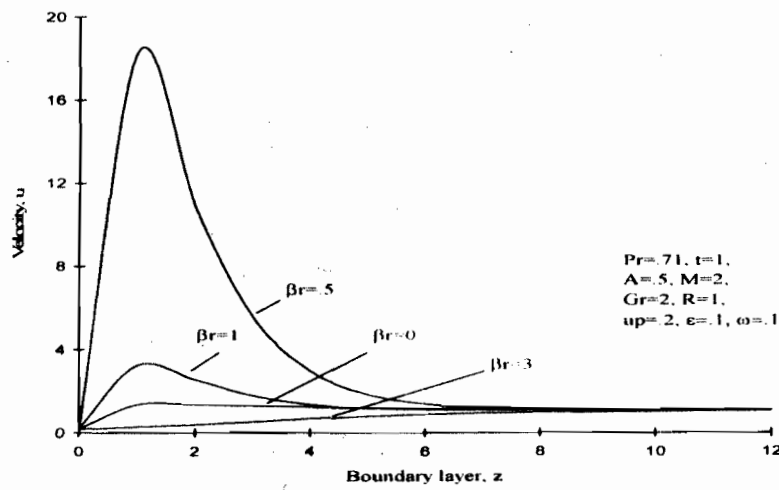


Fig. 9. Velocity profile against the boundary layer for different values of viscosity ratio.

Fig. 9 presents the effect of the viscosity ratio β_r on the velocity profile for the moving plate. The results show lower velocity values for $\beta_r = 0$ as compared to $\beta_r \leq 0.5$. However, the velocity decreased with increase in $\beta_r (> 0.5)$. Again, this is in qualitative agreement with earlier results of Kim (2001) in the absence of radiation.

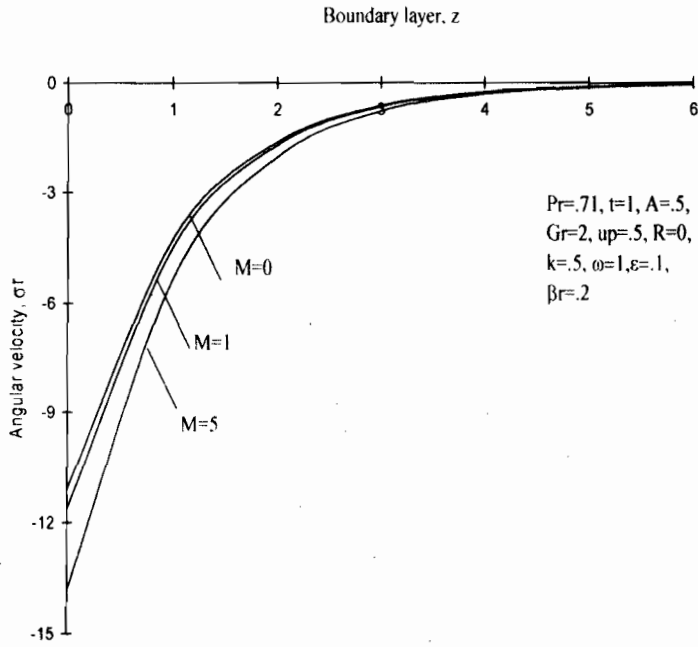


Fig.10 Angular velocity profile against the boundary layer for different values of magnetic parameter.

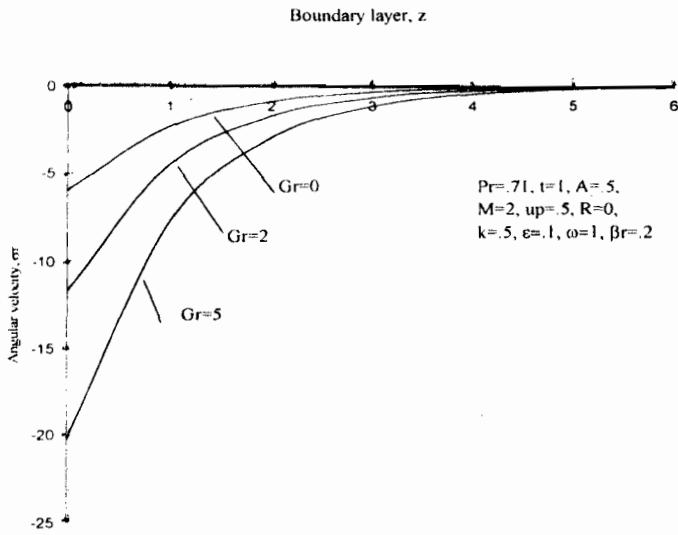


Fig.11 Angular velocity against the boundary layer, z for different values of free convection parameter

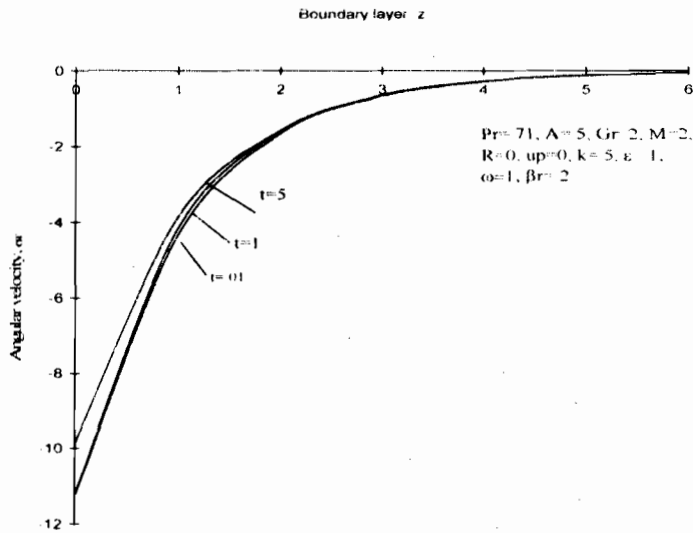


Fig 12 Angular velocity against the boundary layer at different times

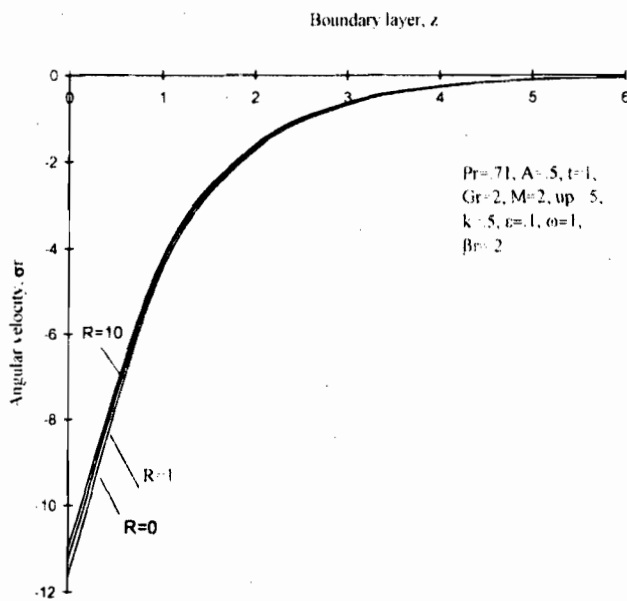


Fig 13 Angular velocity against the boundary layer for different values of radiation parameter

Figs 10 – 13 show the angular velocity distribution of the flow for various material parameters. It is observed that separate increases in the magnetic field and time led to increase in the angular velocity; while increased cooling of the plate is associated with a decrease in the angular velocity profile. Furthermore, increased radiation resulted in a slight growth in the angular velocity (see Fig. 13).

CONCLUSIONS

It has been demonstrated that radiation has some influence on the temperature, linear velocity and angular velocity on the unsteady magnetohydrodynamic flow of polar fluids past a heated continuously moving vertical plate in a porous medium with time-dependent suction, in the presence of a horizontal magnetic field and radiation. We present our conclusion as:

- Increase in radiation and free stream frequency led to a decrease in the temperature profile
- Increase in radiation and magnetic field are associated with decrease in velocity
- Increase in the porosity of the medium as well as the cooling of the plate lead to increase in velocity
- Increase in magnetic field and cooling of the plate lead to a decrease in angular velocity
- Increase in radiation is associated with a slight rise in angular velocity
- Temperature, velocity and angular velocity rose with time, and

- The velocity profiles show a lower velocity values for $\beta_r = 0$ as compared to $\beta_r \leq 0.5$.
However, the velocity decreased with increase in $\beta_r (> 0.5)$.

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