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NULL CONTROLLABILITY OF NON-LINEAR INFINITE DELAY SYSTEMS WITH IMPLICIT DERIVATIVE.

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ABSTRACT

We develop sufficient conditions for the null controllability of a non-linear infinite delay system with implicit derivative. Namely, if the uncontrolled system is uniformly asymptotically stable, and if the linear control system is controllable, then by placing appropriate conditions on the perturbation function f the non-linear perturbed system is null controllable with constraints.

KEYWORDS: Non linear systems, Null controllability, Delayed systems, Controllability matrix, Properness

1. INTRODUCTION

In recent years several authors (0Chukwu, 1979; Dauer, 1976; Hermes and Lasalle, 1969; Onwuatu ,1989) have studied the controllability of the linear system

$$\dot{x}(t) = A(t)x(t) + B(t)\,u(t) \tag{1.1}$$

and the delay system

$$\dot{x}(t) = L(t,x_t) + B(t)\,u(t) \tag{1.2}$$

and independent results obtained. Controllability studies have been extended to nonlinear systems of various kinds (with or without delays), (Klamka, 1978, 1980; Yamatoto, 1977) and results established from these studies.
Several authors Chukwu, 1980, 1987; Dauer and Gahl, 1977; Nse, 2005 have extended the concept of controllability to null controllability with the help of fixed point theorems.
In this paper, we shall consider the null controllability of the nonlinear infinite delay system with implicit derivative given by

$$\begin{aligned}\dot{x}(t) = L(t,x_t) + \int_{-\gamma}^{0} d_sH(t,s)u(t+s) + \int_{-\infty}^{0} A(s)x(t+s)ds +\\ f(t,x(t),\dot{x}(t),u(t))\end{aligned} \tag{1.3}$$

$$x(t) = \phi(t)$$

and develop sufficient conditions for the null controllability of (1.3) by placing condition on the perturbation function f which guarantee that if the linear control system is proper and the uncontrolled linear system uniformly asymptotically stable, then the system (1.3) is null controllable with constraints.

2. BASIC NOTATIONS AND PRELIMINARIES

Let E denote the real line and $J = [t_0, t_1]$ an interval in E. For a positive integer n, we denote by E^n the space of real n-tupples with the Euclidean norm (see Iheagwam, 2003) denoted by $|\cdot|$. Let $h \geq \gamma \geq 0$ be given real numbers (h may be $+\infty$) and the function $\eta: [-h,0] \to [0,\infty)$ be Lebesgue integrable positive and non decreasing on $[-h,0]$ (see Iheagwam, 2003). Let $B = B([-h,0], E^n)$ be the Banach space of functions which are continuous and bounded on $[-h,0]$ and such that $|\phi| = \sup_{s\in[-\gamma,0]} |\phi(s)| + \int_{-h}^{0} \eta(s)\,\phi(s)\,ds < \infty$ (see Sinha, 1985) and for $x:[t-h,t] \to E^n$, let $x_t: [-h,0] \to E^n$ be defined by $x_t(s) = x(t+s)$, $s \in [-h,0]$

We shall consider the infinite delay system given by equation (2.1)

$$\begin{aligned}\dot{x}(t) = L(t,x_t) + \int_{-\gamma}^{0} d_sH(t,s)\,u(t+s) + \int_{-\infty}^{0} A(s)x(t+s)\,ds\\ x(t) = \phi(t)\end{aligned} \tag{2.1}$$

Its free system will be given by equation (2.2)

$$\dot{x}(t) = L(t,x_t) + \int_{-\infty}^{0} A(s)\,x(t+s)\,ds \tag{2.2}$$

and its linear control base system given by equation (2.3)

$$\dot{x}(t) = L(t,x_t) + \int_{-\gamma}^{0} d_sH(t,s)u(t+s) \tag{2.3}$$

where

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$$L(t,\phi) = \sum_{k=0}^{N} A_k \phi(-h_k) \tag{2.4}$$

satisfied almost everywhere on $[t_0,t_1]$. $L(t,\phi)$ is continuous in t, linear in ϕ (see Davies, 2006). A_k is a continuous $n\times n$ matrix function for $0\le h_k \le h$. $A(s)$ is an $n\times n$ matrix whose elements are square integrable on $(-\infty,0]$ and $H(t,s)$ is an $n\times m$ matrix valued function which is measurable in (t,s) and of bounded variation in s on $[-h,0]$ for each $t\in[t_0,t_1]$, the controls u are square integrable with values in the unit cube $C^m = \{u : u\in E^m, \ |u_j|\le 1, \ j=1..........m\}$ (see Nse, 2005).

To obtain the solution of system (2.1) let X satisfy the equation.

$$\frac{\partial X(t,l)}{\partial t} = L(t,X_t(\cdot,l)), \quad t\ge l$$

$$X(t,l) = \begin{cases} 0, & l-h\le t\le l \\ I, & t=l \end{cases}$$

where $X_t(\cdot,l)(s)=X(t+s,l), \quad -h\le s\le 0.$

Then the solution of (2.1) will be given by equation (2.5)

$$x(t,t_0,\phi,u) = x(t,t_0,\phi(0)) + \int_{t_0}^{t} X(t,l)\int_{-\gamma}^{0} d_s H(l,s)u(l+s)dl$$
$$+ \int_{t_0}^{t} X(t,l)\left(\int_{-h}^{0} A(s)x(l+s)ds\right)dl + \int_{t_0}^{t} X(t,l)\, f(l,x(l),\dot{x}(l),u(l))\,dl \tag{2.5}$$

Using the unsymmetric Funini's theorem (see Klamka, 1980) for $t = t_1$, the solution (2.5) of system (2.1) becomes

$$x(t_1,t_0,\phi,u) = x(t_1,t_0,\phi(0)) + \int_{-\gamma}^{0} dH_s\left(\int_{t_0}^{t_1} X(t_1,l)H(l,s)dl\right)$$
$$+ \int_{t_0}^{t_1} X(t_1,l)\left(\int_{-h}^{0} A(s)x(l+s)ds\right)dl + \int_{t_0}^{t_1} X(t_1,l)\, f(l,x(l),\dot{x}(l),u(l))\,dl \tag{2.6}$$

By defining

$$H_t(l,s) = \begin{cases} H(l,s), & for \ \ l\le t_1 \\ 0, & for \ \ l>t_1 \end{cases}$$

equation (2.6) becomes

$$x(t_1,t_0,\phi,u) = x(t_1,t_0,\phi(0)) + \int_{-\gamma}^{0} dH_s\left(\int_{t_0+s}^{t_0} X(t_1,l-s)H(l-s,s)u_{t_0}\,dl\right)$$
$$+ \int_{-\gamma}^{0} dH_s\left(\int_{t_0}^{t_1} X(t_1,l-s)H_{t_1}(l-s,s)u(l)dl\right) + \int_{t_0}^{t} X(t_1,l)\left(\int_{-h}^{0} A(s)x(l+s)ds\right)dl \tag{2.7}$$
$$+ \int_{t_0}^{t_1} X(t,l)\, f(l,x(l),\dot{x}(l),u(l))dl$$

Using again the unsymmetric Fubini's theorem on the order of change of integration (2.7) becomes

$$x(t_1,t_0,\phi,u) = x(t_1,t_0,\phi(0) + \int_{-\gamma}^{0} dH_s\left(\int_{t_0+s}^{t_0} X(t_1,l-s)H(l-s,s)u_{t_0}\,dl\right)$$
$$+ \int_{t_0}^{t_1}\left(\int_{-\gamma}^{0} X(t_1,l-s)\,dH_{t_1}(l-s,s)u(l)dl\right) + \int_{t_0}^{t_1} X(t_1,l)\left(\int_{-h}^{0} A(s)x(l+s)ds\right)dl \tag{2.8}$$
$$+ \int_{t_0}^{t_1} X(t_1,l)f(l,x(l),\dot{x}(l),u(l))dl$$

For brevity, we introduce the following notations

$$Z(t_0,l,s) = \int_{-\gamma}^{0} X(t_1,l-s)dH_{t_1}(l-s,s) \tag{2.9}$$

and

$$G(t,l) = x(t_1,t_0,\phi(0)) + \int_{t_0}^{t_1} X(t_1,l)f(l,x(l),\dot{x}(l),u(l))dl + \int_{t_0}^{t_1} X(t_1,l)\left(\int_h^0 A(s)x(l+s)ds\right)dl \tag{2.10}$$

We now define the $n \times n$ dimensional controllability matrix and other basic definitions of our study.
The controllability matrix of system (2.1) will be given as

$$W(t_0,t_1) = \int_{t_0}^{t_1} Z(t_0,l,s)\, Z^T(t_0,l,s) \tag{2.11}$$

where the symbol T denotes the matrix transpose. The matrix $W(t_0,t_1)$ is symmetric and non-negative definite (see Davies, 2006)

The reachable set for system (2.1) is given by equation (2.12)

$$R(t_1,t_0) = \left\{ \int_t^{t} Z(t_0,l,s)u(l)dl : u \in U \right\} \tag{2.12}$$

Definition 2.1 The complete state of system (2.1) at time t is $z(t) = \{x(t), x_t, u(t)\}$

Definition 2.2: The system (2.1) is said to be null controllable on $[t_0,t_1]$, if for each $\phi \in B([-h,0],E^n)$, there exists a $t_1 \geq t_0$, $u \in L_2([t_0,t_1],U)$, U a compact convex subset of E^m, such that the solution $x(t,t_0,\phi,u)$ of (2.1) satisfies $x_t(t_1,\phi,u) = \phi$ and $x(t_1,t_0,\phi,u)=0$ (see Nse, 2005)

The following propositions on the controllability of system (2.3) are similar to corresponding results for linear control systems of various types including some with delays and some without Hermes and Lasalle, 1969; Onwuatu, 1990 The proofs are therefore omitted

Proposition 1: The following are equivalent

$W(t_0,t_1)$ is non-singular

System (2.3) is completely controllable on $[t_0,t_1]$, $t_1 > t_0$

System (2.3) is proper on $[t_0,t_1]$, $t_1 > t_0$

Proposition 2 System (2.3) is proper on $[t_0,t_1]$ if and only if $0 \in \text{int } R(t_1,t_0)$

3. MAIN RESULTS

Here we give theorems, which summarises result on null controllability of system (2.1)

Theorem 3.1: Assume for system (2.1) that

The constraint set U is an arbitrary compact subset of E^m

The system (2.2) is uniformly asymptotically stable, so that the solution of (2.2) satisfies $\| x(t,t_0,\phi,0)\| \leq M e^{-\alpha(t-t_0)}\|\phi\|$ for some $\alpha > 0$, $M > 0$

The linear control system (2.3) is controllable

The continuous function f satisfies

$$|f(t,x(\cdot),\dot{x}(\cdot),u(\cdot))| \leq \exp(-\beta t)\pi(x(\cdot),u(\cdot)) \text{ for all } (t,x(\cdot),\dot{x}(\cdot),u(\cdot)) \in [t_0,\infty) \times B \times L_2$$

where $\int_t^{\infty} \pi(x(\cdot),\dot{x}(\cdot),u(\cdot))\,ds < K < \infty$ and $\beta - \alpha > 0$

then the system (2.1) is Euclidean null controllable

Proof: By (iii), $W^{-1}(t_0,t_1)$ exists for each $t_1 > t_0$ Suppose the pair of functions x, u from a solution pair to the set of integral equations

$$u(t) = Z(t_0,l,s)^T\, W^{-1}(t_0,t_1)\, G(t,l) \tag{3.1}$$

for suitably chosen $t_1 \geq t \geq t_0$

$$x(t) = \int_{t_0}^{t_1} Z(t_0,l,s)u(l)dl + G(t,l)$$
$$x(t) = \phi(t),\ t \in [t_0 - \gamma, t_0] \tag{3.2}$$

Then u is square integrable on $[t_0, t_1]$ and x is a solution of (2.1) corresponding to u with initial state $z(t_0) = (x(t_0), \phi, \sigma)$ where $u_{t_0} = \sigma$. Also

$$x(t_1) = -\int_{t_0}^{t_1} X(t_1, l) Z^{T}(t_0, l, s) W^{-1}(t_0, t_1)\,[G(t, l)] + G(t, l) = 0 \tag{3.3}$$

We now show that $u : [t_0, t_1] \rightarrow u$ is in the arbitrary compact constraint subset of E^m, that is $|u| \le a_1$, for some constant $a_1 > 0$.

By (ii)

$\left| Z(t_0, l, s)^T W^{-1}(t_0, t_1) \right| \le k_1$ for some $k_1 > 0$, and

$|X(t_1, t_0, \phi, 0)| \le k_2 \exp(-\propto(t_1 - t_0))$ for some $k_2 > 0$. Hence,

$$|u(t)| \le k_1 \left[k_2 \exp(-\propto(t_1 - t_0)) + \int_{t_0}^{t_1} k_3 \exp(-\propto(t_1 - l)\exp(-Bl)\pi(x(\cdot), \dot{x}(\cdot), u(\cdot))dl \right]$$

and therefore

$$|u(t)| \le k_1 [k_2 \exp(-\propto(t_1 - t_0))] + k k_3 \exp(-\propto t_1) \tag{3.4}$$

since $\beta - \propto \ge 0$ and $l \ge t_0 \ge 0$. From (3.4), we see that by taking t_1 sufficiently large, we have $|u(t)| \le a_1$, $t \in [t_0, t_1]$, which proves that u is an admissible control for this choice of t_1.

It remains to prove the existence of a solution pair of the equations (3.1) and (3.2).

Let B be the Banach space of all functions $(x, u) : [t_0, -\gamma, t_1], E^n) \times [t_0 - \gamma, t_1] \rightarrow E^n \times E^m$ where $x \in B([t_0 - \gamma, t_1], E^n)$ and $u \in L_2([t_0 - \gamma, t_1], E^m)$ with the norm defined by $\|(x, u)\| \le \|x\|_2 + \|u\|_2$,

where

$$\|x\|_2 = \left[\int_{t_0 - \gamma}^{t_1} |x(l)|^2 dl \right]^{\frac{1}{2}}, \qquad \|u\|_2 = \left[\int_{t_0 - \gamma}^{t_1} |u(l)|^2 dl \right]^{\frac{1}{2}}$$

Define the operator

$T : B \rightarrow B$ by $T(x, u) = (y, v)$ where

$$v(t) = -Z^T(t_0, l, s) W^{-1}(t_0, t_1) G(t, l), \quad \text{for } t \in J \equiv [t_0, t_1] \tag{3.5}$$

$$v(t) = \sigma(t) \quad \text{for } t \in [t_0 - h, t_0]$$

$$y(t) = \int_{t_0}^{t} Z(t_0, l, s) v(l) dl + G(t, l) \tag{3.6}$$

for $t \in J$ and $y(t) = \phi(t)$ for $t \in [t_0 - h, t_0]$. We have already shown that $|v(t)| \le a_1, t \in J$ and also $v : [t_0 - \gamma, t_0] \rightarrow U$, so $|v(t)| \le a_1$. Hence, $\|v(t)\|_2 \le a_1 (t_1 + \gamma - t_0)^{\frac{1}{2}} = \beta_0$. Next

$|y(t)| \le k_2 \exp[-\propto(t - t_0)] + k_4 \int_{t_0}^{t} |v(s)| ds + kk_3 \exp(-\propto t_1)$ where

$k_4 = \sup |Z(t_0, l, s)|$. Since $\propto > 0$, $t \ge t_0 \ge 0$, we deduce that

$|y(t)| \le k_2 + k_1 \gamma (t_1 - t_0) + k k_3 \quad \beta_1, t \in J$ and $|y(t)| \le \sup |\phi(t)| \equiv d$, $t \in [t_0 - h, t_0]$ Hence if $\lambda = \max [\beta_1, d]$, then $\|y\|_2 \le \lambda (t_1 + \gamma - t_0)^{\frac{1}{2}} = \beta_2 < \infty$ let $r = \max\{\beta_0, \beta_2\}$ Then letting $Q(r) = \{(x, u) \in B : \|x\|_2 \le r\}$ we have shown that $T : Q(r) \rightarrow Q(r)$.

Since $Q(r)$ is closed, bounded and convex, by Riesz's theorem (Kantorovich and Akilov, 1982, p. 297), it is relatively compact under T. Hence, by the Schauder's fixed point theorem (see Nse, 2005) T has a fixed point, and therefore system (2.1) is Euclidean null controllable.

CONCLUSION

Sufficient conditions for the controllability of the perturbed non-linear systems with implicit derivative have been derived. These were derived with respect to the stability of the free linear system and the controllability of the linear controllable base system with appropriate conditions placed on the perturbation f. These results complement and extend known result.

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