GLOBAL JOURNAL OF PURE AND APPLIED SCIENCES VOL. 13 NO. 1 , JANUARY, 2007 : 69 - 75
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CENTRAL DIFFERENCE APPROXIMATION OF HEAT FUNCTION IN FORCED LAMINAR FLOW ON FLAT PLATE WITH VARYING SURFACE TEMPERATURE

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(Recieved 20 September 2004; Revision Accepted 18 August, 2005)

ABSTRACT

Heat function formulation in forced laminar flow over a flat plate with spatial surface temperature variation is considered The variation of the surface temperature rendered the classical analytic approach through similarity transformations intractable In order to circumvent this difficulty, a computational algorithm is presented to approximate the heat function along the plate The surface temperature estimate employed based, on perturbed collocation method recovers some well known discrete method on the grid points namely, the central difference approximations of order two to the derivates, the most accurate p-stable linear multi-step of Ritchmeyer and Morton using a legendre perturbation term Numerical results show that the surface temperature variation has significant effect on the heat transfer. For a given set of parameters, Re, Pr, and ε, a reasonable heat gain is indicated for the spatial surface temperature variation for the range $0 < \eta < 1.0$ considered.

KEYWORDS: Perturbation, Central Difference Flat Plate, Nusselt number

NOMENCLATURE

U,V	= Dimensionless velocity components, u/u_∞ and $vRe^{1/2}/u_\infty$
u,v	= Dimensional velocity component in x - and y-directions, m/see
x,y	= Dimensional horizontal and vertical coordinates, m
u_∞	= Mainstream fluid velocity m/sec
L_c	= Characteristic length of plate, m
R_e, P_r	= Corresponding Reynolds and Prandtl numbers respectively
T	= Fluid temperature, k
T_{mw}	= Mean wall temperature, k
T_∞	= Mainstream fluid temperature, k
N	= Number of spatial divisions along ξ.

Greek Symbol

η, ξ	= Dimensionless horizontal and vertical coordinates, x/L_c and $y Re^{1/2}/L_c$
a	= Thermal diffusivity of fluid
δ^*_T	= Dimensionless thermal boundary layer
θ	= Dimensionless temperature potential, $(T - T_\infty)/(T_{mw} - T_\infty)$
ε	= Perturbation parameter

Subscript

i,j	= Nodal point
w	= Wall values
∞	= Ambient conditions

1.0 INTRODUCTION

The problem of heat transfer in forced laminar flow over a flat plate with varying surface temperature has been investigated by many researchers such as Fage and Falkner (1931), Chapman and Rubesin (1949), Tamaki (1951), Levy (1952) and Punis (1953) This is due to its numerous applications such as in the cooling of electronic equipment design of cover plates for solar collectors and in the design of solar food dryers

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The solution of the Problem consists in the determination of the temperature profile along the surface of the plate and in the thin boundary layer that developed. For a stepwise axial variation of the surface temperature in the direction of flow, Kays and Crawford (1980) concluded that it is possible to construct a solution of the problem by merely breaking up the surface temperature domain into a number of constant surface-temperature subdomains and summing or superposing the constant-surface temperature for each of the subdomains together. This method has been extensively used and has the advantage of being applied not only for turbulent but as well as laminar flow conditions. However, the evaluation of the resulting integral is somewhat tedious.

In this paper therefore, a computational algorithm for heat function in forced laminar flow over a flat plate with spatial variation of the surface temperature is presented. The energy transport equation is discretized following Oluyede (1995) and the temperature profile along the surface is determined based on perturbed collocation method of solving second order ODE formulated by Taiwo (1991). Results indicate a strong variation of the local Nusselt number along the plate with the Reynolds and Prandtl numbers as well as the perturbation term, ε.

2.0 MATHEMATICAL FORMULATION

The physical model and discretized domain with associated Cartesian coordinate is as shown in Figure 1

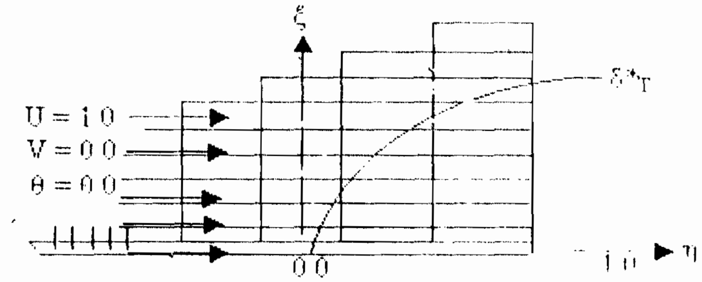

Fig. 1 Schematic for coordinate system, boundary constraints and numerical grids

The fluid flow over the flat plate is assumed to be laminar, constant property and two-dimensional. The effect of viscous dissipation is neglected and the surface temperature is assumed to vary spatially along the plate. Hence, the applicable normalized boundary layer equations are;

$$\frac{\partial U}{\partial \eta} + \frac{\partial V}{\partial \xi} = 0 \qquad (1)$$

$$U\frac{\partial U}{\partial \eta} + V\frac{\partial U}{\partial \xi} = \frac{\partial^2 U}{\partial \xi^2} \qquad (2)$$

$$U\frac{\partial \theta}{\partial \eta} + V\frac{\partial \theta}{\partial \xi} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial \xi^2} \qquad (3)$$

The normalized boundary conditions are,

$$\xi = 0,\ U = V = 0.0,\ \theta = F(\eta) \qquad (4)$$

$$\xi = \delta^*_T,\ U - 1.0 = V = 0,\ \theta = 0.0$$

At the stagnation point, $\eta = \xi = 0.0$

$$U - 1.0 = V = 0.0,\ \theta = 0.0 \qquad (5)$$

The solution of Equation (2) and the mode of discretization of Equation (3) with the associated boundary conditions are available in Oluyede (1995) but have been omitted here for the sake of brevity

2.1 NUMERICAL PROCEDURE

The thermal boundary layer thickness, δ^*_T at η = 1 0 can be estimated on the basis of mass flux conservation to be

$$\delta^*_T = 2/\alpha$$

Where α is a constant found to be 0.3321 (Howarth, 1938) and the grid sizes along the horizontal axis are computed from the stability criteria of equation (3) as follow;

$$\eta_{i,j} = \eta_{i-1,j} + \frac{\alpha(j-1)\Pr(\Delta\xi)^3}{2A} \tag{6}$$

$\Delta\xi = \delta^*_T/N$, $i = j = 2, 3, \ldots, N$

Where A is constant greater than unity to ensure the stability of the numerical scheme and N is the number of spatial divisions on the horizontal axis. Having determined all the grid points, the formal definition of $\Delta\eta_i$ is given as

$$\Delta\eta_i = \Delta\eta_{i+1,1} - \Delta\eta_{i,1}$$

Where i = 1, 2, 3...N-1and $\eta_{i,1} = 0.0$ at the origin

The condition that the surface temperature varies spatially along the plate can be written, following Taiwo (1991), as

$$\theta(\eta) = F(\eta) \quad L\theta(\eta) = \varepsilon\theta''(\eta) = F(\eta) \qquad \text{for } \eta_{i-1} \le \eta \le \eta_{i+1} \tag{7}$$

Where ε is a small parameter and L is an operator defined by

$$L = \varepsilon\frac{d^2}{d\eta^2} + 1$$

Equation (7) is perturbed using Legendre polynomial as

$$\varepsilon\theta''_2(\eta) = f(\eta) + R_2(\eta)$$

and

$$R_2(\eta) = m_1P_1(\eta) + m_2P_2(\eta)$$

m_1, m_2 are constant parameters; P_1 and P_2 are the Legendre polynomials

Hence

$$\varepsilon\theta''_2(\eta) = f(\eta) + m_1P_1(\eta) + m_2P_2(\eta) \quad \text{for } \eta_{i-1} \le \eta \le \eta_{i+1} \tag{8}$$

An order two approximate solution of Equation (8) can be obtained in the form

$$\theta(\eta) \approx \theta_2(\eta) = \sum_{k=0}^{2} a_k Q_k(\eta) \tag{9}$$

Where a_k are constants and $Q_k(\eta)$ are the canonical polynomials which are associated with the operator L.

$$LQ_k(\eta) = \eta^k$$

$$L\eta^k = \varepsilon k(k-1)\eta^{k-2} + \eta^k$$

That is $L[LQ_k(\eta)] = \varepsilon k(k-1)LQ_{k-2}(\eta) + LQ_k(\eta)$

From the linearity of L, and the existence of L^{-1}, we have

$$LQ_k(\eta) = \varepsilon k(k-1)Q_{k-2}(\eta) + Q_k(\eta)$$

Hence,

$$Q_k(\eta) = \eta^k - \varepsilon k(k-1)Q_{k-2}(\eta) \tag{10}$$

Where k = 1, 2.

For the above problem, Taiwo (1991) proposed a continuous scheme for the smooth varying region $\eta_{i-1} < \eta < \eta_{i+1}$ using Legendre polynomial as

$$\theta_2(\eta) = \theta_{i,1} + \frac{(\eta-\eta_i)(\theta_{i+1,1}-\theta_{i,1})}{\eta_{i+1}-\eta_i} + \frac{(\eta-\eta_{i+1})(\eta-\eta_i)(f_{i+2}+4f_{i+1}+f_i)}{12\varepsilon} \tag{11}$$

This formula obtained the temperature profile along the surface of the plate. On the grid points, the scheme recover some well, known discrete methods, namely the central difference approximations of order two to the derivates, the most accurate p-stable linear multi-step of Ritchmeyer and Morton using a legendre perturbation term.

However, the discrete value $\theta(\eta_{i+2})$ is obtained by collocating Equation (11) at $\eta = \eta_{i+2}$ with the condition $\theta_2(\eta) = \theta_{i+2}$

Thus

$$\theta_{i+2,1} = \theta_{i,1} + \frac{(\Delta\eta_{i-1}+\Delta\eta_i)(\theta_{i+1,1}-\theta_{i,1})}{\eta_{i+1}-\eta_i} + \frac{(\Delta\eta_{i+1})(\Delta\eta_{i+1}+\Delta\eta_i)(f_{i+2}+4f_{i+1}+f_i)}{12\varepsilon} \tag{12}$$

2.2 HEAT TRANSFER ANALYSIS

The local coefficient based on the distance from the leading edge of the plate is evaluated as

$$Nu = -\mathrm{Re}^{1/2}\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=0} \tag{13}$$

The temperature profile at any location along the plate can be expressed as a polynomial of the form,

$$\theta = \theta_0 + A_1\xi + A_2\xi^2 \tag{14}$$

At $\xi = 0.0$, $\theta = \theta_0 = \theta_{i,1}$ and at $\xi = \Delta\xi$, $\theta = \theta_{i,2}$ for $1 \le i \le N$. Therefore

$$\theta_{i,2} = \theta_{i,1} + A_1\Delta\xi + A_2(\Delta\xi)^2 \tag{15}$$

At $\xi = 2(\Delta\xi)$, $\theta = \theta_{i,3}$ Hence,

$$\theta_{i,3} = \theta_{i,1} + A_1(2\Delta\xi) + A_2[4(\Delta\xi)^2] \tag{16}$$

Solving Equations (15) and (16) simultaneously for A_1 and A_2 and substituting these values in Equation (14) and then differentiating the resulting equations with respect to ξ, we have.

$$\left(\frac{\partial\theta}{\partial\xi}\right) = \frac{(4\theta_{i,2}-\theta_{i,3}+3\theta_{i,1})}{2(\Delta\xi)} \tag{17}$$

Hence,

$$Nu = \mathrm{Re}^{1/2}\left[\frac{3\theta_{i,1}-4\theta_{i,2}+\theta_{i,3}}{2(\Delta\xi)}\right] \tag{18}$$

Where i = 1, 2, 3..., N and the mean Nusselt number is given by

$$\bar{Nu} = \frac{1}{2}\sum_{i=1}^{N-1}\left[\frac{(Nu_{i,1}+Nu_{i+1,1})(\Delta\eta_i)}{\eta_N}\right] \tag{19}$$

3.0 RESULTS AND DISCUSSION

The effect of the Reynolds number on the local heat transfer coefficient for a given set of
Reynolds number on the local heat transfer coefficient for a given set of parameters is illustrated in Fig. 2. It is observed that as the Reynolds number increases, the local heat transfer coefficient along the plate gradually increase downstream. This is due to the fact that, as the velocity of flow increases, more heat is been convected away from the surface of the plate. This result agreed well with that of Bello-Ochende and Obiajunwa (1990) for Newtonian flows.

In Fig.3, as the Reynolds numbers increase, the average coefficients of heat transfer along the plate increase asymptotically downstream. There is a remarkable effect on the average heat transfer coefficient as heat is convected rapidly from the surface of

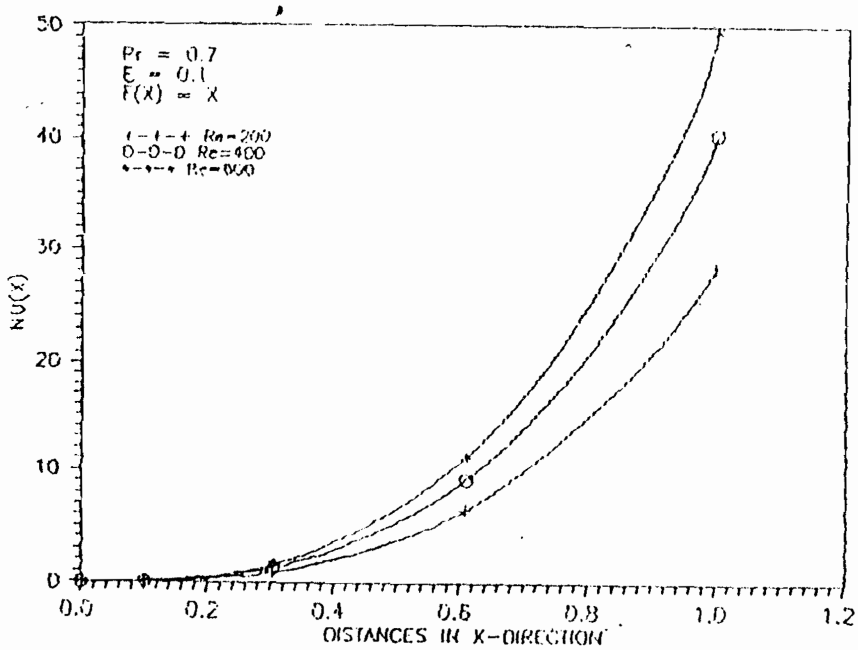

Fig 2. Effect of Reynolds Number on Local Nusselt Number along the plate.

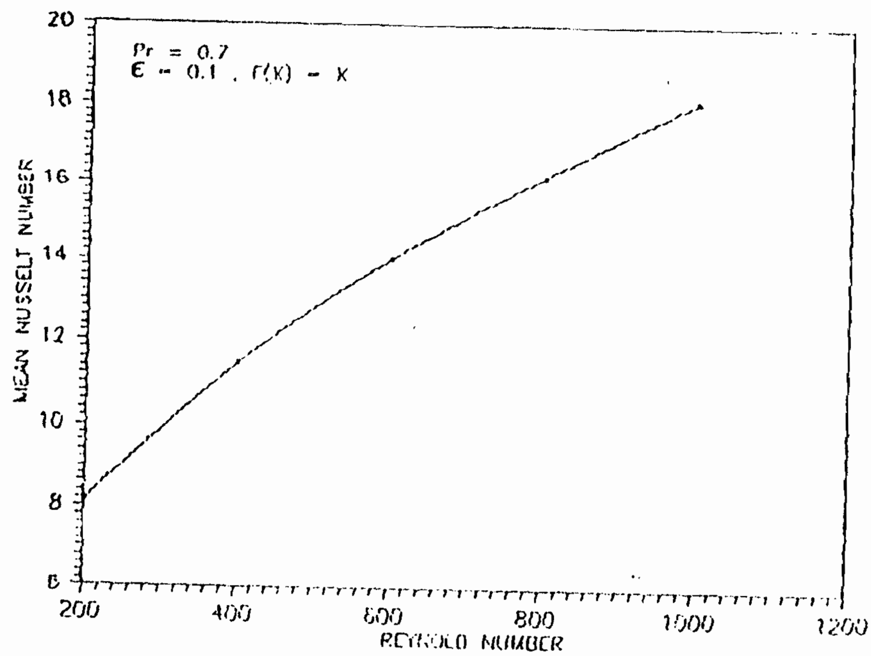

Fig. 3. Effect of Reynolds Number on the mean Nusselt Number

the plate in fast flows than in slow flows.

An observation of Fig.4 shows the effect of the perturbation parameter on the average heat transfer coefficient. As the perturbation parameter increase, there is a remarkable decrease on the average heat transfer coefficient. As the perturbation parameter approaches zero, the average heat transfer coefficient along the plate becomes more pronounced. Figure 5 shows the influence of the Prandtl number on the heat transfer coefficient along the plate. The increase in the Nusselt number with growing Prandtl number changes somewhat as the heat transfer coefficient increase with increasing thermal diffusivity of the fluid.

In Fig.6, the effect of the local Nusselt number for various functions describing the temperature variation along the surface of the plate are compared with that of the corresponding isothermal surface. Since the surface temperature increases spatially from the leading edge, the air stream approaching the trailing edge would have convected relatively low enthalpy from the leading edge, hence more heat is convected at the trailing edge. This is a departure from the isothermal case where the bulk of the heat transfer is concentrated at the leading edge. Although, the isothermal case shows a higher heat transfer rate initially, the spatial surface temperature variation shows an overall heat gain for the same set of parameters.

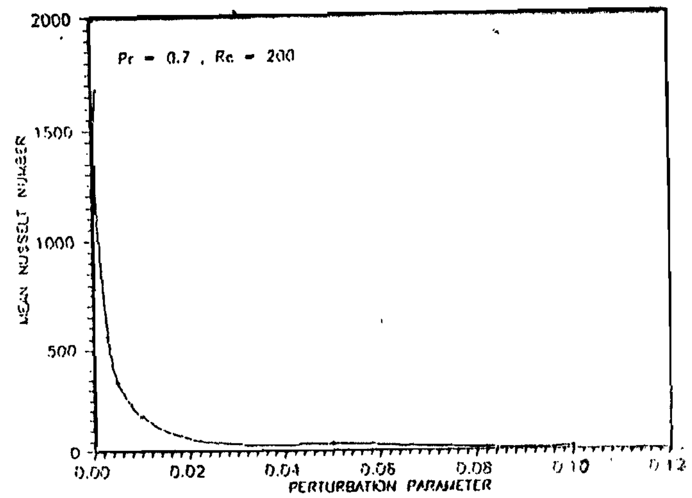

Fig. 4. Effect of perturbation parameter on the mean Nusselt Number.

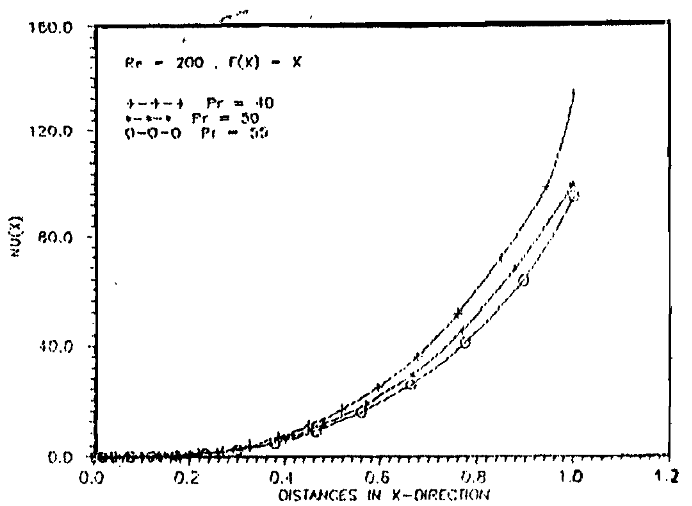

Fig. 5. Effect of the Prandtl Number the local Nusselt Number along the plate

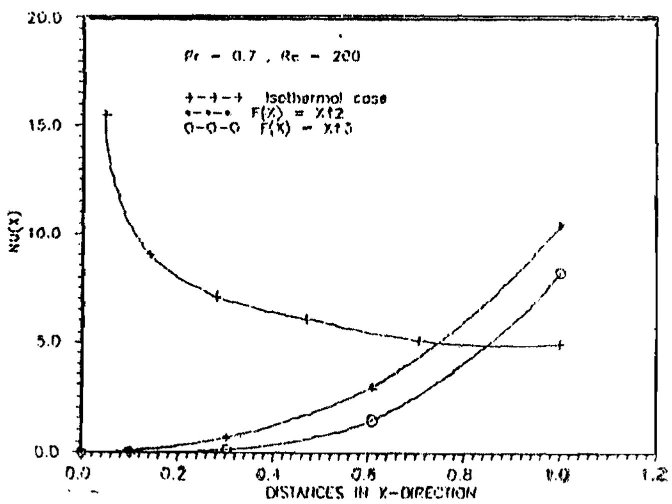

Fig. 6. Comparison of the Local Nusselt Numbers with the corresponding Isothermal case.

4.0 CONCLUSION

In conclusion, it is clear that the surface variation has a significant effect on heat transfer over the flat plate. For a given set of parameters; Re, Pr and ε, a reasonable heat gain for the spatial surface temperature variation is indicated for the range $0 \le \eta \le 1.0$ considered.

5.0 ACKNOWLEDGEMENT

The authors acknowledge with thanks the contributions of Prof. F L Bello-Ochende of Mechanical Engineering Department and Dr O.A Taiwo of Mathematics Department, both of University of Ilorin. Thanks also go to the Faculty of Engineering and Engineering Technology, University of Ilorin for the use of their computer laboratory.

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