

COMMON ERRORS AND PERFORMANCE OF STUDENTS IN JUNIOR SECONDARY THREE MATHEMATICS CERTIFICATE EXAMINATIONS IN CROSS RIVER STATE, NIGERIA

CECILIA OLUNWA EKWUEME and H. JOHNSON NENTY

Received 9 December 2000; Revision accepted 30 April 2001)

ABSTRACT

A survey inferential design and a step-by step procedural analysis of students scripts of 1994 Junior Secondary Certificate Examination in Mathematics, have led to identification of the amount of structural, execution, arbitrary and clerical errors committed by students in Junior Secondary Three Mathematics. No significant sex difference was found in all errors and no significant difference in performance of students whose major type of error was arbitrary, structural, execution or clerical, but amount of arbitrary, structural and clerical error a student commits significantly influences his performance.

KEY WORDS: Error, Mathematics, Performance, Junior Secondary.

INTRODUCTION

A very important aspect of human nature is man's proneness to errors. The effect these errors have on our daily life is dependent on the magnitude of the error committed. A lot of variables come into play when we look at error committed by students in their academic performance especially in the area of mathematics.

Some very good science students sometimes fail woefully in mathematics because of careless errors committed while attempting mathematics questions. A student in 1991 Junior Secondary School Three (JSS 3) mathematics examinations neatly answered all the questions with correct answers but still got a very low grade. One may ask "Why"?

In this contribution, the authors show that it is not true as generally believed that once a student gets the correct answer in mathematics, he has gotten every mark irrespective of how he got the answer. It is shown that some learners with high ability underperform due to fumbling and carelessness (Nenty, 1986). A student who fumbles may fail an easy item more often than their abilities would predict. This paper asks questions on whether the public knows the extent to which students error patterns affect their performance in mathematics. The paper highlights these common mistakes/errors that affect individual student.

Some researchers have identified conceptual knowledge accounting for 50% of

variance connected with problem solving ability (Webb, 1979), also students have difficulty in obtaining related information, mastery pre-requisite skills etc. (Radatz, 1979). Newman (1977) in his study identified that understanding of a problem is a major source of error in mathematics. Rees (1981) noted in her study that there was no significant difference in verbal abilities between boys and girls. It was also found that boys like mathematics more than girls.

A chief examiner also in 1981 May/June WAEC examination lamented on the students poor performance in mathematics due to lack of confidence, usage of wrong units, preparedness for cheating (Educ. News, 1983).

In this study these different errors are grouped into four: arbitrary, structural, execution and clerical errors (Ekwueme, 1997). *Arbitrary* error involves some sort of lack of loyalty to what was given in the question in mathematical context. It also includes ignoring part of the information while acting on the rest. *Execution* error occurs when one understands the problem but does not give the correct answer because of an error made while carrying out a reasonable strategy.

Structural error involves failure to appreciate the relationships involved in a problem, such as selecting the wrong operation, algorithm or formula.

Clerical error is a combination of all the other errors that cannot be classified like boredom,

psychological or physical state, etc.

RESEARCH DESIGN AND METHODOLOGY

The research design adopted for this study is survey inferential design. This is a type of design where the amount of different errors committed by students and the effect the errors have on their performance are investigated and inferences are drawn based on the results of the data collected.

A population of all junior secondary school students in Cross River State who sat for the junior secondary certificate examination in 1994 was used and from this, five schools were randomly selected. Sixty scripts were selected from each of the five selected school making a total of three hundred (300) scripts. These three hundred scripts were marked, coded and kept separately from other scripts since they have been sampled out for the study. This sample of 300 students' scripts were selected from a population of scripts from one boys' school, one girls' school and three mixed schools. In all, one hundred and fifty three male students' scripts and one hundred and forty seven female students' scripts were selected respectively.

There was no special instrument developed for this study, rather the data were generated through a critical examination of the scripts of the three hundred students marking the different types of error committed at each stage with the redesigned marking scheme. The theory and objective part of the answer scripts were used. The theory part showed the response pattern of the students. This method of using the students answer scripts was to ensure that all the students errors were identified under the categories adopted from Donaldson's (1963) classification and redesigned by the researcher to include other classifications. Each examinee was given a zero (0) for each error committed and a one (1) if no error is committed for the theory part of the question. For the response pattern (objective) correct answer was coded '1' mark while incorrect answer was coded '0' mark in the coding section. The responses in the objective items were later arranged in a matrix form with the highest scored item and the highest scorer written first horizontally in order according to the magnitude of each item score (rank ordered) and the examinees' scores were also arranged vertically in order of magnitude (person-by-item matrix). This was followed by the calculations of the examinees' caution indices as well as the item difficulty and item discriminations.

Errors committed (arbitrary error, structural, execution and clerical errors) in

each of the questions were noted. The questions were made up of the following skill areas:-mensuration, ratio/percentage, quadratic table completion. On the objective part all the questions were grouped under eight skill areas (approximation, statistics, mensuration, ratio/percentages, angles/polygon, factorization, symbolical expression/simple fraction, probability/number base. All the information were coded in the sheet to facilitate data analysis. The coding sheet was partitioned into serial number, variables, code for each variable and the column(s).

RESULTS AND INTERPRETATION OF DATA

The various common errors associated with mathematical calculations were identified. The total number of students committing each type of error was grouped according to the error they committed most, using the coding 1,2,3, and 4 for arbitrary, structural, execution and clerical (other types of errors) as the case may be. All the means and standard deviations of the performance on both objective and theory items, person's caution index were also obtained. A general description of these variables in terms of means and standard deviation is as shown in Table 1. Also, the inter-correlation matrix for variables in the study shown in Table 2. The analysis used were an analysis of variance (ANOVA), multiple linear regression analysis and pearson correlation coefficient.

The result of the analysis of variance (ANOVA) of the effect of major type of error of a student is classified into, on their performance level revealed no significant effect. This was tested at an alpha level of .05 with degree of freedom 3 and 296 (calculated F-values =2.242, critical F-value=2.63). This is as shown in Table 3. This tells us that the performance of students whose major type of error is arbitrary might differ from the one whose major type of error is structural, execution or clerical but not significantly.

When these errors were singly correlated to the students performance, the relationship was significantly very high and negative ($r = -.346, -.289$ and $-.249$ for arbitrary, structural, execution and clerical error respectively with critical $r = .114$). The results of the correlation analyses of the effect on performance of the various types of errors committed while solving mathematics examination problems revealed generally highly significant relationships among the variables. It was also found that structural error had the

Table 1: General Description of Statistics Derived from Measurement of Variables Under Study

VARIABLES	X	S _x
1. Performance on objective items	22.820	6.153
2. Performance on theory items	20.103	5.761
3. Total Performance	42.947	9.207
4. Person Caution index (C...)	0.416 ²	0.179
5. Amount of arbitrary error	2.623	0.640
6. Amount of structural/conceptual error	4.177	1.127
7. Amount of execution/conceptual error	2.550	1.200
8. Amount of clerical error	1.833	1.069
9. Total error	10.817	2.602
10. Error or Mensuration	2.797	1.350
11. Error on ration/percentage	4.180	1.163
12. Error on quadratic table completion	0.773	0.794
13. Error on factorization	3.070	0.935
14. Score on mensuration	6.040	2.619
15. Score on Ratio/Percentage	3.357	2.576
16. Score on quadratic table completion	7.197	1.959
17. Score on factorization	3.367	1.777

Table 2: Inter-Correlation Matrix for Variable in the Study

Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1 Performance on objective items	1.00																		
2 Performance on theory items	.201**	1.00																	
3 Total performance	.790**	.713*	1.00																
4 Person caution index (C _p ²)	.096	.008	0.91	1.00															
5 Amount of arbitrary error	-.138*	-.448**	-.346**	-.023	1.00														
6 Amount of structural/conceptual error	.4215**	-.804**	-.606**	.011	.371**	1.00													
7 Amount of execution/computational error	-.040	-.453**	-.289**	-.010	-.293**	-.353*	1.00												
8 Amount of clerical error	-.031	-.460**	-.249**	-.010	.211**	.288**	.205**	1.00											
9 Total error	-.179**	-.809**	-.600**	-.038	.609**	.751**	.540**	.611**	1.00										
10 Error on mensuration	-.216**	-.548**	-.648**	-.031	.364**	.459**	.421**	.524**	.677**	1.00									
11 Error on Ratio/Percentage	-.224**	-.600**	-.506**	-.037	.586**	.517**	.463**	.054	.645**	.251**	1.00								
12 Error on quadratic table completion	.061	-.433**	-.204**	.038	.082	.415**	.149**	.515**	.420**	.166**	-.024	1.00							
13 Error on factorization	0.27	-.361**	-.208**	-.010	.413**	.458**	.207**	.413**	.552**	.083	.194**	.161**	1.00						
14 Score on mensuration	.386**	.628**	.605**	.063	-.191**	-.460**	-.261**	-.311**	-.469**	-.706**	-.169**	-.100	.010	1.00					
15 Score on Ratio/percentage	.100	.727**	-.488**	.015	-.450**	-.595**	-.446**	-.091	-.614**	-.277**	-.814**	-.058	-.238**	.257**	1.00				
16 Score on quadratic table completion	.035	.514**	.325**	-.013	-.111*	-.461**	-.163**	-.494**	-.455**	-.172**	-.026	-.952**	-.197**	.174**	.093	1.00			
17 Score on factorization	-.134*	.611**	.281**	-.058	-.352**	-.527**	-.286**	-.207**	-.506**	-.093	-.349**	-.157**	-.662**	-.069	.446**	.208**	1.00		
18 Ability level	-.905**	-.183**	-.718**	.128*	.121*	-.194**	.091	.013	.155**	.189**	.207**	-.063	-.039	-.366**	-.099	-.028	-.120*	1.00	

*P < .05, df = 298, Critical r = .114, **p < .01, df = 298, Critical r = .148.

highest negative effect on performance from the comparison of the degree of relationship between other types of error and performance. Also a significant relationship was found between performance in the theory and total error based on the theory part of the examination (calculated r-value = -.809, critical

r = .114)(Table 4). This showed that about 65% of variability in students performance in mathematics was accounted for by variability of the amount of total error they commit. It was also discovered that type of error has no sex influence, tested at an alpha level of .05(calculated F = .395, critical

$F = 10.128$), the influence of sex on different types of errors (arbitrary, structural, execution and clerical errors) showed no significant difference for all the types of error (Table 5). Using Pearson product moment correlation analysis at an alpha level of .05, it was found that there is no significant relationship between the caution index of students' response pattern and their performance in junior secondary three mathematics (calculated $r = .091$, critical $r = .114$). The interpretation of the result of the analysis of inter-relationships that exist among difficulty index and discrimination index showed that only the difficulty index and the degree of irregularity (caution index) had significant inter-relationship. This confirms Nenty's (1987) observation that the more

difficult an item is the more irregular the response pattern tends to be. Some students, due to guess work, will score very highly in the most difficult question and score very low on the easy items and the pattern will definitely be irregular.

DISCUSSION

In mathematics, students tend to appreciate certain branches of mathematics more (arithmetic, algebra, geometry, trigonometry, statistics and probability) due to interest and understanding towards such branches. They also tend to perform poorly in some other branches and this creates room for a lot of errors in such branches. The fact that an individual is classified as making more of

Table 3: Analysis (ANOVA) of the Influence of Type of Error on Students Performance in Mathematics

Performance in Mathematics					
Type of Error	n	x	Sx		
Arbitrary	164	42.829	8.775		
Structural	63	41.175	10.910		
Execution	58	45.414	8.263		
Clerical	15	42.133	8.280		
Total	300	42.947	9.207		
Sources of Variation	SS	df	Ms	F	P
Between Groups	563.045	3	187.682	2.242	.84
Within Groups (error)	24782.841	283	83.726		
Total	25345.886	299			

$P > .05$; $df = 3, & 296$; Critical $F = 2.63$

Table 4: Pearson Correlation Analysis of the Relationship Between Students' Total Error Based on Theory (X) and Performance on the Theory Part of the Junior Secondary Three Mathematics Examination (Y).

Variable	ΣX	ΣX^2	ΣXY	r
	(ΣY)	(ΣY^2)		
Total amount of error based on theory (X)	3245.99	61.652	61605	.809*
Total score on the theory Part of the examination (Y)	6031	131177		

* $p < .05$, $df = 298$; Critical $r = .114$

Table 5: A Two-way Analysis of Variance (ANOVA) OF Sex Influence in the Amount of Different Types of Error Committed by Students while Solving Mathematics Examination Problems in Junior Secondary Three.

Source of Variation	SS	df	Ms	F
Between Groups (Rows)	684.5	1	684.5	0.000
	5695373	3	1898457.7	0.395
Between Groups (Columns)	14425126.5	3		
Total	20121184	7		

$p < 0.05$; $df = 1, 3$; critical F -ratio = 10.128

$p < 0.05$; $df = 3, 3$; critical F -ratio = 9.2766

arbitrary error does not mean that the student will not perform comparably well as those that have structural or execution or clerical error as their major type of error.

This study showed that the degree of relationship between structural error and performance was high and negative. This shows the seriousness and effect of committing a structural error because structural error has to do with the understanding of the basic concept in mathematics. Most students do not have deep understanding of most of the mathematical concepts due to shallow foundation laid for them. This agrees with Easterday (1980) who commented on learners's lack of understanding of some basic topics.

It could be noted that arbitrary error deals with ignoring part of the information while acting on the rest and structural error involves failure to appreciate the relationship involved in a problem. Having discovered these in a problem-solving situation, performance will be greatly affected. For instance, if a student does not consider the instruction given, such as to approximate to certain significant figures and also does not use the correct formula or step, definitely the performance will be hampered. The analysis of the students' mistakes showed lack of understanding of the basic concepts in mathematics (structural error) and carelessness especially for girls though not significant. Personal observation from the students' junior secondary certificate examination answer scripts in mathematics used for this study revealed that most of the students especially girls had the idea about how to tackle the mathematics problems but owing to probably distraction, lack of interest or lack of confidence they conclude a well started

problem wrongly.

It was discovered that the errors recorded for a student on these different skill areas (mensuration, ratio/percentages,) becomes higher, the score expected from that particular skill area becomes less. Some students concentrate on getting the right answer not minding the procedure. And without the procedure, one is bound to commit one or more errors thereby lowering the score for that particular problem.

As was observed from the students' scripts used for this research, majority of the students found it difficult to write down the correct formula for the volume of a cylinder that was required in the theory part. This may have been as a result of poor teaching of the concept of volume of solid objects. In the normal class situation, some students may find it difficult to effectively calculate the circumference of a circle, etc. because of sticking to rote method where students merely memorize the formula blindly. For instance, the concept of pie (π), the area and circumference of a circle, surface areas of solid shapes, Pythagoras theorem, trigonometric ratios which can all be practically demonstrated are all memorized by most of the students. Most students cannot relate 2 radii to diameter. This problem could also be from the way the way the students were taught. Some teachers merely tell the students that the formula for the circumference of a circle is $2\pi r$, etc, without explaining with concrete examples how the 2π came about so that they could easily derive the formula on their own anytime they are required to do so. Also in algebra, the way easy equations are taught by some teachers makes it difficult for students to really use the worked examples or teachers' examples to solve

similar problems on their own. For instance, $2x = 6$, $x = 3$ is correct but when some problems like $2x - 1 = 5$ or $3 = x + 1$ are given, the students' finds difficult to relate them to the examples the teacher gave possibly because the teacher did not make them to understand the role of division when multiplication is given in an equation and that of addition when subtraction is given in an equation.

CONCLUSION

If mathematics is to bring about the desired improvement in students performance bearing in mind our circumstances, teachers should constantly update their knowledge of mathematics as a growing subject. A mathematics teacher should be current with the new developments in mathematics. Because of the special role of mathematics in science and technology, it has become imperative that both the Federal and State Governments should take the teachings of mathematics as a priority. Mathematics teachers should be regularly exposed to workshops on current issues and methods of teaching mathematics. This has become inevitable if this country is to join other countries in the race for technological development.

ACKNOWLEDGEMENT

This paper is part of the M. Ed Thesis of C. O. E under the supervision of H. J. N. The first author is grateful to Prof. Barth N. Ekwueme for scholarly advice and financial support during the project at University of Calabar, Nigeria. The paper benefitted from critical reviews by two anonymous Referees of Global Journal of Pure and Applied Sciences.

REFERENCES

- Donaldson, M. A., 1963. A study of children's thinking. Tanslock. London
- Easterday, K., 1980. Students' error pattern in studying square root. *School sciences and Mathematics*. LXXX(2): 141- 147.
- Education News 1983. Poor performance in SC/GCE result. 1(2): 21.
- Ekwueme, C. O., 1997. Error pattern and students performance in junior secondary three mathematics examination in Cross River State. Unpubl. M.ED Thesis, University of Calabar, 108pp.
- Nenty, H. J., 1986. Diagnostic evaluation technique for effective academic guidance and counseling. *Nigerian Jour. Counseling and Development*. 5: 50 -61.
- Radatz, H., 1979. Error analysis in mathematics education. *Jour. of Research in Mathematics Education*, 10(3): 163 - 172.
- Rces, R., 1974. An investigation of some common mathematical difficulties experienced by students. *Mathematics in School*. 3(1): 93.
- Webb, N. M., 1979. Process conceptual knowledge and mathematics problem solving ability. *Jour. Research in Mathematics Education*, 10(2): 83 - 93.