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THE HYDROGEN ATOM IN A MAGNETIC FIELD

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ABSTRACT

Using the reduced orbit method the rapid transition to chaoticity of positive-energy hydrogen atom in a slowly varying magnetic field is deduced to be energy of the system rather than the magnetic field.

INTRODUCTION

Classical conservative Hamiltonian system of degree $n \geq 2$ is known to be chaotic even though governed by deterministic laws. Thus in spite of exact reversibility of the equations of motion, the inherent stability in the orbits of these systems lead to instability of initial conditions (Casati, 1984). As a result of this extreme sensitivity of initial conditions, chaos has been branded to be the property of simple systems since it is extremely difficult to specify initial conditions and then map the initial trajectories for many-particle system (Kleppner, 1991).

The hydrogen atom is one of such simple experimentally viable systems consisting of one electron and proton (Friendrich and Wintgen, 1989). For this reason, we examine another facet of hydrogen atom in a magnetic field (Ajala, 1998).

The main objective of this paper is to know what contributes what and what is exactly responsible for rapid transition to chaoticity in this simple system. It is discovered that the energy of the system is responsible for rapid transistion to chaoticity. This was achieved through the reduced orbit method (ROM) (Ajala, 1998) by fixing the energy in the positive energy regime and slowly varying the magnetic field.

THEORY

The Hamiltonian of the atom in a magnetic field (along z-axis) is (Delande et al, 1991).

$$H = \frac{1}{2}p^2 - \frac{\gamma^2\rho^2}{8} + \frac{\gamma}{2}L_z \qquad (1)$$

where γ is the dimensionless field strength parameter and the angular momentum in the z-axis, L_z, the coordinates, r and the conjugate momenta, p (Ajala, 1998) are defined as follows

$$L_z = xp_y - yp_z$$
$$p^2 = p_x^2 + p_y^2$$
$$r^2 = \rho^2 + z^2$$

For high fields ($\gamma >> 1$) and neglecting coupling terms $(\rho^2+z^2)^{-1/2}$, the system is integrable i.e.

$$\frac{p^2}{2} + \frac{\gamma^2\rho^2}{8} = \frac{p_x^2}{2} + \frac{(\gamma/2)^2}{2}x^2 + \frac{p_y^2}{2} + \frac{(\gamma/2)^2}{2}y^2$$
$$= (n_1 + \frac{1}{2})\gamma/2 + (n_2 + \frac{1}{2})\gamma/2$$
$$= (\frac{n_1 + n_2}{2} + \frac{1}{2})\gamma \qquad (2)$$

Thus, we can write (Delande et al, 1991)

$$E_N = (N + \frac{1}{2})\gamma \qquad (3)$$

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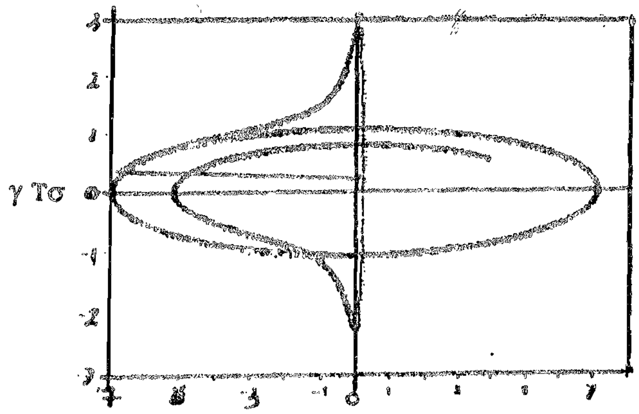

Fig. 1: Phase Portraits for $\gamma = 0.6$

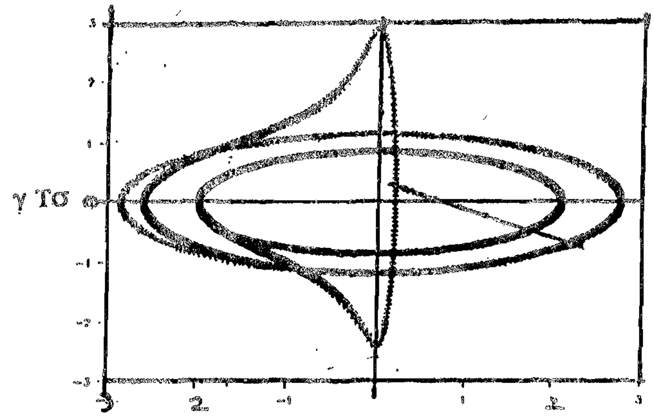

Fig. 2: Phase Portraits for $\gamma = 0.8$

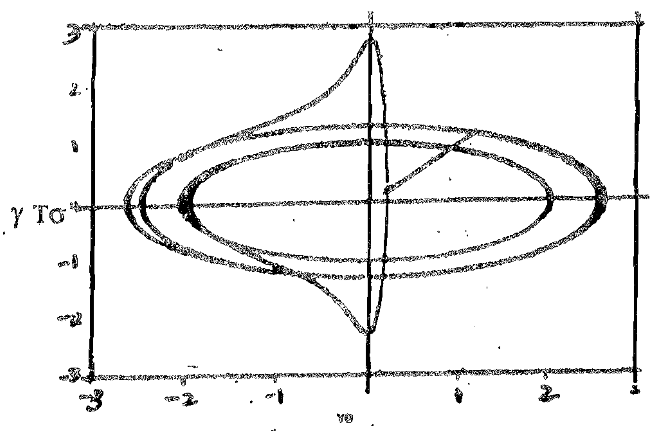

Fig. 3: Phase Portraits for $\gamma = 1.0$

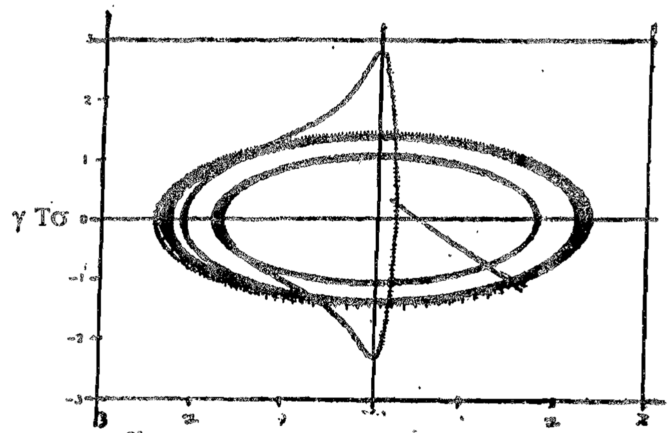

Fig. 4: Phase Portraits for $\gamma = 1.2$

where

$$E_N = \frac{p^2}{2} + \frac{\gamma^2 \rho^2}{8}$$

and

$$N = \frac{n_1 + n_2}{2}$$

E_N are the eigenvalues and N, the principal quantum number. Equation (3) is otherwise referred to as Landau oscillator.

Thus at high fields, the system behaves like a free-particle due to adiabatic separation of the slow motion along the magnetic field. However at low fields ($\gamma<1$) such separation does not apply.
The Kamiltonian (Ajala, 1998) is

$$K = [2E' - (p^2 - \frac{2}{r} + \frac{\gamma^2 \rho^2}{4})]$$

where E' is assumed small. The Lyapunov analysis of this system has been shown to be positive depicting the instability of the system.

RESULTS AND DISCUSSION

The chaoticity of this system under a slow variation is depicted by the self-intersection and self-similarity of the trajectories in Figures 1 to 4.

Comparing Figures 1 to 4 with the earlier ones (Ajala, 1998), it is discovered that chaoticity occurs in the former within a wide interval as the energy increases than in the latter which has a narrow interval. Furthermore, at small values of γ, the ellipse around the fixed point broadens but the self-intersection is small. As γ increases the ellipse decreases in size as the self-intersection increases. The attractor basin also broadens at the sides in place of the absorbing point. This simply shows that although this system is bounded by γ (Ajala, 1998), the rapid transition to chaoticity is governed by the energy.

CONCLUSION.

The slow variation of the magnetic field with the energy fixed has been discussed. It is concluded that although Hydrogen atom in a magnetic field (and in positive energy regime) is bounded by γ, the rapid transition to chaoticity is due to the energy of the system. It is further discovered that the absorbing point with almost sure probability in our earlier result gave way to an expanded attractor basin.

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