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ESTIMATION OF THE PARAMETERS OF LINEAR REGRESSION MODEL WITH AUTOCORRELATED ERROR TERMS WHICH ARE ALSO CORRELATED WITH THE REGRESSOR

J. O. OLAOMI

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ABSTRACT

This study used the Monte–Carlo method to investigate the performance of five estimators: Ordinary Least Squares (OLS), Cochrane Orcutt (CORC), Hildreth Lu (HILU), Maximum Likelihood (ML) and Maximum Likelihood Grid (MLGRID) in estimating the parameters of a single equation linear regression model in which the autoregressive independent variable is also correlated with the autoregressive error terms.

The simulation results, under the finite sampling properties of Bias, Variance and Root Mean Squared Error, show that all estimators are adversely affected as autocorrelation coefficient (ρ) is close to unity. In this regard, the estimators rank as follows in descending order of performance: OLS, MLGRID, ML, HILU and CORC.

The estimators conform to the asymptotic properties of estimates considered. This is seen when the level of autocorrelation is mild (i.e. $\rho \leq 0.8$) at all significant levels. The estimators' rank in decreasing order in conformity with the observed asymptotic behaviour as follows: OLS, HILU, MLGRID, CORC and ML. Most of the criteria used in studying the relative performance of the five estimators have not exhibited any remarkable sensitivity to the number of replications. Increasing the number of replications has tended to confirm the stability of the study.

The results also suggest that OLS should be preferred when autocorrelation level is relatively mild $(\rho = 0.4)$ and the regressor is significantly correlated (at least at 5%) with the autocorrelated error terms.

This result helps in the choice of estimators in empirical work when the regressor and the error terms are not well behaved. It also allows correct inferences in linear models plagued by autocorrelated disturbances, which are also significantly correlated with the independent variable.

KEYWORDS: Monte-Carlo Experiment, GLS Estimators, Autocorrelated Error Terms, Linear Regression

1. INTRODUCTION

Consider the standard classical statistical linear regression model

$$Y = X\beta + U \tag{1}$$

where Y is a (N x 1) vector of observations, X is a known (N x K) non-stochastic design matrix of rank K, β is a (K x 1) fixed vector of unknown parameters and U is a (N x 1) vector of unobservable random variable with zero mean and finite covariance matrix. When the error terms follow a first order autoregressive (AR) process, we have

$$U_t = \rho U_{t-1} + \varepsilon_t, \qquad |\rho| < 1, \quad \varepsilon_t \to NID(0, \sigma^2),$$

$$U_0 \to N\left(0, \frac{\sigma^2}{1-\rho^2}\right)$$

Assumptions in the classical normal linear regression model include that of lack of autocorrelation of the error terms and the zero covariances between the independent variable and the error terms. This paper examines the estimation of the parameters of the linear models when the above two assumptions are violated.

These violations are seen in widespread applications in operations research, like in queuing theory and econometrics, where the usual assumption of independent error terms may not be plausible in most cases. These violations are also observed when using time series data on a number of micro-economic units, for example, households; and also in service oriented channels, where the stochastic disturbance terms in part reflect variables which are not included explicitly in the model and which may change slowly over time[17]. Cochrane and Orcutt[5] have shown that the error terms in most current formulations of economic relations are highly positively autocorrelated. Rao and Griliches[20] have shown that there is much to gain and little to lose by considering alternatives to the independent error assumption of the classical linear regression model.

Many models with autocorrelated error terms have been discussed in the literature. These include the works of Anderson[1], Cochran and Orcutt[5], Durbin and Watson[6,7,8], Rao and Griliches[20], Beach and Mackinnon[2], Kramer[13,14], Busse, Jeske and Kramer[4], Kramer and Hassler[15], Nwabueze[17], Kleiber[12], Kramer and Marmol[16], Butte[3] and Olaomi[18]. Tests for detecting the presence of autocorrelation and alternative consistent methods of estimating linear models with autocorrelated disturbance terms have been proposed. However, in spite of these tests and estimation methods, a number of questions in connection with the estimation of the classical regression linear model with autocorrelated error terms and non-zero covariance between the independent variable and the error terms remained unanswered. These include the most appropriate estimation method in the above named specification of the independent variable, the effect of the degree of correlation of the disturbance term, the effect of the degree of correlation of independent variable and the error terms, the effect of replications and sample size and the sampling properties of the various estimation methods.

The answers to most of these questions would allow for correct inferences to be made in linear models plagued by the scenario depicted above.

The rest of this paper discusses the methodology, the model and the data generation procedure in section 2, section

J. O. Olaomi, Department of Statistics, University of Ibadan, Ibadan, Nigeria

3 presents the simulation results, and section 4 presents the discussions, while we conclude in section 5.

2. METHODOLOGY

This study used the Monte-Carlo approach for the investigation due to the fact that real life observations on economic variables are in most cases plagued by one or several of the problems of non-spherical disturbances (a problem where the disturbance term U in any period is correlated with any other value U in the series, that is, serial correlation of the random variable U), measurement error and specification error. Also when the covariance between the independent variable and the error terms is non-zero, the problem is near intractable by analytical procedure.

The following four Generalised Least Squares (GLS) estimators {Cochrane and Orcutt (CORC), Hildreth and Lu (HILU), Maximum Likelihood (ML) and Maximum Likelihood Grid (MLGRID)} and Ordinary Least Squares (OLS) estimation methods, choosing in the light of the previous studies, are used. These estimators are equivalent with identical asymptotic properties[15]. But in small samples, such as in this study, Park and Mitchell[19] have argued that those that use the T transformation matrix (ML, MLGRID) are generally more efficient, in terms of estimation of the parameters of the model, than those that use T* transformation matrix (CORC, HILU).

The degree of autocorrelation affects the efficiency of the estimators[17]. Consequently, we investigated the sensitivity of the estimators to the degree of autocorrelation by varying rho $(\hat{\rho})$ from 0.4, to 0.8 and 0.9. We also found out the effect of the correlation of the independent variable and the error terms at significant level (α) 1%, 2% and 5% on the estimators. The effects of sample size (N) and replication (R) on the estimators were also investigated by varying the sample size from 20, 40 to 60 each replicated 30, 40 and 50 times. Evaluation of the best estimator(s) was then done using the finite sampling properties of Minimum Bias (BIAS), Minimum Variance (VAR), and Minimum Root Mean Squared Error (RMSE).

2.1 THE MODEL

We assume a simple linear regression model:

$$Y_t = \beta_0 + \beta_1 X_t + U_t \qquad (2)$$

$$U_t = \rho U_{t-1} + \varepsilon_t, \qquad |\rho| < 1,$$

$$X_t = \lambda X_{t-1} + V_t, \quad V_t \to N(0,1) \quad \lambda = 0.8$$

$$U_t \to N\left(0, \frac{\sigma^2}{1-\rho^2}\right), \qquad X_t \to N\left(0, \frac{\sigma^2}{1-\lambda^2}\right).$$

$t = 1, 2, \ldots N, \quad \beta = (1,1)$

where Y_t is the dependent variable and the first order autoregressive X_t is the independent variable with U_t also autoregressive of order one. ε_t and V_t are standard normal distributed. ρ and λ are stationarity parameters while the model parameters are assumed to be unity. Spitzer[21], Nwabueze[17] and Olaomi[18] had used this independent variable specification. It is chosen to allow for comparison of results.

2.2 DATA GENERATION

A total of 81 data sets spread over three sample sizes (20, 40 and 60) and three replication numbers (30, 40, and 50) were used in generating the data for this study. Using model (2), a value U_0 was generated by drawing a random value ε_0 from N(0,1) and dividing by $\sqrt{(1-\rho^2)}$. Successive values of ε_t drawn from N(0,1) were used to calculate U_t. X_t was similarly generated. Correlation between U_t and X_t was then computed and its absolute value tested for significance at say 1%. If this value is significant, it is chosen, otherwise it is discarded. This procedure is repeated as many times as are necessary to obtain specified number of replications for a desired autocorrelation level, significance level and sample size. Y_t is thus computed for the chosen U_t and fixed X_t using equation (2). The computations are made using the Excel package; different estimation methods are then applied to the data using the AR procedure of the TSP[20] package.

3 SIMULATION RESULTS

The finite sampling properties of estimators we used include the Bias (BIAS), Variance (VAR) and the Root Mean Squared Error (RMSE). Additionally, we calculated and displayed the Sum of Bias (SBIAS), Variances (SVAR) and the Root Mean Squared Error (SRMSE).

The results are summarized for SBIAS, SVAR and SRMSE as shown in Tables 1, 2, and 3 respectively for sample sizes 20 and 60 replicated 30, 40 and 50 times each. (The result for sample size forty (N = 40) is omitted for page constraint, though, it is used in the explanation of results)

In the discussion of the results, attention is focused on comparison of the following attributes of the estimates yielded by each of the five estimators:

(i) the sum of the bias of the two estimated parameters (SBIAS) with particular emphasis on sensitivity of these magnitudes to α, ρ, R and N, and

(ii) the sum of variances and the sum of the RMSE of the estimates also with emphasis to their sensitivity to α, ρ, R and N as in (i) above.

Observing the trends followed by the estimates as α and ρ varies, for each of Bias, Variance and the RMSE, it could be observed that all estimators are adversely affected as autocorrelation coefficient (ρ) is close to unity when the regressor is significantly correlated with the error term. This is evidenced by the optimum (ρ, α) combinations of (0.4, 0.01) as ρ increases and (0.4, 0.05) as α decreases using both the Variance and the RMSE criteria while the Bias criterion give the optimum combination of (0.8, 0.05) with minimum occurring at ρ=0.8. There is absence of the combinations (0.9, 0.01), (0.9, 0.02), (0.9, 0.05), (0.8, 0.01) and (0.8, 0.02) which shows that the estimators perform less as ρ → 1.

The performance of the estimators rank as follows in descending order based on trends of Bias, Variance and RMSE: OLS, MLGRID, ML, HILU, and CORC.

We also investigated the asymptotic behaviour of the estimators in our experiment (i.e. as N increases). The five estimators rank as follows in decreasing order of conformity with the observed asymptotic behaviour of Bias, Variance and RMSE: OLS, HILU, MLGRID, CORC, and ML.

In most Monte – Carlo studies, magnitudes such as bias, variance and root mean squared error are not usually remarkably sensitive to the number of replications. We did not assume this a priori; hence the possible effects of different numbers of replication on these magnitudes are investigated.

The five estimators rank as follows in decreasing order of conformity with the observed replication effect of bias, variance and RMSE: OLS, CORC, ML, MLGRID, and HILU. The results of replication effect suggest that the behaviour of bias, variance and root mean squared error are remarkably less sensitive to replication numbers than sample sizes. Also, the optimum trend occurred at the same replication number 50 for bias, variance and RMSE, which suggests that the replications actually confirmed the stability of our results.

Table 1: Sum of Absolute Bias for Estimators of β for N=20 and N=60 (All replications)

Significant Level	Estimator	ρ = 0.4			ρ = 0.8			ρ = 0.9		
		N=20, R=30	N=20, R=40	N=20, R=50	N=20, R=30	N=20, R=40	N=20, R=50	N=20, R=30	N=20, R=40	N=20, R=50
0.01	OLS	2.00672	2.01345	2.001397	2.019522	2.011611	1.979407	1.94924	1.976075	1.945679
	CORC	2.009557	2.007646	1.98372	2.099689	2.085352	2.117686	2.007161	1.983825	1.969557
	HILU	2.010673	2.009382	1.985357	1.971174	1.989489	2.05607	1.821298	1.797405	1.786586
	ML	2.002887	2.009204	1.99943	1.954783	1.949677	1.970245	1.965999	1.960984	1.990792
	MLGRID	2.002931	2.009446	1.99984	1.946327	1.944887	1.939896	2.047327	2.04501	2.020209
0.02	OLS	2.035417	2.027578	2.012331	0.058988	0.087823	0.053624	2.100742	2.102724	2.088479
	CORC	2.071447	2.052589	2.05112	0.044664	0.072496	0.038231	2.164015	1.85121	1.850582
	HILU	2.018253	2.023326	2.027572	0.069359	0.101422	0.061715	1.995361	1.938694	1.980895
	ML	2.002364	1.985979	1.971339	0.053895	0.077344	0.050747	2.01348	1.975195	1.975273
	MLGRID	1.989597	1.989994	1.992195	0.088088	0.102886	0.092551	2.059795	2.019704	1.998286
0.05	OLS	1.999429	2.003305	2.000829	0.0561496	0.0334392	0.0664662	2.049742	1.968029	1.982158
	CORC	1.979828	1.983376	1.994258	0.1011124	0.2348154	0.1686729	2.082196	2.082721	2.089437
	HILU	2.003201	2.001	2.008555	0.088639	0.0888913	0.0529243	2.010138	2.032735	2.104394
	ML	1.985907	1.993163	1.995076	0.087603	0.025485	0.0056	1.966716	1.922908	1.947758
	MLGRID	1.987827	1.994408	1.996452	0.105352	0.0893324	0.0565083	1.980556	1.938704	1.966893

Significant Level	Estimator	ρ = 0.4			ρ = 0.8			ρ = 0.9		
		N=60, R=30	N=60, R=40	N=60, R=50	N=60, R=30	N=60, R=40	N=60, R=50	N=60, R=30	N=60, R=40	N=60, R=50
0.01	OLS	0.01296	0.01045	0.02452	0.020918	0.016128	0.009589	0.091205	0.072217	0.072096
	CORC	0.05750	0.04417	0.04124	0.017076	0.020482	0.007208	0.037332	0.011274	0.024551
	HILU	0.04027	0.03354	0.0327	0.017481	0.00989	0.0025244	0.09573	0.06727	0.06797
	ML	0.06089	0.04544	0.04065	0.033145	0.015499	0.015663	0.045336	0.057495	0.086133
	MLGRID	0.03861	0.02787	0.0273	0.029097	0.012525	0.031493	0.018391	0.043605	0.046388
0.02	OLS	0.00211	0.001035	0.001623	0.026568	0.012778	0.008919	0.04865	0.06253	0.06218
	CORC	0.016565	0.010162	0.011069	0.025391	0.008719	0.014044	0.253943	0.217335	0.194355
	HILU	0.015469	0.009561	0.010425	0.016038	0.012195	0.01728	0.070292	0.061384	0.024102
	ML	0.008936	0.00884	0.005808	0.017933	0.018277	0.02101	0.180547	0.113496	0.040749
	MLGRID	0.041458	0.026908	0.024178	0.02666	0.013897	0.021114	0.153787	0.098974	0.046591
0.05	OLS	0.002809	0.003382	0.003315	0.020296	0.026675	0.007642	0.064927	0.038539	0.034141
	CORC	0.013529	0.01119	0.00572	0.05402	0.07636	0.0602	0.124921	0.023225	0.026179
	HILU	0.016459	0.014448	0.017928	0.0513	0.07616	0.06263	0.145867	0.062353	0.051439
	ML	0.015143	0.009646	0.010106	0.065352	0.085669	0.067492	0.180878	0.121945	0.068846
	MLGRID	0.014375	0.008302	0.009429	0.044863	0.067861	0.072527	0.150453	0.093332	0.055344

Table 2: Sum of Variances of Bias for Estimators of β for N=20 and N=60 (All replications)

Significant Level	Estimator	ρ = 0.4			ρ = 0.8			ρ = 0.9		
		N=20, R=30	N=20, R=40	N=20, R=50	N=20, R=30	N=20, R=40	N=20, R=50	N=20, R=30	N=20, R=40	N=20, R=50
0.01	OLS	0.2917708	0.2770682	0.2600066	0.7306049	0.6730976	0.6360087	1.2950126	1.2009676	1.1410226
	CORC	0.2831761	0.2656922	0.2505263	0.8270936	0.6898761	0.6527618	1.0194169	0.8708283	0.7657319
	HILU	0.2833661	0.2660475	0.2510355	0.6192805	0.5347448	0.5481383	0.9735029	0.892521	0.8196442
	ML	0.2818188	0.2630449	0.2469734	0.4580663	0.4167675	0.4139603	1.2254239	1.1767008	1.1163396
	MLGRID	0.2815797	0.2628911	0.2466277	0.4414921	0.4043617	0.4023914	1.1775677	1.1154045	1.0340933
0.02	OLS	0.1393636	0.1359369	0.1344286	0.2058534	0.2172545	0.2089693	0.5903161	0.6773591	0.6634187
	CORC	0.1342936	0.136505	0.1354404	0.1778809	0.1591779	0.1500322	2.192935	3.7476047	3.1649482
	HILU	0.1636967	0.1523624	0.1479018	0.2136975	0.1905953	0.1762796	0.3998344	0.3735938	0.3769719
	ML	0.1369034	0.154189	0.1673877	0.231402	0.2150166	0.200496	0.4249026	0.4181026	0.4358225
	MLGRID	0.1528453	0.1425759	0.1396	0.2543282	0.2314688	0.2223818	0.4372153	0.4426205	0.4567298
0.05	OLS	0.1202272	0.1175814	0.1155771	0.2482037	0.2294696	0.2231072	0.5956265	0.6403861	0.5958598
	CORC	0.0957375	0.0970671	0.097874	0.5256693	1.4968368	1.2303207	1.5197523	1.5531538	1.267398
	HILU	0.0960903	0.097347	0.098252	0.4367708	0.4688644	0.3981557	0.2512618	0.2622081	0.3752807
	ML	0.0955999	0.0960047	0.0925097	0.2188795	0.2088446	0.1958921	0.42803	0.4470161	0.4230614
	MLGRID	0.0946084	0.0949742	0.0915859	0.2556906	0.2419552	0.2236712	0.4201651	0.4348797	0.4139272

Significant Level	Estimator	ρ = 0.4			ρ = 0.8			ρ = 0.9		
		N=60, R=30	N=60, R=40	N=60, R=50	N=60, R=30	N=60, R=40	N=60, R=50	N=60, R=30	N=60, R=40	N=60, R=50
0.01	OLS	0.176368	0.158899	0.1642362	0.517286	0.459528	0.4374466	1.394178	1.191885	1.1364449
	CORC	0.1057766	0.095094	0.0916026	0.072525	0.068183	0.0692881	0.220702	0.191118	0.1819406
	HILU	0.107703	0.09647	0.0926124	0.081027	0.074721	0.0765582	0.184252	0.150104	0.1455753
	ML	0.122844	0.105467	0.9812212	0.073347	0.06987	0.0683459	0.182376	0.153864	0.1607063
	MLGRID	0.102877	0.090692	0.0865706	0.074704	0.071007	0.0868902	0.1312	0.119674	0.1122606
0.02	OLS	0.079563	0.074993	0.0736084	0.176403	0.173018	0.1666772	0.45346	0.402187	0.391828
	CORC	0.055579	0.053211	0.0526387	0.044403	0.039953	0.0366614	1.499516	1.106077	0.9052559
	HILU	0.054644	0.052807	0.0521946	0.047191	0.041455	0.0374849	0.080429	0.105857	0.1002894
	ML	0.078449	0.068465	0.0643163	0.041674	0.040717	0.038563	0.121253	0.132635	0.1546279
	MLGRID	0.081523	0.071261	0.0665087	0.046266	0.046218	0.0451247	0.126422	0.122912	0.1265501
0.05	OLS	0.066635	0.064482	0.0636957	0.137192	0.132738	0.130665	0.337234	0.353237	0.3362023
	CORC	0.050026	0.065585	0.0632818	0.038182	0.033103	0.0327175	0.075943	0.19537	0.1909612
	HILU	0.050365	0.044886	0.0468954	0.039983	0.034771	0.033373	0.058576	0.094843	0.0935037
	ML	0.043962	0.039757	0.041602	0.048031	0.040245	0.0377033	0.183619	0.171501	0.1967271
	MLGRID	0.04497	0.040363	0.0421535	0.037801	0.032457	0.0428181	0.13729	0.140487	0.1479234

Table 3: Sum of RMSE of Bias for Estimators of β for N=20 and N=60 (All replications)

Significant Level	Estimator	ρ = 0.4			ρ = 0.8			ρ = 0.9		
		N=20, R=30	N=20, R=40	N=20, R=50	N=20, R=30	N=20, R=40	N=20, R=50	N=20, R=30	N=20, R=40	N=20, R=50
0.01	OLS	2.1463831	2.1457388	2.126307	2.3499986	2.3172336	2.2729925	2.5031266	2.490157	2.4173733
	CORC	2.1452809	2.1355531	2.105471	2.462485	2.3940988	2.4069304	2.4647014	2.3835384	2.3268551
	HILU	2.1463955	2.1373491	2.1072543	2.2629714	2.2424531	2.3075708	2.3034195	2.2433428	2.2028399
	ML	2.1381025	2.1356164	2.1185449	2.1743471	2.1504862	2.1672798	2.5181316	2.4926103	2.4889467
	MLGRID	2.1379813	2.1357456	2.118759	2.1569248	2.138691	2.1317296	2.5544234	2.5272226	2.4754435
0.02	OLS	2.1030006	2.0937159	2.0779932	0.6282435	0.644284	0.6307173	2.361669	2.3984561	2.3807055
	CORC	2.1354608	2.1180829	2.1161594	0.5984365	0.5693603	0.5492479	2.8817568	3.1000047	2.9631635
	HILU	2.0978876	2.0974097	2.0993738	0.6575598	0.6256405	0.5969537	2.1823687	2.1198413	2.1585131
	ML	2.0692482	2.0619373	2.0543535	0.6794952	0.6574901	0.6328284	2.2053012	2.1680719	2.1764571
	MLGRID	2.0646949	2.0600925	2.0607954	0.7132915	0.6836332	0.6690777	2.2486543	2.2150724	2.203685
0.05	OLS	2.0584924	2.060925	2.0575592	0.6950253	0.666721	0.6563293	2.3185126	2.268946	2.2600278
	CORC	2.0276345	2.0317627	2.0427828	0.9593283	1.4901532	1.3663103	2.6426014	2.6621549	2.575431
	HILU	2.0504206	2.0489425	2.0567887	0.8954525	0.9140499	0.8510265	2.1306611	2.1568205	2.2757302
	ML	2.0333824	2.040713	2.0408009	0.6642964	0.6435349	0.625306	2.1660587	2.1406656	2.1520211
	MLGRID	2.0348269	2.0414671	2.041734	0.7247571	0.6995756	0.670853	2.1751938	2.1497901	2.163981

Significant Level	Estimator	ρ = 0.4			ρ = 0.8			ρ = 0.9		
		N=60, R=30	N=60, R=40	N=60, R=50	N=60, R=30	N=60, R=40	N=60, R=50	N=60, R=30	N=60, R=40	N=60, R=50
0.01	OLS	0.432837	0.417545	0.510646	0.799578	0.771886	0.758931	1.478458	1.390382	1.361902
	CORC	0.364423	0.348959	0.343160	0.358727	0.348169	0.351424	0.608653	0.575812	0.563226
	HILU	0.365166	0.350318	0.344064	0.388868	0.374297	0.379615	0.596516	0.544424	0.536552
	ML	0.371134	0.347982	0.337455	0.36433	0.356154	0.351744	0.566072	0.532515	0.544283
	MLGRID	0.338586	0.322367	0.315461	0.369397	0.36049	0.392958	0.492167	0.479582	0.466345
0.02	OLS	0.289108	0.287262	0.287064	0.499281	0,499618	0.487889	0.924996	0.88138	0.887076
	CORC	0.262365	0.261914	0.263569	0.294048	0.282924	0.271163	1.347693	1.195065	1.093986
	HILU	0.260199	0.260856	0.262112	0.302649	0.288182	0.274514	0.38191	0.451763	0.436026
	ML	0.292932	0.278016	0.26989	0.284472	0.28511	0.277913	0.481006	0.482905	0.534099
	MLGRID	0.301111	0.284509	0.275033	0.299567	0.301843	0.298131	0.47942	0.467303	0.468303
0.05	OLS	0.274168	0.273427	0.270894	0.446598	0.442921	0.439856	0.79876	0.834229	0.81285
	CORC	0.25861	0.29465	0.289281	0.281603	0.271409	0.265086	0.378365	0.536427	0.538272
	HILU	0.2604	0.25065	0.256657	0.286694	0.277325	0.268568	0.358037	0.405429	0.406265
	ML	0.237609	0.227659	0.229851	0.313517	0.297066	0.283936	0.555445	0.531554	0.554473
	MLGRID	0.245459	0.233951	0.235508	0.275394	0.26423	0.296059	0.475221	0.476191	0.465926

4.0 DISCUSSION OF RESULT

The simulation results, under all the finite properties considered show that all estimators are adversely affected as autocorrelation coefficient (ρ) is close to unity when the regressor is significantly correlated with the autocorrelated error terms. This conforms to literature when there is no correlation between the regressor and the error terms. (See Green[9], Verbeck[23], Johnston and DiNardo[11], Nwabueze[17]). In this regard, the estimators rank as follows in descending order: OLS, MLGRID, ML, HILU and CORC.

The results suggest that OLS should be preferred when autocorrelation level is relatively mild $(\rho = 0.4)$ and the regressor is significantly correlated (at least at 5%) with the autocorrelated error term. This seems plausible because the corrective measures incorporated into the GLS (CORC, HILU, ML, MLGRID) estimators make use of the 'badly behaved regressor' (regressor correlated with error terms) and these may adversely affect the performance of these estimators. The OLS estimator does not correct for autocorrelation and is therefore, not affected by this problem.

We found that the estimators conform to the asymptotic properties of estimates considered. This is seen when the level of autocorrelation is mild (i.e. $\rho \leq 0.8$) at all significant levels. The estimators' rank in decreasing order of conformity with the observed asymptotic behaviour is as follows: OLS, HILU, MLGRID, CORC and ML. This ranking is contrary to that of Nwabueze[17] when there is independence between the regressor and the error terms.

Most of the criteria used in studying the relative performance of the five estimators have not exhibited any remarkable sensitivity to the number of replications. Increasing the number of replications has tended to confirm the stability of the study.

We also note that ML and MLGRID have very similar behavioural pattern, the same for CORC and HILU as observed in the finite sampling properties of Bias, Variance and the RMSE.

5.0 CONCLUSION

We have shown that when there is correlation between the regressor and the error terms in a classical simple linear regression estimation problem, OLS estimation method should be used based on the finite sampling criteria used in this experiment. It is also shown that all the estimators are still asymptotically behaved based on the criteria used, with OLS estimation method performing best.

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