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ON THE COMBINED EFFECT OF THE CHEMICAL REACTION AND A HIGHER ORDER TEMPERATURE PROFILE ON THE VELOCITY OF A STRETCHED VERTICAL PERMEABLE SURFACE IN THE MAGNETOHYDRODYNAMIC (MHD) FLOW IN THE PRESENCE OF HEAT GENERATION AND ABSORPTION

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ABSTRACT

In this work, it has been assumed that heat generation is non-linear, and this extends previous results in literature. It has further been assumed that the coefficient of non-linearity depends on a small parameter $\in = \alpha(T_w - T_\infty)$ where T_w is the wall temperature and T_∞ is the temperature at infinity. Velocity, temperature and concentration have been expanded asymptotically We provide solutions for θ_0, θ_1, θ_2 U_0, U_1, and U_2 where θ_0, and U_0, ...are the respective initial temperature and velocity while θ_1, θ_2, U_1, and U_2, are higher order temperature and velocity profiles. It is shown that θ_1 has a maximum which shows that the non-linearity has significant effect on the heat generation.

KEYWORDS: Magnetohydrodynamic, Permeable Surface, Asymptotic Expansion, Boundary Layers, Ambient Fluid

INTRODUCTION

Magnetohydrodynamic (MHD) flow of electrically conducting fluids in the presence of magnetic field is encountered in many problems in geophysics and astrophysics. Many industrial processes involve fluid flow, heat and mass transfer in the boundary layers induced by a surface moving with a uniform velocity. Many of such processes involve handling materials along boundary layers conveyors, the extrusion of plastic sheets, the cooling of finite metallic plate in a cooling bath, glass blowing etc.

There has been increased interest in the study of MHD flow and heat transfer due to the effect of magnetic fields on the flow using electrically conducting fluids such as liquid metal, water mixed with acid and others.

Herbert (2003) investigated the similarity solutions of the boundary layer flow along a vertical plate. He suggested that the similarity solutions of the difference between the temperature of the plate and the temperature of the ambient fluid is inversely proportional to the distance from the leading edge of the plate. He used the modified boundary-layer equations in describing incompressible mixed convection flow along a vertical plate.

Gbadeyan and Dada (1998) considered the effect of variable fluid properties and radiative magnetohydrodynamic (MHD) flow of a fluid in a vertical channel. The problem concerned the flow of electrically conducting fluid inside an infinite vertical channel formed by two parallel plates of distance 2L apart. They analysed the effect of temperature dependent fluid properties on a radiative MHD flow of a fluid in an open- ended vertical channel.

Gbadeyan and Andi (1999) carried out a study on the combined effect of radiation and viscous dissipation on hydromagnetic fluid flow in a vertical channel. Ogunsola (2000) in the study of unsteady non-isothermal free boundary flows in porous media used the series method of solution to find a similarity solution for the equation of energy transfer in a porous medium.

The heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream was investigated by Vajravelu and Hadjinicolaou (1993). They reported that heat generation/absorption effects in a moving fluid is important. While Gupta and Gupta (1997) studied heat and mass transfer on a stretching sheet with suction or blowing, Crane (1970) investigated the flow induced by a surface moving with constant velocity in an ambient fluid.

Recently, Muthucumaraswamy (2001) reported the effects of heat generation/absorption and magnetic effects on a moving isothermal infinitely long surface with suction while Chamkha (2003) studied a generalization of the Muthucumaraswamy's problem.

Ayeni et al (2004) provided a higher order correction to the previous temperature field obtained by Chamkha. They discovered that the temperature field obtained by him did not exist where the scaled heat generation/absorption coefficient is greater than a quarter of the Prandtl number.

This work reported analytical solution for the problems of heat and mass transfer by the steady flow of an electrically conducting and heat generating/absorbing fluid in a uniformly moving permeable surface in the absence of a magnetic field. The problem is derived from the first order chemical reaction.

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FORMULATION OF THE PROBLEM

Assumptions used in the formulation

The flow is assumed steady, laminar and two-dimensional. The concentration of species is assumed to be infinitely long (i.e. dependent variables are not dependent on the vertical or axial coordinate). It is also assumed that the applied transverse magnetic field is uniform and that the magnetic Reynolds number is small so that the induced magnetic field is negliglected. In addition, there is no applied electric field and all of the Hall effect, viscous dissipation and Joule heating are neglected.

The Governing Equations

The governing equations in literature are

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} - \tau c, \tag{2}$$

$$\rho c_p v\frac{\partial T}{\partial y} = k\frac{\partial T}{\partial y} + Q(T - T_\infty) \tag{3}$$

$$v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_c(c - c_\infty) - \frac{\gamma\beta_o^2}{\rho}u \tag{4}$$

where

y =	horizontal or transverse coordinate
u =	axial velocity
v =	transverse velocity
T =	fluid temperature
c =	species concentration
ρ =	density
g =	gravitational acceleration
β_T =	coefficient of thermal expansion
β_c =	coefficient of concentration expansion
μ =	dynamic viscosity
γ =	fluid electrical conductivity
Q =	heat generation/absorption coefficient
D =	mass diffusion
τ =	chemical reaction parameter
k =	fluid thermal conductivity

The physical boundary conditions for the problems are

$$\left.\begin{aligned} &u(0) = u_w,\ v(0) = -v_w\ \ T(0) = T_\infty\ \ c(0) = c_w \\ &y \to \infty\ \ u \to 0\ \ T \to T_\infty\ \ c \to c_\infty \end{aligned}\right\}, \tag{5}$$

where u_w (a constant) is the surface velocity
v_w; the suction velocity
T_w; the surface Temperature
and c_w is the concentration

The equation (3) is modified as

$$\rho c_p v\frac{\partial T}{\partial y} = k\frac{\partial T}{\partial y} + Q(T - T_\infty) + Q\alpha(T - T_\infty)^2 \tag{6}$$

so that the heat absorption term be non-linear.
The term $Q\alpha(T - T_\infty)^2$ in (6) extends previous models in literature.
The solution to (1) subject to the physical boundary conditions is $v = -v_\infty$
Using this result and variables

$$Y = y\left(\frac{v_w}{\nu}\right), \quad U = \frac{u}{u_w}; \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}; \quad C = \frac{c - c_\infty}{c_w - c_\infty}$$

equations (2), (4) and (6) are transformed respectively to dimensionless ones as

$$\frac{d^2C}{dY^2} + Sc\frac{dC}{dY} - K\,Sc\,C = 0 \quad (7)$$

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} + Gr_T\,\theta + Gr_c\,C - M^2U = 0, \quad (8)$$

$$\frac{d^2\theta}{dY^2} + Pr\frac{d\theta}{dY} + Pr\,\phi\,\theta + Pr\,\phi\in\theta^2 = 0 \quad (9)$$

where θ = dimensionless Temperature
U = dimensionless velocity
C = dimensionless concentration

$\phi = \frac{Qv}{\rho c_p v_w^2}$ = dimensionless heat generation/absorption coefficient

$K = \frac{\tau v}{v_w^2}$ = dimensionless chemical reaction parameter

$Pr = \frac{\mu c_p}{k}$ = Prandtl number

$Sc = \frac{v}{D}$ = Schmidt number

$Gr_T = \frac{g\beta_T v(T_w - T_\infty)}{u_w v_w^2}$ = Thermal Grashof number

$Gr_C = \frac{g\beta_C v(c_w - c_\infty)}{u_w v_w^2}$ = Mass Grashof number

$M = \frac{\gamma\beta_o^2}{\rho v_w^2}$ = Hartmann number

$\in = \alpha(T_w - T_\infty)$

The non-dimensional physical boundary conditions are

$$\left.\begin{aligned} &U(0) = 1; \quad \theta(0) = 1; \quad C(0) = 1 \\ \text{as}\quad &Y \to \infty;\ U \to 0,\ \theta \to 0 \text{ and } C \to 0 \end{aligned}\right\} \quad (10)$$

Method of Solutions

The major contribution of this work is the case $\in \neq 0$ in (9)

We seek the asymptotic solutions for the equations.

Let $0 < \in << 1$; $\theta = \theta_0 + \in\theta_1 + \in^2\theta_2\ldots$; $U = U_0 + \in U_1 + \in^{2U}{}_2\ldots$; and

$C = C_0 + \in C_1 + \in^2 C_2\ldots$;

We obtained

$$u_1(Y) = a_6(e^{2mY} - e^{nY}) + a_7(e^{mY} - e^{nY})$$

$$u_2(Y) = a_8(e^{mY} - e^{\lambda Y}) + a_9(e^{2mY} - e^{\lambda Y}) + a_{10}(e^{3mY} - e^{\lambda Y})$$

where $m = -\frac{1}{2}\left(\Pr + \sqrt{\Pr^2 - 4\Pr\phi}\right)$; $n = \frac{1}{2}\left(1 + \sqrt{1 + 4M^2}\right)$

$$\lambda = -\frac{1}{2}\left(Sc + \sqrt{Sc^2 + 4KSc}\right) \quad a_1 = \frac{-\Pr\phi}{4m^2 + 2m + \Pr\phi};\ a_2 = \frac{-2a_1\Pr\phi}{9m^2 + 3m\Pr + \Pr\phi}$$

$$a_3 = \frac{2a_1\Pr\phi}{4m^2 + 2m\Pr + \Pr\phi};\ a_4 = \frac{-Gr_T}{m^2 + m - M^2};\ a_5 = \frac{-Gr_C}{\lambda^2 + \lambda - M^2}$$

$$a_6 = \frac{-a_1 Gr_T}{4m^2 + 2m - M^2};\ a_7 = \frac{a_1 Gr_T}{m^2 + m - M^2};\ a_8 = \frac{(a_1 - a_3)Gr_T}{m^2 + m - M^2}$$

$$a_9 = \frac{-a_3 Gr_T}{4m^2 + 2m - M^2};\ a_{10} = \frac{-a_2 Gr_T}{9m^2 + 3m - M^2}$$

Hence, by $U = U_0 + \in U_1 + \in^2 U_2 \ldots$ we have

$$U(Y) = e^{nY} + a_4(e^{mY} - e^{nY}) + a_5(e^{\lambda Y} - e^{nY}) + \in[a_6(e^{2mY} - e^{nY}) + a_7(e^{mY} - e^{nY})] +$$
$$\in^2[a_8(me^{2mY} - \lambda e^{\lambda Y}) + a_9(2me^{2mY} - \lambda e^{\lambda Y}) + a_{10}(3me^{3mY} - \lambda e^{\lambda Y})]$$

Graphical Solutions

The graphical solutions of equations (17), (18) and (17 and (18) combined for varying values of ϕ are displayed in figures 1, 2, and 3 respectively while the effects of the varying chemical reactions (K) are displayed in figures 4 and 5.

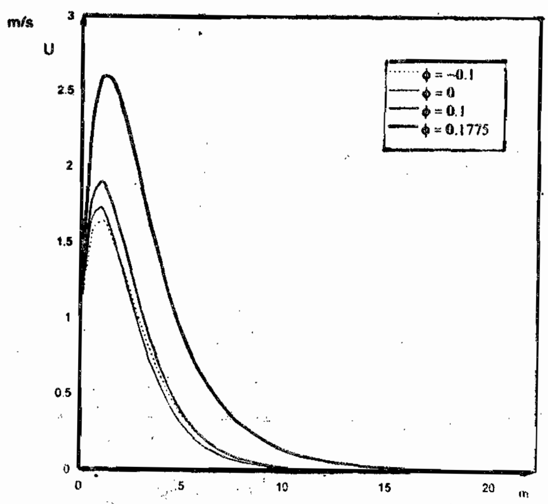

Figure 1: Effect of heat generation/absorption ϕ on axial velocity U_0 for equation (16) when Gr_T = 1.0 Gr_C = 1.0; K = 0.0; Pr = 0.71; Sc = 0.6;

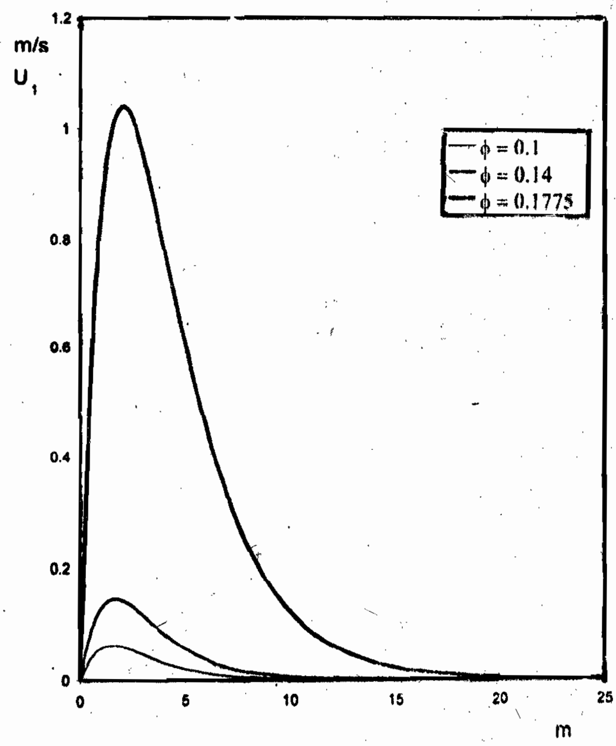

Figure 2: Effect of heat generation/absorption ϕ on axial Axial Velocity U_1 for equation (17) when $Gr_T = 1.0$ $Gr_C = 1.0$; $K = 0.0$; $Pr = 0.71$; $Sc = 0.6$;

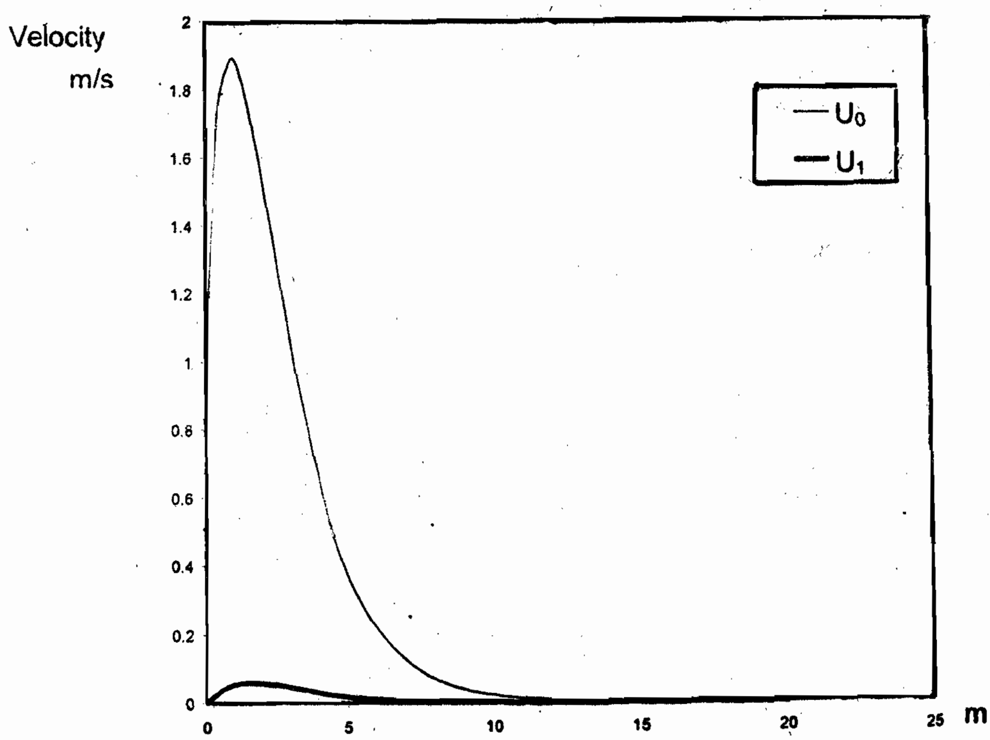

Figure 3: Graph of velocity profile for $K = 0.1$ and $\phi = 0.1$ for equations (16) and (17) combined $Gr_T = 1.0$ $Gr_C = 1.0$; $K = 0.0$; $Pr = 0.71$; $Sc = 0.6$; $M = 0$

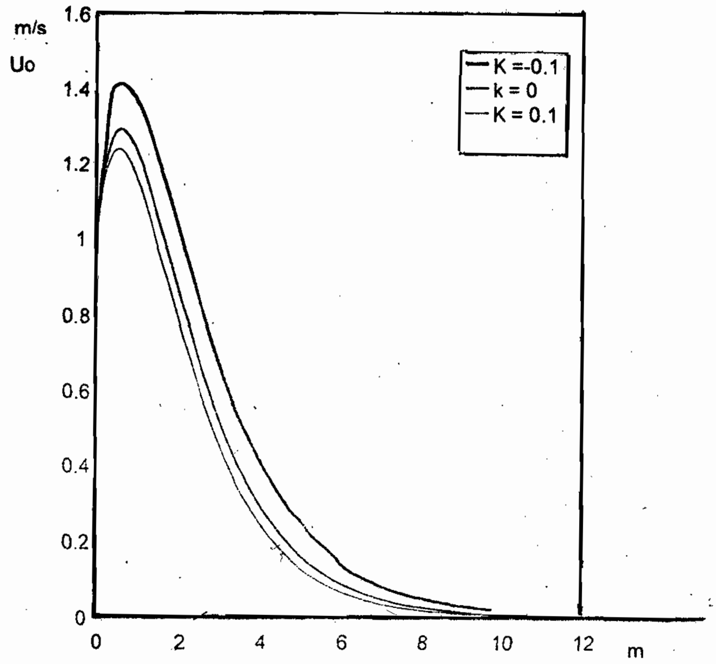

Figure 4: Effect of varying chemical reactions (K) when $Gr_T = 1.0$ $Gr_C = 1.0$; $Pr = 0.71$; $Sc = 0.6$; $M = 0$

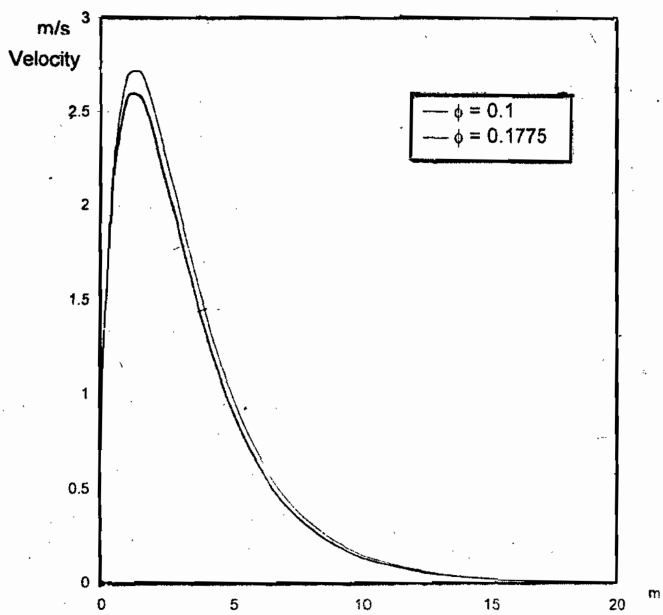

Figure 5: Effect of varying level of heat absorption (ϕ) on the velocity when $Gr_T = 1.0$ $Gr_C = 1.0$; $Pr = 0.71$; $Sc = 0.6$; $M = 0$

RESULTS AND DISCUSSION

For the purpose of this work, a constant Prandtl number $Pr = 0.71$; Schmidt number $Sc = 0.6$; Mass Grashof number $Gr_T = 1.0$ and Thermal Grashof number $Gr_T = 1.0$ were considered.

The figures present results for the various combinations of the parameters K and ϕ on the velocity (U). Note that

K < 0 indicates a generative Chemical reaction
K = 0 indicates no Chemical Reaction
K > 0 indicates a destructive Chemical Reaction
ϕ < 0 indicates Heat Generation
ϕ = 0 indicates no Heat Generation
ϕ > 0 indicates Heat Absorption

Figure 1 and 2 depict the influence of heat generation/absorption in the absence of chemical reaction (K = 0.0) on the velocity profiles U_0 and U_1 respectively. In all cases, there is an initial increase in the velocity of the fluid flow only to start decreasing with distance after the peak has been attained.

Figure 3 displays the effects of both a destructive chemical reaction (K = 0.1) and heat absorption (ϕ = 0.1) on the velocity profiles U_0 and U_1. The combination of a destructive chemical reaction and heat absorption has little effect on U_1.

Figure 4 shows the effects of a generative chemical reaction (K=-0.1), the absence of chemical reaction (K = 0.0) and a destructive chemical reaction (K = 0.1) on the velocity profile U_0 in the absence of the heat generation (ϕ = 0.0) There was an initial rise in the velocity. This was followed by a steady fall in the velocity.

Figures 5 displays the effects of the heat absorption (ϕ = 0.1) and (ϕ = 0.1775) in the presence of a destructive chemical reaction (K = 0.1) on the velocity. gain, there exist no significant difference between U_0 and U for ϕ = 0.1 and a significant difference is noticed for the maximum value of ϕ = 0.1775.

Conclusion

An analytical solution for heat and mass transfer in the boundary layer induced by a steady laminar flow of an electrically conducting and heat generation/absorption fluid on a uniformly moving vertical permeable surface in the absence of a magnetic field and a first-order chemical reaction has been reported.

Based on the obtained graphical results, it is concluded that fluid velocity increases during a generative chemical reaction and decreases during a destructive one. Also, the presence of heat generation effects increases the fluid velocity while the presence of heat absorption effects decreases it.

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