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PERFORMANCES OF ESTIMATORS OF LINEAR MODELS WITH AUTOCORRELATED ERROR TERMS WHEN THE INDEPENDENT VARIBLE IS AUTOREGRESSIVE.

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ABSTRACT

The performances of five estimators of linear models with Autocorrelated error terms are compared when the independent variable is autoregressive. The results reveal that the properties of the estimators when the sample size is finite is quite similar to the properties of the estimators when the sample size is infinite although much also depends on the error terms and the individual coefficient being estimated.

KEY WORDS: Autocorrelated, Error terms, Independent variable, model.

INTRODUCTION

Autocorrelation of the error terms included in econometric models has remained a major characteristics of most time series data, (Johnston, J and Dinardo J. (1997). Many models incorporating autocorrelated error terms have been discussed in the literature. The variety of scenarios in which time series observations can be plagued by autocorrelated disturbances are so many that inspite of numerous analytical and empirical contributions already made on this subject, the available diagnostic procedures and competing corrective estimation methods leave many questions yet to be answered.

Although some authors like Chipman (1979), Kadiyala (1968) and Kramer (1980) have argued that the efficiency of the estimators at the asymptotic level depends much on the specification of the independent variable used in the experiment. There is still much need to investigate the finite sampling properties of these estimators because most of the authors investigated on the asymptotic properties of the estimators while their sampling properties are yet to be well investigated and understood.

Some researchers like Godfrey (1978) have tried to give a general approach to the treatment of autocorrelation when they occur in linear models, however, the treatment of each type as it occurs specifically in a model have always produced better result (Spitzer (1979)).

Therefore, this study shall have as its main focus, the performances of estimators of linear models with first order autoregressive disturbance terms when the independent variable is also autoregressive. Rao and Grilliches (1969) gave one of the earliest known Monte Carlo works on this study. They used a linear model with autocorrelated error terms as in equation (1.1).

$$\left.\begin{array}{l} Y_t = \beta X_t + U_t;\ X_t = \lambda X_{t-1} + V_t \qquad U_t = \rho U_{-1} + \varepsilon_t \\ E(V_t) = E(\varepsilon_t) = E(V_t\varepsilon_t) = E(\varepsilon_t\varepsilon_{t-1}) = E(V_t V_{t-1}) = 0 \\ E(V_t^2) = \sigma^2_V,\ E(\varepsilon_t^2) = \sigma^2_\varepsilon,\ |\lambda| < 1,\ |\rho| < 1.\ t = 1, \ldots, T \end{array}\right\} \text{---- (1.1)}$$

Rao and Grilliches (1969) in their study found out that for samples of size 20, the relative efficiency of OLS is considerably below that of Durbin and Prais Wintein estimators for the values of ρ close to 1, a result which is quite surprising. Kramer (1980) noted that there are some contradictions in the findings of Rao and Grilliches (1969) and went a head to explain that the apparent contradiction may be found in the

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TABLE 1: USING BIAS PROPORTION TO COMPARE THE ESTIMATORS

		T = 20			T = 40			T = 60		
ρ	Estimator	β_1	β_2	SBIAS	β_1	β_2	SBIAS	β_1	β_2	SBIAS
	OLS	-0.013536	-0.030800	0.044336	-0.0.49570	-0.019460	0.069030	0.006635	-0.025860	0.032495
	COC	-0.011068	-0.054586	0.066270	-0.052870	-0.065330	0.118157	0.006483	-0.033630	0.040113
0.4	HILU	0.001047	-0.031608	0.032655	-0.053633	-0.064485	0.118118	0.006655	-0.033350	0.040005
	MLGRID	-0.007859	-0.029337	0.037196	-0.061050	-0.055180	0.116230	0.009207	-0.028420	0.037627
	ML	-0.00770	-0.030090	-0.037860	0065350	-0.054570	0.119860	0.009590	-0.028780	0.038370
	OLS	-0.045780	-0.015606	0.061386	0.037890	-0.016882	0.054722	0.015993	-0.047540	0.063533
	COC	-0.075991	-0.047005	0.122996	-0.041421	-0.019017	0.150438	0.001823	-0.035190	0.036370
0.8	HILU	-0.064965	-0.035380	0.100345	-0.036600	-0.107945	0.144545	0.004090	-0.032280	0.036370
	MLGRID	0.072087	-0.030528	0.102615	-0.071430	0.032983	0.104413	0.017001	-0.026610	0.043611
	ML	0.063707	-0.030410	0.094117	-0.081316	-0.101460	0.182776	0.021410	-0.027970	0.049380
	OLS	0.130017	-0.018560	0.148577	-0.026570	-0.019870	0.046440	0.040595	-0.067230	0.107825
	COC	-0.057908	-0.028760	0.086668	0.004460	-0.114930	0.119390	0.013770	-0.029969	0.043739
0.9	HILU	-0.053360	-0.042270	0.095630	0.012260	-0.114900	0.127160	0.107100	-0.030415	0.047515
	MLGRID	0.135964	-0.034600	0.170564	-0.092190	-0.113970	0.206160	0.039029	-0.032871	0.071900
	ML	0.139136	-0.047590	0.186726	-0.050394	-0.126490	0.176884	0.038136	-0.023808	0.061944

experimental design they adopted. He noted that Rao and Grilliches (1969) confined their experiment to only one exogenous variable and it is not clear from their presentation whether they were estimating β_2 in the model $Y_t = \beta_1 = \beta_2 X_t + U_t$ or $Y_t = \beta_2 X_t + U_t$

MODEL SPECIFICATION

The following econometric single equation model with auto correlated disturbances was assumed. $Y_t = \beta_1 + \beta_2 X_t + U_t$; $U_t = \rho U_{t-1} + \varepsilon_t$, $|\rho| < 1$, $t = 1, 2, \ldots, T$ (2.1). where Y_t is the dependent variable and X_t the independent variable. The U's are the random error terms which are assumed to be autocorrelated. The independent variable X_t was assumed to be autoregressive of the first order given as
$X_t = \lambda X_{t-1} + V_t$ -----(2.2)
where V_t is the error term of the independent variable.

SIMULATION PROCEDURE

In econometrics, while asymptotic properties of estimators obtained by using various econometric techniques are deduced from postulates an approach that is often described as analytical, small sample properties of such estimators have always been studied from simulated data known as Monte Carlo Approach.

The parameter values of β_1 and β_2 in equation (2.1) were fixed at (1,1). To generate multivariate normal vectors to be used for this study, the autoregressive error term $U_t = \rho U_{t-1} + \varepsilon_t$ was first generated. Then the independent variable $X_t = \lambda X_{t-1} + V_t$ was also generated. There after, the multivariate normal dependent vector Y was computed using equation (2.1).

The generation of the error terms, the independent variable and the computations of the dependent variable are done using a Time Series Processor (TSP, 1983) package for econometric studies on an IBM Computer.

The simulation experiment was replicated 50 times. The sample sizes were varied from 20 t0 40 to 60 in order to study the effect of sample size on the performance of the estimators since the study is investigating the performance of the estimators when the error term is autocorrelated, Three different estimates of the error term p were used for the study namely when ρ=0.4, 0.8 and 0.9. The value of λ in the independent variable was fixed at 0.8 because a high autoregressive coefficient of the error term in the independent variable would go a long way to highlight the effects of autocorrelation on the properties of the estimators of our study.

Thereafter, different estimation methods were applied to the data using the AR(1) functions of the TSP software package on an IBM computer at the center for econometric and Allied research (CEAR), university of Ibadan. The deviations of the simulated values from the original data series based on the estimators are being assessed using some statistics. The statistics used in assessing the performances of the estimators in this study are: Bias, sum of bias of both the intercept and slope coefficients (SBIAS), the variance, sum of variance of both the intercept and slope coefficients (SVARS), the root mean square error (RMSE), and the sum of root mean square error (SRMSE) of both the intercept and slope coefficients.

The estimators used for this study are: Ordinary least squares (OLS), Hildreth and Lu estimator (HILU), Cochrane and Orcutt estimator (COC), the maximum likelihood grid estimator (MLGRID), and the maximum likihood estimator (ML). All the estimators apart from the ordinary least squares estimator (OLS), are called Generalized least squares estimators (GLS). They are usually compared with the ordinary least squares estimator (OLS).

RESULTS OF THE SIMULATION EXPERIMENT.

We noticed that for all the sample sizes considerated, the ordinary least squares estimator (**OLS**) does not dominate the Generalized least squares estimators (GLS) using Bias criterion even as the size of autocorrelation increases. It is apparent from table 1 that both the OLS and the GLS methods underestimate β_2, thus resulting in negative bias for β_1 but our result suggest that some estimators overestimate the intercept while some underestimate it although, this observation is more relevant at a lower sample value than at a higher sample values. On the whole, using the criterion of SBIAS, no method is remarkably superior to the other for both small and large autocorrelation especially when the sample size is large.

An evaluation of the estimators using the least variance criterion in table 2 reveals that the estimators fall

TABLE 2: THE USE OF VARIANCE PROPORTION TO COMPARE THE ESTIMATORS

		T = 20			T = 40			T = 60		
p	Estimator	β_1	β_2	SVAR	β_1	β_2	SVAR	β_1	β_2	SVAR
	OLS	0.012200	0.041080	0.053280	0.002474	0.026530	0.029004	0.001136	0.014688	0.015824
	COC	0.064728	0.049680	0.114408	0.002866	0.026140	0.029006	0.001390	0.013580	0.014970
0.4	HILU	0.061730	0.039860	0.101590	0.002866	0.026140	0.029006	0.001397	0.013550	0.014947
	MLGRID	0.008901	0.034674	0.043575	0.002583	0.025507	0.028090	0.001145	0.013417	0.014562
	ML	0.009150	0.034660	0.043810	0.002900	0.024190	0.001106	0.001106	0.013190	0.014296
	OLS	0.101022	0.160213	0.261235	0.040996	0.100510	0.141506	0.017360	0.079090	0.096450
	COC	0.670854	0.060358	0.731212	0.017566	0.028807	0.046373	0.007620	0.026960	0.034580
0.8	HILU	0.241124	0.054717	0.295841	0.016855	0.029024	0.045879	0.008500	0.027300	0.035800
	MLGRID	0.108223	0.061679	0.169902	0.042221	0.024830	0.067051	0.028020	0.026500	0.054520
	ML	0.110911	0.061400	0.712311	0.040071	0.025200	0.065271	0.029987	0.026350	0.056337
	OLS	0.739889	0.200573	0.940462	0.310105	0.149820	0.149820	0.135960	0.195696	0.331656
	COC	1.065600	0.058500	1.124100	0.108780	0.024190	0.132970	0.051306	0.024990	0.076296
0.9	HILU	0.313733	0.057160	0.370898	0.124700	0.023500	0.148200	0.055803	0.023740	0.079543
	MLGRID	0.673300	0.071800	0.745100	0.337096	0.022001	0.359097	0.240518	0.020979	0.261497
	ML	0.648077	0.072860	0.720937	0.327998	0.032140	0.360138	0.242909	0.024204	0.267113

TABLE 3: THE USE OF RMSE TO COMPARE THE ESTIMATORS

ρ	Estimator	β_1 (T = 20)	β_2 (T = 20)	SRMSE (T = 20)	β_1 (T = 40)	β_2 (T = 40)	SRMSE (T = 40)	β_1 (T = 60)	β_2 (T = 60)	SRMSE (T = 60)
	OLS	0.111280	0.205001	0.317801	0.070222	0.164039	0.234261	0.034352	0.123922	0.158274
	COC	0.254658	0.229477	0.484135	0.075241	0.174379	0.249620	0.037842	0.121888	0.159730
0.4	HILU	0.248457	0.202136	0.450593	0.073888	0.172236	0.246124	0.037964	0.121088	0.159052
	MLGRID	0.095971	0.188506	0.283178	0.076988	0.155926	0.232914	0.035068	0.119267	0.154335
	ML	0.095971	0.188588	0.284559	0.084680	0.164807	0.249487	0.034612	0.118399	0.153011
	OLS	0.321120	0.400570	0.721690	0.205989	0.317480	0.523469	0.132724	0.285219	0.417943
	COC	0.822574	0.250135	1.072709	0.138859	0.201722	0.340581	0.087312	0.167936	0.255248
0.8	HILU	0.495323	0.236577	0.731900	0.134887	0.201683	0.336570	0.092286	0.168351	0.260637
	MLGRID	0.336778	0.250222	0.587000	0.217539	0.160990	0.378529	0.168253	0.164949	0.333202
	ML	0.339072	0.249649	0.588721	0.216063	0.188400	0.404463	0.174486	0.164719	0.339205
	OLS	0.869939	0.448238	1.318177	0.557507	0.387576	0.945080	0.370956	0.447445	0.818411
	COC	1.033902	0.243572	1.277474	0.329848	0.193388	0.523236	0.226927	0.160898	0.387825
0.9	HILU	0.562655	0.242790	0.805445	0.353342	0.191578	0.544920	0.236845	0,157051	0.393896
	MLGRID	0.831737	0.270180	1.101917	0.587873	0.187057	0.77490	0.491977	0.148524	0.640501
	ML	0.816968	0.274089	1.091057	0.574924	0.219408	0.794332	0.494331	0.157388	0.651719

into two categories: OLS, MLGRID and ML on one hand, COC and HILU on the other. The first category have an edge over the second for small sample size. As ρ increase, both the variance of OLS and GLS methods increase for the intercept coefficient although the var (β_1) (OLS) show marked superiority over the var (β_2) (COC) or var (β) (HILU) when ρ = 0.8 and T=20 and var (β_1) (MLGRID) and var (β_1) (ML) shows marked superiority over the var (β_1) (OLS).

On the other hand, as ρ increases, the var β_2 (GLS) increases very slowly for all the GLS methods while var β_1 (OLS) increases very sharply with increasing value of ρ and it is always larger than var β_2 (GLS). We obtain var β_2 (OLS) as 0.041080, 0.160213 and 0.200573 and var β_2 (COC) as 0.049680,0.060358 and 0.058500, var β_2 (HILU) as 0.039860,0.054717 and 0.057160,var β_2 (MLGRID) as 0.034674,0.061679 and 0.071800 and var β_2 (ML)as 0.034660,0.061400 and 0.072860 for T=20 and ρ = 0.4,0.8 and 0.9 respectively. Our simulation results show that OLS consistently demonstrates superiority over COC and HILU in estimating β_1, when ρ = 0.8 but when ρ = 0.9, HILU D) performs better than OLS in respect of VAR property. But the MLGRID and ML show superiority over OLS in estimating β_1 and β_2 especially when ρ is large and the sample size T is small. There is no ambiguity in ran king the estimators MLGRID and ML based on SVAR property because ML always shows some slight edge over MLGRID especially when ρ is large. When the size of the sample is increased to 60, COC and HILU methods perform better than the OLS especially for large values of ρ under SVAR criterion.

The same conclusions reached under variance are also reached when the estimators are evaluated using the root mean square error criterion (RMSE) in table 3.

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