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COLLISIONAL KINETIC ENERGY DISSIPATION IN THE SUPARAURORAL IONOSPHERE FOR DRIFTING MAXWELLIANS

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ABSTRACT

The coulomb collision transfer rates for momentum and energy exchange between drifting Maxwellians is investigated theoretically. The developed time span and energy exchange or heating rate is numerically solved for plasma parameter relevant to the auroral region. It is shown that the coulomb collision between the hot precipitating electron beam and the auroral plasma results in an up going electron beam as is observed experimentally. It was observe that there is increase in temperature of the hot precipitating electron beams after collision instead of it thermalizing with the background plasma.

KEYWORDS: coulomb collision, kinetic energy dissipation, suparauroral –ionosphere

INTRODUCTION

Collisional transport in plasma is well understood in the case of small deviations of particle distributions from thermodynamic equilibrium. Transport properties have been derived in detail for the situation where the deviations (caused by gradients and fields) of the distribution functions from a Maxwellian and small, i.e., only of the order of the comparatively short mean – free- path divided by the gradient scale height, which is assumed to be large. Based on the Chapman – Enskog expansion procedure (Chapman and Cowling, 1952) various transport coefficients have explicitly been calculated. There are classical reviews on this topic by many authors (Braginskii, 1966; Spitzer and Harm, 1953; Hinton 1983). The coulomb collision operator in the form of the Landau integral (Rosenbluth et al, 1957) or the Fokker – Planck equation (Chandrasehkar, 1943), however, not only applies to the situation of dense, weakly inhomogeneous plasmas close to thermodynamic equilibrium but it is also appropriate for the rather dilute plasmas as they occur in interplanetary space or planetary magnetospheres.

The exact coulomb collision transfer rates for energy and momentum exchange between various species depend, of course, on the detailed underlying distribution functions. Rigorous expressions for these rates have only been worked out within the test particle approach, where one species is described by a delta function in velocity space (Spitzer, 1962) or for drifting Maxwellians (Tanenbaum, 1957; Trubnikor, 1965). On the basis of these references, Schunk, 1977 and Barakat and Schunk, 1982, have advanced a transport theory for various collision processes and for multicomponent isotropic space plasma based on drifting bi-Maxwellian distribution functions. In a special case of coulomb collisions, their energy exchange or heating rates are equal to the result obtained from the Fokker – Planck equation (Salat, 1974).

Observations in the auroral region by several reporters have revealed the presence of field – aligned electron beams (feb). Sharp et al (1980) and Lin et al. 1982 reported counter streaming electrons using data from S3 – 3 and DE – 1 satellites, respectively. Klumpar and Heikkila 1982 discovered field – aligned up flowing electrons at low altitude, using data from the ISIS – 2 satellites. Collin et al. 1982 made a statistical study on the occurrence and characteristics of field – aligned electron beams based on S3 – 3 observation while Miyake et al. 1998 reported a statistical study of field – aligned electron beams associated with ion conics events based on Exos – D (Akebono) satellite. These electrons are aligned to within 10° of the local magnetic field and have energies of several tens of eV to a few Kev.

In this present paper, we do not intend to calculate transport coefficients, but instead we investigate the coulomb collisional heating rate or energy exchange between drifting maxwellians in the auroral region. Our aim is to demonstrate the heating and acceleration of electrons due to coulomb collision in the auroral region.

2. THEORETICAL CONSIDERATION

We consider our model for the auroral acceleration region to consist of a three component: plasma hot precipitating electron beams as the test particle which streams downwards in a direction opposite to the ions with drift velocity U_{he}, temperature T_h and density N_h; ion beam moving upward away from the Earth along the auroral field lines with drift velocity U_{hi}, temperature T_i and density N_i and cold background electrons with temperature T_c and density N_c. The external equilibrium geomagnetic field is pointing upwards along the Z – direction, ie $\mathbf{B_o} = B_o\mathbf{Z}$.

2.1 Coulomb Collision Operators

The starting point of our calculation is the Fokker – Planck equation (Cairns, 1985):

$$\frac{\delta f}{\delta t} = -\frac{\delta}{\delta \mathbf{V}}\left(\mathbf{A_a} f_a - \frac{1}{2}\frac{\delta}{\delta \mathbf{V}} . \mathbf{D_a} f_a\right) \qquad (1)$$

The first term on the right hand side comes from the slowing down of the particle owing to collisions, and is referred to as the friction term (drag force $\mathbf{A_a}$), while the second term represents diffusion of particles (diffusion tensor $\mathbf{D_a}$) in velocity space. In plasma the equilibrium Maxwellian distribution results from a balance between the friction term, tending to concentrate the particles around the mean flow velocity of the plasma, and the diffusion term with tends to spread particles in velocity space. The Fokker – Planck

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coefficients $A_a = \Sigma_b A_{ab}$ and $D_a = \Sigma_b D_{ab}$ can be expressed by derivatives of the Rosenbluth potentials and are the sums over individual terms each of which represents the scattering effects of particles of species b on particles of species a. It is a matter of straightforward calculation from Equation (1) to show that the internal energy and momentum transfer rates between two species are generally given by (Ichimaru, 1973)

$$F_{ab} = \int d^3v\, m_a v \frac{\delta f_a}{\delta t} = m_a \int d^3v\, \mathbf{A_{ab}}(v)\, f_a(v) \qquad (2)$$

and

$$Q_{ab} = \int d^3v\, m_a \frac{(V - U_a)^2}{2} \frac{\delta f_a}{\delta t}$$

$$= m_a \int d^3v \left\{ \mathbf{A_{av}}(v).\,(v - u_a) + \tfrac{1}{2} T_r\, \mathbf{D_{ad}}(v) \right\} f_a(v). \qquad (3)$$

where u_a is the bulk velocity of the species a in an arbitrary frame of reference.

For drifting isotropic distributions, the drag force and diffusion tensor are given as (Marsch and Livi, 1985).

$$A_{ab}(v) = -\frac{1}{t_{ab}} v_b \bar{y}\, \nu_F(y) \frac{1}{2} \left(1 + \frac{M_a}{M_b} \right) \qquad (4)$$

$$D_{ab}(v) = \frac{1}{t_{ab}} \frac{1}{2} v_b^2 \left[\nu_L(y)\, \bar{y}\, \bar{y} = \nu_T(y)\,(1 - \bar{y}\,\bar{y}) \right] \qquad (5)$$

where u_b is the drift velocity of species b, $v_b = (2K_B T_b/m_b)^{1/2}$ is the thermal speed, $y = (v-u_b)/v_b$, $\bar{y} = y/y$ and the time scale involved is

$$t_{ab}^{-1} = 8\pi e^2{}_a e^2{}_b \ln \Lambda\, n_b / (m_a^2 v_b^3). \qquad (6)$$

The friction rate ν_F and longitudinal ν_L and transverse diffusion rates ν_T are given by derivatives of the Rosenbluth potentials. These rates for the isotropic "self – similar" and "kappa" distributions (generalized Gaussian and Lorentzian) have been calculated by Marsch and Livi, 1985 as:

$$y\, \nu_L(y) = \nu_F(y) = (\phi - y\,\phi) / y^2 = -(\phi / y)^1 \qquad (7)$$

and

$$\nu_T(y) = \phi/y - \tfrac{1}{2} \nu_L(y). \qquad (8)$$

The symbol $\phi(y)$ represents the standard error function

$$\phi(y) = 2/\pi^{1/2} \int_0^y dz\, e^{-z2}$$

which according to Stephenson, 1995 can be expressed as

$$\phi(y) = y - \frac{y^3}{3.1!} + \frac{y^5}{5.2!} - \frac{y^7}{7.3!}$$

The error function $\phi(y)$ can also be expressed in terms of the Maxwell integral $\mu(y)$ as (Trubnikor, 1965):

$$\mu(y) = \phi(y) - y\phi^1(y). \qquad (9)$$

Approximate expressions for $\mu(y)$ for two limiting cases are:

$$\mu(y) = \begin{cases} \frac{4y^{3/2}}{3\sqrt{\pi}} \left(1 - \frac{3}{5} y + \frac{3}{14} y^2 - \ldots\right) (y<<1) \\ 1 - \frac{2}{\sqrt{\pi}} e^{-y} \sqrt{y} \left(1 + \frac{1}{2y} - \frac{1}{4y^2} + \ldots\right) (y>>1) \end{cases} \quad (10)$$

Recently, Duwa, 2002 had made use of generalized Lorentzian (Kappa) drift distribution to develop lower hybrid wave instability by electron beams precipitating in the auroral zone.

2.2 Collisional energy and momentum transfer rates between drifting Maxwellian

Burgers, 1969 and Salat, 1974 have calculated the transfer rates for drifting Maxwellians:

$$F_{ab}(v) = \frac{n_{a,b}}{\pi^{3/2} V^3_{a,b}} \exp\{-(v-u_{a,b})^2/V_{ab}^2\} \quad (11)$$

for which Equation (4)- (8) is directly applicable. Their calculations shall not be repeated here, rather the role-played by velocity space friction and diffusion in the interspecies energy and momentum transfer as presented by Hernandez and Marsch, 1985 is considered. The combined thermal speed is defined as

$$w_{ab}^2 = v_a^2 + v_b^2 \quad (12)$$

and the normalized differential speed as

$$x = (u_a - u_b) / w_{ab}. \quad (13)$$

Finally, we obtain the momentum transfer rate in compact form by making use of equations (4) – (8) as

$$F_{ab} = n_a m_a \hat{A}_{ab} \quad (14)$$

where $\hat{A}_{ab} = -\frac{1}{\tau_{ab}} w_{ab} \mathbf{x} \; \nu_F(x)$ (15)

and

$$\tau_{ab}^{-1} = 4\pi e^2_a e_b^2 \ln\Lambda \, n_b / (\mu m_a w_{ab}^3) \quad (16)$$

Recently, the same equation as that of equation (16), was used by Chike – Obi et al, 2000 to determine the life times of precipitating ionosphere electron beams. When comparing Equation (15) and (4), it is obvious that the frictional deceleration A_{ab} between two drift ting Maxwellians resembles the frictional force any particle of speed V suffers in an ambient medium of density n_b and thermal speed V_b. The difference being that V is to be replaced by $u_a - u_b$ and V_b by w_{ab} as far as the interspecies interaction is concerned. Since by inspection $n_a m_a / T_{ab} = n_b m_b / T_{ab}$, we find that $F_{ab} = -F_{ba}$ according to Equation (14) and (15), which is consistent with Newton's third law.

For the heating rate Q_{ab} the calculations are straightforward but somewhat tedious and can be conveniently expressed as (Hernandez and Marsch, 1985):

$$Q_{ab} = n_a m_a \left[(v_a/w_{ab})^2 (u_b - u_a). \tilde{A}_{ab} + \frac{m_b v_b^2 - m_a v_a^2}{(m_a + m_b) w_{ab}^2} T_r \mathbf{D}_{ab}\right] \quad (17)$$

Where the diffusion tensor is

$$\mathbf{D}_{ab} = \frac{1}{\tau_{ab}} \frac{1}{2} w_{ab}^2 [\nu_L(x) \mathbf{XX} + \nu_T(x) (\mathbf{I} - \mathbf{XX})] \quad (18)$$

and its trace is given by the expression below making use of equation (8) as

$$T_r \mathbf{D}_{ab} = w_{ab}^2 / T_{ab} \, \phi(x)/x.$$

For particle number conservation during collision, we have

$$\partial n_a/\partial T \Big|_c = 0 \quad (19)$$

Equation (14) may be rewritten as (Trubnikov, 1965):

$$\left.\frac{\partial u_a}{\partial t}\right|_c = -\frac{1}{\tau_{ab}} (u_a - u_b)\, \nu_L(x) \qquad (20)$$

The collisional charge of the test particle temperature T_a is related to Q_{ab} by

$\frac{3}{2} n_a \frac{\partial T_a}{\partial t} = Q_{ab}$. Hence, one obtains

$$\left.\frac{\partial T_a}{\partial t}\right|_c = -\frac{1}{\tau_{ab}} \left\{ (T_a - T_b) \frac{m_a}{m_a + m_b} \frac{2}{3}(\nu_L + 2\nu_T) - T_a \frac{4}{3} x^2 \nu_L(x) \right\} \qquad (21)$$

The second term of Equation (21) is caused by the work done by the drag force on the differential speed. The kinetic energy dissipated (Equation 21) in this way always tends to increase the temperature of the test particle. T_a. This trend is opposite to the one, which leads to temperature equalization with T_b as discussed, in most classical work of coulomb collision. Equations (19) and (21) can be readily generalized to multi-component plasma by simply summing over the index b, which is what we are going to do in this report.

3. APPLICATION TO THE AURORAL REGION

In the auroral acceleration region, between 1 to 2 R_E altitudes we take typical values according to Lakhina, 1993 of the hot precipitating electron component as the test particle which streams downwards in a direction opposite to the ion with drift velocity U_{he} = 3657 km/s, temperature T_h = 1 Kev and density N_h = 1.0 cm^{-3}; ion beam moving upward away from the Earth along the auroral field lines with drift velocity U_{hi} = 1545 km/s, temperature T_i = 10ev and density N_i = 11 cm^{-3} and cold background electrons with temperature T_c = 1 ev and density N_c = 10cm^{-3}. Inserting these values into Equation (16) and finally Equation (21), the kinetic energy dissipated by coulomb collision of the multicomponent plasma is predicted by Equation (21) to be

$$\left.\frac{\partial T_a}{\partial t}\right|_c = 1.7907 \times 10^{-15}\, \text{eV / sec.}$$

The temperature of the precipitating hot electron beam after a time span of T_{ab} = 2.9067 x 10^{17} sec during collision is calculated as T_a = 520.503 eV.

According to observations in auroral acceleration region (Lakhina, 1993), the hot precipitating electron beams have their energy E_h ~ (0.1 to 5) keV. Comparing the energy of the hot precipitating electron beams from the plasma sheet and that of the energy exchange or heating rate, it is convent to say that the precipitating electron beams undergo elastic collision with the background plasma since there is little or no energy exchange during the coulomb collisions. The energy of the resultant up going electron beam after collision is approximately 521 ev. This energy value is within the range (~ 100 ev to 1 kev) of what has been observed experimentally by so many reporters (sharp et al, 1980; Lin et al, 1982; Klumpar and Heikkila, 1982; Malinger et a; 1992; Miyake et al, 1998 and Miyake et al, 2001). Again, the temperature of the test particle (hot precipitating electron beams) T_a, is much greater than that of the background plasma. As against temperature equalization with the background plasma has reported in most classical coulomb collision.

4. CONCLUDING REMARKS

We have developed the coulomb collision transfer rates for momentum and energy. Solving the energy exchange or heating rate numerically as applied to the auroral region, we found out that the scattering of the hot precipitating electron beams by the suparauroral plasma leads to up going field aligned electron beams as observed experimentally. Hence, we can conclude that coulomb collision in the suparauroral ionosphere in combination with other forms of acceleration mechanism could be responsible for the ionospheric plasma acceleration. Again, from our calculation we observed that; the temperature of the hot precipitating electron beam instead of thermalizing with the ionospheric plasma increases in value as up going field aligned electron beam when compared with the temperature of the background ionospheric plasma.

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