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THERMODYNAMIC QUANTITIES OF TWO-DIMENSIONAL ONE COMPONENT PLASMA

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ABSTRACT

Thermodynamic quantities of two-dimensional one-component plasma are calculated analytically based on the Mayer's giant cluster expansion. The quantities are calculated exactly to the order of $\varepsilon = 2\Gamma^2 = 2\pi ne^4T^{-2}$ .The results show the importance of the effects of the discreteness associated with individuality of particles in two-dimensional plasmas

1.0 INTRODUCTION

One-component plasmas (OCP) consist of a single species of charged particles immersed in a uniform neutralizing background of oppositely charged particles (Ichimaru, 1982; Ichimaru, et al 1987). The OCP model can be used to describe different physical systems occurring in nature, e.g. stellar interior (Kremer, et al 1996), liquid metals, electron layers on the surface of liquid helium (Golden, et al 1992; Grimes, et al 1979),electrolytes and charged-stabilized colloids, laser-cooled ions in cryogenic traps (Dubin and O'Neil, 1988) and particles in dusty plasmas (Murray and Winkle, 1987, Thomas, et al 1994; Totsuji et al. 2004)

In the case of pure Coulombic interaction, the properties of the classical OCP are exclusively determined by the dimensionless plasma coupling parameter (Ichimaru, et al 1987) $\Gamma = (Ze)^2/4\pi\varepsilon_0 aK_BT$ expressing the ratio of the potential energy of particles to their kinetic energy. Ze is the charge, K_B is the Boltzmann's constant, T is the temperature and a is the Wigner-Seitz (ion-sphere) radius defined by $a = (3/4\pi n)^{1/3}$ (3D) and $a = (1/\pi n)^{1/2}$ (2D) Here n is the number density of the plasmas. For $\Gamma << 1$, the plasma is said to be weakly coupled, while $\Gamma >> 1$ represents the strong coupling regime where the potential energy of the particles dominates over their kinetic energy We define the screening parameter by $\xi = a/\lambda$, where λ is the screening length.

A two-dimensional classical OCP is characterized by dimensionless parameters ε or Γ defined by $\varepsilon = 2\Gamma^2 = 2\pi ne^4T^{-2}$. As in the case of a three-dimensional plasma parameter ε in the two-dimensional system is closely related to the discreteness associated with individuality of the particles (Totsuji, 1975; Totsuji, et al 2004)

In this paper, we present the exact calculation of thermodynamic quantities of two-dimensional one-component plasma to the first order in ε . The results may serve to provide a useful theoretical guideline when one proceeds to consider the cases with ε >1; such procedure was quite successful in the case of three dimensional plasmas and two-dimensional dusty plasmas (Totsuji, et al 2004).

2.0 Thermodynamic Quantities by Cluster Expansion

Thermodynamic quantities of three-dimensional one component plasmas have been calculated by the method of Mayer's giant cluster expansion (Abe, 1959; Mayer, 1950). The method is equally applicable to two-dimensional plasmas. The pressure and the interaction energy density are given by

$$\frac{P}{nK_BT} - 1 = \frac{E_c}{2nT} = -n\frac{\partial}{\partial n}(W_0 + W_p) \qquad \ldots (2.1)$$

where W_0 represents the contribution of all ring-type clusters. W_p defined by

$$W_p = \sum_{\ell=2}^{\infty}\frac{n^{\ell-1}}{\ell!}\int\cdots\int d\vec{r}_2\cdots d\vec{r}_\ell \sum_{\ell\geq i>j>1}\prod\left(-u(r_{ij})/T\right)^{k(i,j)/k(i,j)!} \qquad \ldots (2.2)$$

is the contributions of prototype graphs. Prototype graphs are diagrams composed of interaction lines (bonds) and the vertices (junctions) to which three or more bonds are connected. In prototype graphs each bond represents the screened interaction u(r) given by

$$u(r) = \frac{e^2}{r}\int_0^{\infty}dx\frac{x}{x + K_D r}J_0(x), \qquad \ldots (2.3)$$

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where $K_D = 2\pi ne^2/T$; $J_0(x)$ is the Bessel function of the zeroth order; ℓ and k (i,j) denote the number of junctions and the multiplicity of the bond between junctions i and j, respectively. The contribution of the ring diagram W_0 is calculated as

$$W_0 = \frac{1}{2n}\sum_k \left[-\ln\{1 + n\beta v(k)\} + n\beta v(k)\right] \qquad \ldots (2.4)$$

where

$$v(r) = \frac{e^2}{r} = \sum_k v(k)\exp(i\vec{k}\cdot\vec{r}) \qquad \ldots (2.5)$$

and $\beta = 1/T$. In the two-dimensional case, W_0 diverges logarithmically in the short range domain The leading contribution arising from W_p is that of prototype graphs with two junctions, $W_p^{(2)}$, given by

$$W_p^{(2)} = \frac{n}{2}\int dr\left\{e^{-\beta u} - 1 + \beta u - \frac{1}{2}(\beta u)^2\right\} \qquad \ldots (2.6)$$

Equation (2.6) has logarithmic divergence which makes $W_0 + W_p^{(2)}$ finite. Thus, we introduce the term W' defined by

$$W' = \frac{n\beta^2}{4}\int d\vec{r}u(r)v(r) = \frac{n\beta^2}{4}\sum_k u(k)v(k) \qquad \ldots (2.7)$$

We then calculated $W_0 + W_p^{(2)}$ as the sum of $W_0 - W'$ and $W_p^{(2)} + W'$ To evaluate $W_0 - W'$, we use the expression of W' in the Fourier space and obtain

$$W_0 - W' = \frac{1}{2n}\sum_k\left[-\ln\{1 + n\beta v(k)\} + n\beta v(k) - \frac{1}{2}n^2\beta^2 u(k)v(k)\right] = \frac{1}{8}\varepsilon. \qquad \ldots (2.8)$$

To evaluate $W_p^{(2)} + W'$, we use the expression of W' in the real space and divide the integral into short- range part I_1 and long-range part I_2 at radius r_0 so that $e^2/T \le r_0 \le 1/K_D$. In calculating I_1, we note that the effect of screening is small for $r \le r_0 << 1/K_D$ and obtain

$$I_1 \approx \frac{\varepsilon}{4}\left\{-\ln\left(\frac{\beta e^2}{r_0}\right) - \gamma + \frac{3}{2} + \frac{1}{3}\frac{\beta e^2}{r_0} - \int_0^{K_D r_0} dy\int_0^{\infty} dx\frac{1}{x+y}J_0(x)\right\}. \qquad \ldots (2.9)$$

Here $\gamma = 0.5772\cdots$ and the last term in the parentheses is of the order of $|K_D r_0 \ln(K_D r_0)|$.

In calculating I_2, we note that $\beta u << 1$ for $e^2/T << r_0 << r$ and obtain

$$I_2 \approx \frac{\varepsilon}{4}\left\{-\ln(K_D r_0) - \ln 2 - \gamma - \frac{1}{3}\frac{\beta e^2}{r_0} + \int_0^{K_D r_0} dy\int_0^{\infty} dx\frac{1}{x+y}J_0(x)\right\}. \qquad \ldots (2.10)$$

See Appendix for the details. From (2.9) and (2.10), we have

$$W_p^{(2)} + W' \approx \frac{1}{4}\varepsilon\left(-\ln 2\varepsilon - 2\gamma + \frac{3}{2}\right). \qquad \ldots (2.11)$$

Substituting (2.8) and (2.11) into (2.1), we have

$$\frac{P}{nK_BT}-1=\frac{E_c}{2nT}=\frac{\varepsilon}{4}\{\ln 2\varepsilon-1+2\gamma\}=\frac{\varepsilon}{4}\left[\ln\left(2\frac{\beta e^2}{1/K_D}\right)-1+2\gamma\right]. \qquad \ldots(2.12)$$

The Helmoltz free-energy F is separated into the ideal gas part F^{ideal} and the interaction part as

$$F=F^{ideal}+NK_BTf(\Gamma,\xi) \qquad \ldots(2.13)$$

f is a dimensionless function of dimensionless quantities Γ and ξ. Since

$$\frac{P}{nK_BT}-1=-V(\partial/\partial V)_{T,N,\lambda}f(\Gamma,\xi)=n(\partial/\partial n)_{T,\lambda}f(\Gamma,\xi), \qquad \ldots(2.14)$$

we have

$$f(\Gamma,\xi)=-\frac{\varepsilon}{4}\left[-\ln\left(2\frac{\beta e^2}{\lambda}\right)-2\gamma+\frac{3}{2}\right]=-\frac{\Gamma^2}{2}\left[-\ln(2\Gamma\xi)-2\gamma+\frac{3}{2}\right]. \qquad \ldots(2.15)$$

The nonideal part of the entropy ΔS given by $\Delta S=-(\partial\Delta F/\partial T)_{N,V}$ is calculated as

$$\Delta S=NK_B\frac{\varepsilon}{4}\left[\ln\left(2\frac{\beta e^2}{\lambda}\right)+2\gamma-\frac{1}{2}\right]=NK_B\frac{\Gamma^2}{2}\left[\ln(2\Gamma\xi)+2\gamma-\frac{1}{2}\right]. \qquad \ldots(2.16)$$

The internal (correlation or cohesive) energy U given by $U=\Delta F+T\Delta S$ is calculated as

$$U=NK_BT\frac{\varepsilon}{2}\left[\ln\left(2\frac{\beta e^2}{\lambda}\right)+2\gamma-1\right]=NK_BT\Gamma^2[\ln(2\Gamma\xi)+2\gamma-1]. \qquad \ldots(2.17)$$

In the case of Coulomb interaction $\lambda\to\infty$ or $\xi=0$, they are given by

$$f(\Gamma,\xi=0)=-\frac{\varepsilon}{4}[-\ln(2\varepsilon)-2\gamma+2]=-\Gamma^2[-\ln(2\Gamma)-\gamma+1], \qquad \ldots(2.18)$$

$$\Delta S(\lambda\to\infty)=NK_B\frac{\varepsilon}{4}[\ln(2\varepsilon)+2\gamma]=NK_B\Gamma^2[\ln(2\Gamma)+\gamma], \qquad \ldots(2.19)$$

and

$$U(\lambda\to\infty)=\Delta F+T\Delta S=NK_BT\frac{\varepsilon}{2}[\ln(2\varepsilon)+2\gamma-1]=NK_BT\Gamma^2[2\ln(2\Gamma)+2\gamma-1]. \qquad (2.20)$$

3.0 CONCLUSION

Thermodynamic quantities of two-dimensional one component plasmas in the domain of weak coupling have been obtained using analytical method based on Mayer's giant cluster expansion. We have shown that the short-range correlation makes significant contribution to thermodynamic quantities. The results will be useful in investigating two-dimensional system including the single-layered dust particles in dusty plasmas. This work is in progress.

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APPENDIX: Calculations of eqns (2.9) and (2.10)

We rewrite I_1 into $I_1 = I_{11} + I_{12}$

Where

$$I_{11} = \pi n \int_0^{r_0} drr\left(e^{-\beta v} - 1 + \beta v\right). \quad (1)$$

and

$$I_{12} = \pi n \int_0^{r_0} drr\left\{e^{-\beta v}\left(e^{-\beta(u-v)} - 1\right) + \beta(u-v) - \frac{1}{2}\beta^2 u(u-v)\right\}. \quad (2)$$

Here I_{12} represents the effect of screening. The integral I_{11} is calculated to be

$$I_{11} = \frac{\varepsilon}{4}\left\{-\ln\left(\frac{\beta e^2}{r_0}\right) - \gamma + \frac{3}{2} + \frac{1}{3}\left(\frac{\beta e^2}{r_0}\right) + \cdots\right\} \quad (3)$$

To evaluate I_{12}, we note that

$$\beta\{u(r) - v(r)\} = -\frac{\pi}{2}\varepsilon\{H_0(K_D r) - N_0(K_D r)\}; \quad (4)$$

where $N_0(x)$ and $H_0(x)$ are the Neumann's and Sturve's function of zeroth order respectively (Watson, 1944). For $K_D r << 1$, we have

$$\beta\{u(r) - v(r)\} = \varepsilon\{\ln(K_D r) + \gamma - \ln 2 + \cdots\} \quad (5)$$

Since the effect of screening in the short –range domain is thus small, we put

$$\exp(-\beta v) \times \left[\exp\{-\beta(u-v)\} - 1\right] \approx -\exp(-\beta v)\beta(u-v) \text{ and } \beta^2 u(u-v) \approx \beta^2 v(u-v) \quad (6)$$

in (2). We have

$$I_{12} = -\frac{\varepsilon}{4}\int_0^{K_D r_0} dy \int_0^{\infty} dx \frac{1}{x+y} J_0(x) + \pi n \beta \int_0^{r_0} drr(u-v)(1 - \beta v - e^{-\beta v}) \quad (7)$$

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