GLOBAL JOURNAL OF PURE AND APPLIED SCIENCES VOL. 11, NO. 4, SEPTEMBER 2005: 567 - 573
COPYRIGHT(C) BACHUDO SCIENCE CO. LTD. PRINTED IN NIGERIA. ISSN 1118-0579

INTERACTION OF SOLITARY PULSES IN SINGLE MODE OPTICAL FIBRES

A. USMAN, J. OSMAN and D. R. TILLEY

(Received 28 February, 2004; Revision accepted 8 December, 2004)

ABSTRACT

We attempt to classify interaction of nonlinear optical pulses that propagate in form of solitary waves in single mode optical fibres. Two solitary waves launched, by way of incidence, into an optical fibre form a single pulse if the pulses are in-phase as understood from results of inverse scattering transform method applied to the cubic nonlinear Schrödinger equations, (CNLSE's). The single CNLSE is then understood to describe evolution of coupled pulses identified with one optical wavelength. More general physical implications abound such that coupled nonlinear Schrödinger equations, (NLSE's), have to be used to describe dynamics of the coupled pulses as in the effects of cross-phase modulation, (XPM), and phenomenon of birefringence. Our governing NLSE's have the quintic terms. Using a finite-difference method that combines forward- and central-difference approximation schemes, we give results of numerical simulations of coupled in-phase pulses in single pulse system of the NLSE's. Simulations are also given for the XPM in presence of walk-off from analytical results.

KEYWORDS: Nonlinear Schrodinger Equations, solitons, optical pulses, fibre optics

INTRODUCTION

Certainly, analyses of coupled pulses, in monomode optical fibres, constitute a wider area of research efforts. The efforts are necessitated by phenomenon of dynamics of coupled nonlinear optical pulses which have more device modelling and manufacturing applications, particularly, in optical communication systems (Agrawal, 1995; Chaudhry and Moris, 2000). Considerable attention, in this area, continues to yield many more newer theoretical (Yeh and Bergman, 1999) and experimental results (Starodumov, et al., 1998).

It is inferred from theoretical studies that the nonlinear Schrödinger equations, (NLSE's), may be single or coupled depending on the dynamics of the pulses involved (Usman, 2000). A single NLSE is more appropriate to describe the physics of two or more coupled pulses if it is assumed theoretically or experimentally that the pulses have the same magnitude of carrier optical wavelength. In this case, it may be supposed that the pulses form a single beam of solitary waves or soltions providing that the optical fibre allows the pulses to remain in the same state of wave polarisation. According to the inverse scattering method, (ISM) as discussed in (Agrawal, 1995; Desam and Chu, 1992), which has been confirmed by the variational approach (Anderson and Lisak, 1985), the pulses will interact with each other as classical particles. Numerical approach gives credence to this, with an inferable proviso that the pulses will have to be in-phase (Hermasson and Yavic, 1983; Desam and Chu, 1992).

From practical stand point, however, there are many more physical situations such that coupled NLSE's are inevitable to explain the physics of the coupled pulses (Agrawal, 1995; Usman, 2000; Desam and Chu, 1992). These can also be subclassified into two: cross-phase modulation phenomenon that is associated with coupled pulses that propagate with different optical wavelengths and thus tend to walk-off from one another; phenomenon of birefringence is the other one of which two optical pulses would copropagate with the same optical wavelength, but differ in polarisation states. The former can be practically realised in pump-probe experiments (Agrawal, 1995; Starodomov, et al., 1998; Agrawal, et al., 1989) or as in communication systems of pulse amplification wherein the second pulse is generated internally as a consequence of the gain sustained by stimulated Raman scattering (SRS), (Agrawal, 1995; Desam and Chu, 1992). The later is also ubiquitous in practical applications (Usman, 2000).

Based on the forgone brief classification of pulse coupling, the purpose of this communication is to illustrate both the in-phase coupling of two pulses in single NLSE and the coupling of two pulses that have different optical wavelengths thus requiring coupled NLSE's. For the single NLSE, simulation results will be compared as obtained from data of cubic nonlinear Schrödinger equation, (CNLSE), and cuibc-quintic nonlinear Schrödinger equation, (CQNLSE) with application of a finite-difference scheme (Usman, 2000; Cowan, et. al., 1986). Simulations will also be given to illustrate how walk-off and XPM balance each other, in maintaining coupled optical pulses distortionless in profiles, from analytical results of a new coupled system of CQNLSE's (Usman, 2000).

A. USMAN, Department of Physics, Federal University of Technology, Yola, Adamawa State, Nigeria. e-mail: aausman@yahoo.co.uk
J. OSMAN, School of Physics, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia. e-mail: junaidah@usm.my.
D. R. TILLEY, School of Physics, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia. e-mail: drt@pd.jaring.my

Propagation equations

The governing equations for the physics of propagating pulses are derivable from the Maxwell's equations, in differential forms, by several methods (Agrawal, 1995; Kumar, 1990). In dimensionless form the equations are

$$i\frac{\partial U}{\partial \xi}-\frac{\delta}{2}\frac{\partial^2 U}{\partial \tau^2}-\alpha_0 U+|U|^2U+v_{NL}|U|^4U=0 \quad (1)$$

$$i\frac{\partial U_1}{\partial \xi}+\frac{1}{2}\frac{\partial^2 U_1}{\partial \tau^2}-\alpha_{1r}U_1+N^2\left(|U_1|^2+2|U_2|^2\right)U+\mu_{1r}\left(|U_1|^4+3|U_2|^4+6|U_1|^2|U_2|^2\right)U_1=0 \quad (2a)$$

$$i\frac{\partial U_2}{\partial \xi}+iL_r\frac{\partial U_2}{\partial \tau}+\frac{\beta_r}{2}\frac{\partial^2 U_2}{\partial \tau^2}-\alpha_{2r}U_2+\omega_r N^2\left(|U_2|^2+2|U_1|^2\right)U_2$$

$$+\mu_{2r}\left(|U_2|^4+3|U_1|^4+6|U_1|^2|U_2|^2\right)U_2=0 \quad (2b)$$

Equation (1) is the CQNLSE for the single pulse system; equations (2) are the new coupled CQNLSE's for two copropagating pulses of differing optical wavelengths (Usman, 2000).

In (1), $\delta = -1$ would be used for anomalous dispersion propagation, U is the dimensionless complex field obtained from $U(\xi,\tau) = A(\xi,\tau)/A_0$ where $A(\xi,\tau)$ and A_0 are respectively the subsequent complex field of pulse, and the input pulse whose magnitude is given by $A_0 = N_s(|\beta_2|\lambda A_{eff}/2\pi n_2)^{1/2}/\tau_0$ in which $|\beta_2|$ is the magnitude of the group velocity dispersion parameter, A_{eff} is the effective mode cross-sectional area of the single mode optical fibre; $N_s \equiv (L_D/L_N)^{1/2}$, the soliton order as related to dispersion length, L_D, and nonlinear length L_N, so that $N_s = 1$ implies fundamental solitary pulse; τ_0 is the real pulse duration, $\omega \equiv 2\pi c/\lambda$, the angular frequency where c is the speed of light and λ is the optical wavelength of the pulse, $n_2 \equiv 3\chi^{(3)}/(8n_0)$, the third-order nonlinear refractive index where $\chi^{(3)}$ is the corresponding third-order nonlinear susceptibility and n_0 is the linear refractive index. The dimensionless propagation distance $\xi \equiv z/L_D$ where z is the actual distance of propagation in metres; the shifted dimensionless time relates to the actual time, t, by $\tau = (t - z/v_g)/\tau_0$ where v_g denotes the group velocity. The dimensionless constant, $\alpha_0 \equiv L_D\lambda\varpi^2/(4\pi)$ where ϖ is a propagation constant of separation for modal and amplitude propagation equations (Usman, 2000); $v_{NL} \equiv (4n_4|\beta_2|\lambda)/(3\pi n_2^2\tau_0^2)$ where $n_4 \equiv 5\chi^{(5)}/(16n_0)$ is the fifth-order nonlinear refractive index as related to $\chi^{(5)}$ the fifth-order nonlinear susceptibility of the optical fibre.

In equations (2), parameters L_r, β_r and ω_r have the following definitions: $L_r \equiv (\varepsilon_{GVD}L_D)/L_W$ where L_W is the walk-off length between the pulses as a result of group velocity mismatch parameter $\varepsilon_{GVD} \equiv (v_{g1} - v_{g2})/(|v_{g1} - v_{g2}|) = \pm 1$ such that for $v_{g1} > v_{g2}$ one gets $\varepsilon_{GVD} = +1$ and $\varepsilon_{GVD} = -1$ if $v_{g1} < v_{g2}$. The optical wavelengths of the copropagating pulses are λ_1 and λ_2; $\beta_r \equiv |\beta_{22}|/|\beta_{21}|$ where $|\beta_{21}|$ and $|\beta_{22}|$ are the group velocity dispersion parameters of the respective two copropagating pulses; $\omega_r \equiv \omega_2/\omega_1 = \lambda_1/\lambda_2$. The walk-off length L_W is given by

$$L_W = \frac{\tau_0}{\left|\left(1/v_{g1}\right)-\left(1/v_{g2}\right)\right|} \quad (3)$$

All variables and other parameters in (2) have the same meanings corresponding to the single pulse system of equation (1).

Solitary wave solutions

In equation (1), by putting $n_4 = 0$, $n_2 \neq 0$, the CNLSE is obtained of which solitary wave solutions are well known (see ref. 10 of Usman, et al., 1998). For $n_2 \neq 0$ and $n_4 \neq 0$, solitary wave solutions of (1) are also available (Usman., et al., 1998). Here the relevant one of the solutions has an envelope profile of the form

$$|U(\tau)| = \frac{\sqrt{2}}{2\tau_p}\left\{1+\left(1-\frac{4}{3}|v_{NL}|U_0\right)\cosh\frac{\tau-\tau_c}{\tau_p}\right\}^{-1/2} \quad (4)$$

where U_0 is the input pulseheight, τ_c is the centre of the solitary wave, and τ_p the dimensionless pulsewidth is given by (Usman, et al., 1998)

$$\frac{1}{\tau_p^2}=4\left(U_0^2-\frac{2}{3}|\nu_{NL}|U_0^4\right) \tag{5}$$

Observe that the input pulse differs from the actual pulseheight for equation (1) in contrast to the case of CNLSE.

By applying the symmetry/antisymmetry conditions (Usman, 2000; Wadati, et al., 1992) given by $U_1(\tau,\xi)=\pm\alpha_{ev}U_2(\tau,\xi)$, $\alpha_{ev}>0$, where α_{ev} is referred to as pulse envelope factor, the solitary wave solutions of the coupled CQNLSE's (2) are

$$U_1(\tau,\xi)=U_{01}\exp[i\mathrm{v}_1(\tau-\mathrm{v}_1\xi)]\left\{1+B_1\cosh\frac{\tau-\tau_{d1}-\mathrm{v}_1\xi}{T_{01}}\right\}^{-1/2} \tag{6a}$$

$$U_2(\tau,\xi)=U_{02}\exp\left[\frac{i}{\beta_r}(\mathrm{v}_2-L_r)(\tau-\mathrm{v}_2\xi)\right]\left\{1+B_2\cosh\frac{\tau-\tau_{d2}-\mathrm{v}_2\xi}{T_{02}}\right\}^{-1/2} \tag{6b}$$

where the dimensionless pulsewidths (also referred to as solitonlengths (Usman, et al., 1998a)) have the following expressions

$$T_{01}=\left[2|\mathrm{v}_1(2-\mathrm{v}_1)|\right]^{-1/2} \tag{7a}$$

$$T_{02}=\beta_r\left[2\left|2\beta_r\mathrm{v}_2-(\mathrm{v}_2-L_r)^2\right|\right]^{-1/2} \tag{7b}$$

Other parameters are

$$U_{01}=\frac{\alpha_{ev}}{2T_{01}}\left[\frac{2}{\alpha_{ev}^2+2}\right]^{1/2} \tag{8a}$$

$$U_{02}=\frac{1}{2T_{02}}\left[\frac{\beta_r}{\omega_r N^2(1+2\alpha_{ev}^2)}\right]^{1/2} \tag{8b}$$

$$B_1=\frac{\left|4\mu_{1r}(\alpha_{ev}^4+6\alpha_{ev}^2+3)+3T_{01}^2(\alpha_{ev}^2+2)^2\right|^{1/21}}{\sqrt{3}T_{01}(\alpha_{ev}^2+2)} \tag{9a}$$

$$B_2=\frac{\left|2\mu_{2r}\beta_r^2(1+3\alpha_{ev}^4+3)+3\beta_r T_{02}^2\omega_r^2N^4(1+2\alpha_{ev}^2)\right|^{1/2}}{(3\beta_r)^{1/2}T_{02}\omega_r N^2(1+2\alpha_{ev}^2)} \tag{9b}$$

It should be noticed that the dimensionless peak amplitudes are

$$U_{10}=U_{01}[1+B_1]^{-1/2} \tag{10a}$$

$$U_{20}=U_{02}[1+B_2]^{-1/2} \tag{10b}$$

That is, U_{01} and U_{02} defined by equations (8) are not the peak amplitudes as in CNLSE.

Numerical simulations

For a single pulse system, the dynamics of pulse envelopes, in the absence of loss, is governed by equation (1) when $n_4 \neq 0$, in a monomode optical fibre. If $n_4 = 0$, however, the CQNLSE (1) becomes CNLSE (Agrawal, 1995; Kumar, 1990; Hermasson and Yavic, 1983). The pulse envelope of the former has expression given by (4). The corresponding expression for CNLSE is in (Agrawal, 1995; Kumar, 1990; Hermasson and Yavic, 1983).

If the optical fibre of cross-sectional area A_{eff} = 13.5 μm^2, n_0 = 1.5, n_2 =1·2×10^{-22} (m/V)^2; n_4 = −4·4×10^{-37}(m/V)^4 is exited by two first-order solitary waves (i.e., solitons) at a carrier optical wavelength λ =1·4 μm, with β_2 = −12·5 ps^2/km, the coupled system of the pulses has an envelope described by the expression

$$|U(\tau)| = 1.3597\left\{1+0.8487\cosh\left(\frac{\tau-\tau_d}{0.52006}\right)\right\}^{-1/2} + 1.3597\exp[i\theta_p]\left\{1+0.8487\cosh\left(\frac{\tau+\tau_d}{0.52006}\right)\right\}^{-1/2} \quad (12)$$

With $n_4 = 0$, the envelope expression is given by

$$|U(\tau)| = \sec h(\tau-\tau_d) + \exp[i\theta_p]\sec h(\tau+\tau_d) \quad (13)$$

where in (12) and (13), θ_p denotes the relative phase between the two pulses (Hermasson and Yavic, 1983). The parameters, λ and β_2 yield an input power $P_0 \approx$ 0·624 W if the pulse duration τ_0 = 1·0 ps. The nonlinear coefficient $\nu_{NL} \approx -0{\cdot}113472$. Thus, the separation of the pulses is $2\tau_d$ in dimensionless units. The pulses are in-phase if $\theta_p = 0$, as will be applicable here.

To have simulacra of the interactions between the solitary pulses, we have used the finite difference scheme detailed in (Usman, 2000; Cowan, et al., 1986). We have implemented the scheme with equations (13) and (12) as the initial sources of data implying the input pulses for $n_4 = 0$ and $n_4 \neq 0$ respectively. The results, to be discussed in the next section, are displayed in Figs. 1 and 2.

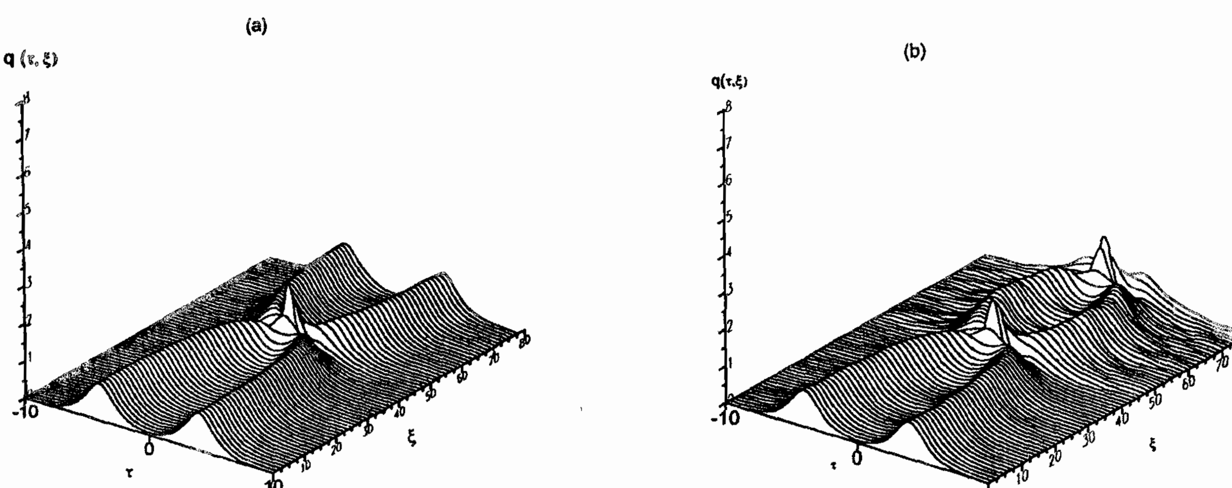

Fig. 1. Evolution plots of amplitude profiles for in-phase solitary wave interaction of two pulses, $q(\tau,\xi) = |U(\tau,\xi)|$: (a) interaction of pulses for $\nu_{NL} = 0$, i.e., $n_4 = 0$ in equation (1) implying CNLSE with equation (13) as the input coupled pulses; (b) interaction of pulses for $\nu_{NL} = -0{\cdot}044$ in equation (1) with equation (12) as the input coupled pulses.

Figs. 3, display numerical simulations from analytical results given by equations (6) - (10) which are the solitary wave solutions of the coupled CQNLSE's (2) describing the dynamics of interaction of two solitary waves from different sources of distinct optical carrier wavelengths, λ_1 and λ_2, in the monomode optical fibre. The parameters n_0, n_2, n_4, λ_1 and $|\beta_{21}|$ are the same as those of the single pulse system. The mode cross-sectional area $A_{eff} \approx$ 12·566 μm^2, $\lambda_2 \approx$1·6μm and $|\beta_{22}| \approx$ 35·0 ps^2/km. For a pulse duration of 1.0 ps, the input power $P_{02} \approx$ 3·7186 W if the first pulse is considered to be the probe pulse. With those parameters known, other relevant parameters appearing in coupled CQNLSE's (2) are obtained for fundamental propagation, i.e., N = 1. These are: $\beta_r \sim 2.8$, $\omega_r \sim 0.875$, $\mu_{1r} \sim -0{\cdot}22694$, and $\mu_{2r} \sim -0{\cdot}19858$; $L_r = -4$ has been used for $L_D = 4L_W$ and $\varepsilon_{GVD} = -1$ as most likely in experimental setup where the pump pulse moves faster. Intuitively, therefore, one can use $v_1 = 1$ and $v_2 = 0.25$, i.e., $v_1 = 4v_2$ is assumed.

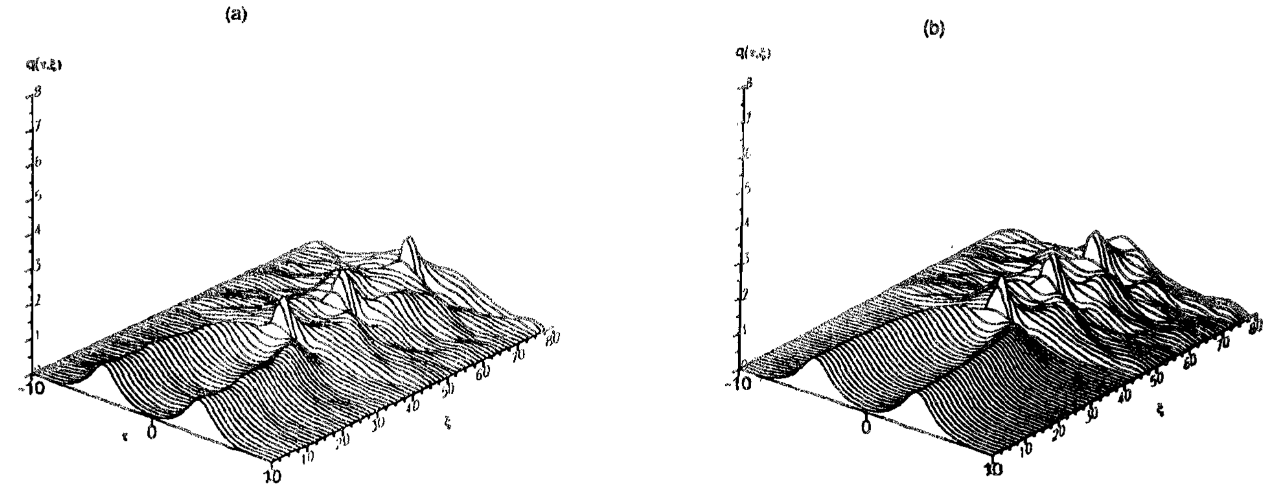

Fig. 2. Interaction of solitary in-phase pulses with v_{NL} = − 0·113472: (a) equation (13) as the input coupled pulses; (b) equation (12) as the input coupled pulses

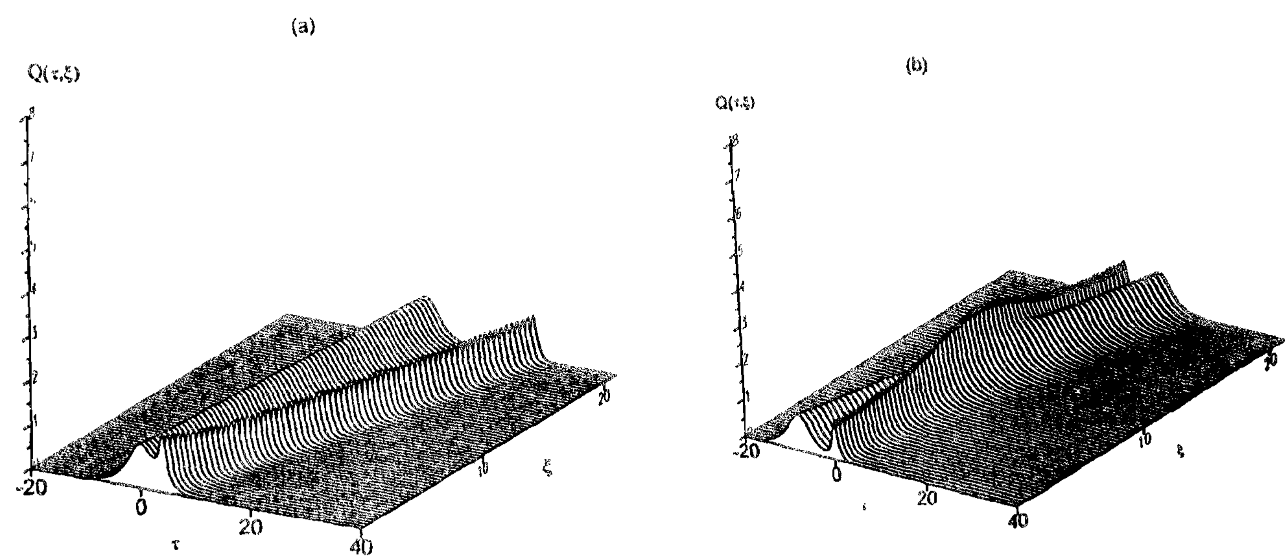

Fig. 3. Interaction of pulses due to walk-off and XPM. Data were obtained from $Q(\tau,\xi) = |U_1(\tau,\xi)| + |U_2(\tau,\xi)|$: (a) v_1 = 1.0, v_2 = 0.25, τ_{d1} = 0·0 and τ_{d2} = −4·0; (b) v_1 = 0.25, v_2 = 1.0, τ_{d1} = 8·0 and τ_{d2} = 0·0.

To implement the equations for the simulacra, a short distance is first advanced followed by calculation of data for the resultant pulse envelope. In other words, the implementation requires some intuition of the vectorial behaviours of the pulses.

DISCUSSION OF NUMERICAL RESULTS

We first consider the single system of coupled pulses depicted in Figs. 1 and 2. In all of the Figures, the centre of each pulse has been taken to be τ_d = 4·0 units so that the initial separation $2\tau_d$ = 8.0 units. As the propagation distance increases from ξ = 0 the pulses attract each other and they coalesce at $\xi \sim$ 46·0 units corresponding to actual distance of ~ 3·68 km (i.e., $z = L_D\xi = \xi\tau_0^2/|\beta_2|$). It may be said that soliton has appeared enroute to reappear in the next period. Fig 1a depicts coupled in-phase solitons of the CNLSE obtained from equations (1) by symbolically requiring n_4 = 0 for the 1.0 ps pulse duration. In Figs 1b and 2, value of n_4 given earlier was used.

In Fig 2a, with the value of the nonlinearity coefficient v_{NL} as given earlier, equation (13) was used as the input pulse profile while in Fig 2b, equation (12) was the input pulse profile. A closer look at the Figures is required to observe some structural difference especially, at the output, i.e., at the end of the propagation segment corresponding to the fibre length ξ_L = 78·0, though, they at first appear to be identical. The more obvious physical implication, when Figs 2 and Fig. 1a are compared, is that the period of pulse coalescence is greatly reduced for values of $v_{NL} > 0$ (i.e., $n_4 \neq 0$) depending on magnitudes of carrier wavelength λ, dispersion parameter $|\beta_2|$, and ultimately, the pulse duration τ_0. That is, these parameters could be adjusted or even chosen at any particular instance of experimental condition to give various values of v_{NL}. Just as could be seen in Fig 1b where $v_{NL} \approx -0{\cdot}044$ was used with separate runs of the finite-difference scheme for the two input pulse profiles given by equations (12) and (13), the Figure depicts larger distance of coalescence as compared to anyone of Figs 2.

The fibre length used, $\xi_L = 78{\cdot}0$ units corresponds to actual output length $L \approx 6{\cdot}24$ km. Beyond this length scale, chaotic structures develop as a result of divergence inherent in the scheme (Cowan, et al., 1986). A method called beam propagation method, (BPM), (Agrawal, 1995; Desam and Chu, 1992) is able to tolerate larger distance. It was used for the CNLSE in (Desam and Chu, 1992) with some details in (Agrawal, 1995), from which it could be estimated that the coalescence length is ~ 50.4 units whereas in Fig. 1b, the length is ~ 30.0 units due to inclusion of $n_4 \neq 0$. Fig. 1a closely compares with Fig. 1 of (Hermasson and Yavic, 1983). A notable difference, however, is that the distance traversed by the coupled solitary pulses is lager in Fig. 1a.

Theoretically, it is observed that the dynamics of the coupled solitary waves as described by the pair of CQNLSE's (2) is aptly, in fact exactly, given by the solutions expressed by equations (6) - (10) in anomalous dispersion regime of propagation (Agrawal, 1995). With values of v_1 and v_2 assumed based on the intuition of tendency of pump pulse to move faster, it could be seen that in Fig. 3a, at the input, (i.e., $\xi = 0$), the pulses are seen already walking away from each other after, in retroaction, they have interacted. But, the interaction effect has persisted at the input which is graphically manifested by overlap of the pulse profiles as a remnant of XPM effect through the walk-off parameter L_r. The delay time between the pulses is another factor: here $\tau_{d2} = -4$ units and $\tau_{d1} = 0$ implying a delay of 4.0 units of dimensionless shifted time. If the delay time is increased, the overlap, at the input, would be observed to be more prominent. In Fig. 3b, the envelope velocity values are interchanged with $\tau_{d1} = 8.0$ units and $\tau_{d2} = 0{\cdot}0$ which thus produced prolonged length of overlap of the two pulses before walk-off could be reached. Interaction can occur once only due to the walk-off as observed in Figs. 3, (Agrawal, 1995; Agrawal, et al., 1989).

CONCLUSION

Interaction of solitary pulses can be modelled either in single NLSE's or coupled NLSE's. Single NLSE's are used when the pulses evolve in a monomode optical fibre, with the same carrier wavelength and the condition operating is that they have the same polarisation. When the pulses originate from different optical sources or differ in polarisation states, coupled NLSE's describe the dynamics.

Here, based on results of ISM, (Agrawal, 1995; Kumar, 1990; Desam and Chu, 1992) which can be confirmed by the variational method (Anderson and Lisak, 1985), we have shown that the period of overlapping is larger when a finite difference scheme is used than in the results of BPM (Hermasson and Yavic, 1983; Desam and Chu, 1992) for coupled pulses that are in-phase in single system of CNLSE. In the single system of CQNLSE's, after the first overlapping (i.e., coalescence), identical to CNLSE the subsequent overlapping period is reduced. It would be noted, however, that this in-phase interaction is undesirable in many actual optical devices, (Agrawal, 1995; Hermasson and Yavic, 1983; Desam and Chu, 1992) most especially in communication applications (Desam and Chu, 1992; Dianov, et al., 1986).

One way to avoid the interaction is to put the pulses out of phase (i.e., $\theta_p = \pi^c$). There are other methods of overcoming the interaction (Dianov, et al., 1986). Our efforts here are to understand the physics involved in the interaction with an ultimate aim to find more practical methods of circumventing the effect from the interactions.

As depicted in Figs. 3, the descriptions of dynamics of coupled NLSE's (2) for two pulses that have distinct values of optical carrier wavelengths, which we have done here are yet to be completed. That is, the studies are still in progress on simulation for which all possible theoretical and numerical methods would be applied and results therein compared. Thus we aim to communicate further analyses.

ACKNOWLEDGEMENTS

This work was done with the facilities of Theoretical Unit at School of Physics, Universiti Sains Malaysia, (USM) from a grant credited to JO by IRPA (Malaysian Government). AU acknowledges, with gratitude, his partaking in the grant and his PhD studentship by MTCP/JPA of Malaysian Government.

REFERENCES

Agrawal, G. P., 1995, Nonlinear Fiber Optics, Academic Press.

Agrawal, G. P., Baldeck, P. L., and Alfano, P. R., 1989, Temporal and spectral effects of cross-phase modulation on copropagating ultrashort pulse in optical fibers, Phys. Rev. A40, p5063.

Anderson, D., and Lisak, M., 1985. Bandwidth limit due to incoherent soliton interaction in optical fiber communication systems, Phys. Rev. A32, p2270.

Anderson, D., and Lisak, M., 1985. Bandwidth limit due to incoherent soliton interaction in optical fiber communication systems, Phys. Rev. A32, p2270.

Cowan, S., Enns, R. H., Ragnekar, S. S., and Sanghera, S. S., 1986, Quasi soliton and other behaviour of the nonlinear cubic-quintic Schrödinger equation, Can. J. Phys. 64, p311.

Desam, C., and Chu, P. I., 1992. Soliton-soliton interactions, in Optical Solitons — Theory and Experiment, ed J. R. Taylor, Cambridge, p 107.

Dianov, E. M., Nikonova, Z. S., and Serkin, V. N., 1986, Influence of frequency modulation on the interaction of pulses in fiber waveguide, Sov. J. Quantum Electron. 16, p1148.

Hermasson, B., and Yavick, D., 1983. Numerical investigation of soliton interaction, Electron. Lett. 19, p570.

Kumar, A., 1990. Soliton Dynamics in a monomode optical fibre, Phys. Rept. 187, p63.

Starodumov, A. N., Barmenkov, Y. O., Martinez, A., and Torres, I., 1998. Nonlinear optical switch based on stimulated Raman scattering, Opt. Fiber Technol. 4, p285.

Usman, A., 2000, Studies of nonlinear Schrödinger equations for electromagnetic pulse propagation in optical fibres, PhD thesis, Uni. Sci. Malaysia.

Usman, A., Osman, J., and Tilley, D. R., 1998. Explicit variational solution of the cubic-quintic nonlinear Schrödinger equation for pulse propagation in optical fibres, J. Nonl. Opt. Phys. Mater. 7, p461.

Usman, A., Osman, J., and Tilley, D. R., 1998a. Analytical solitary wave solutions for nonlinear Schrödinger equations with denominating saturating nonlinearity, J. phys. A: Math. Gen. 31 p8397 and reference [1] therein.

Wadati, M., Iizuka, T., and Hiskado, M., 1992. A coupled nonlinear Schrödinger equation and optical solitons, J. Phys. Soc. Japan, 61, p2241.

Yeh, C., and Bergman, L. A., 1999. Existence of optical solitons on wavelength division multiplexed beams in a nonlinear fiber, Phys. Rev. E60, p2306.