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# AN ALMOST UNBIASED RATIO-CUM-PRODUCT METHOD OF ESTIMATION

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**ABSTRACT**

In the present paper, we have proposed an almost unbiased ratio-cum-product method of estimation using mean-per-unit, ratio and product estimators. These estimators have been mixed by making the use of two design parameters. The optimal values of these parameters have been chosen in such a way that the proposed estimator can be considered to be almost unbiased. An empirical simulation study has been carried out for comparing these estimators in terms of their relative efficiency and bias-wise relative betterment.

KEY-WORDS : First order large sample approximation, Mean square error, Sampling bias.

## INTRODUCTION

Murthy(1964) recommended the use of ratio estimator $\overline{Y}(R)=(\overline{y}/\overline{x})\overline{X}$ when $G > 1/2$ and that of product estimator $\overline{Y}(P)=(\overline{y}\,\overline{x})/\overline{X}$ when $G < -1/2$. Here $G=\rho.C(Y)/C(X)$: $\overline{Y}$ and $\overline{X}$ ($\overline{y}$ and $\overline{x}$) are respectively, population(sample) means of the main and the auxiliary variables; $C(Y)$ and $C(X)$ the respective coefficients of variation and $\rho$ the coefficient of correlation for the two variables. It is presumed that $\overline{X}$ is known and, for the simplicity of study, sample is a simple random one of size, say $n$. While the above recommendation is based on first order ($O(1/n)$) large sample approximation to the mean square error (MSE) of the estimators, an implicit use of a good guess, say $g$, of $G$ is to be made for its implementation. An explicit use of $g$ not only improves upon the ratio and product estimators but also provides an efficient alternative to $\overline{y}$ when the range of $G$ is [-1/2, 1/2] and both ratio and product estimators fail to do so. The same is accomplished by many ratio-cum-product estimators in the literature, such as those by Sahai(1979) and Ray et.al.(1979) given below, for illustration.

$$\overline{Y}(Sa.)=\overline{y}.\{(1+g)\overline{X}+(1-g)\overline{x}\}/\{(1+g)\overline{x}+(1-g)\overline{X}\}$$

and

$$\overline{Y}(P.Ray)=(1+g)\overline{y}-g\overline{Y}(P),$$
$$\overline{Y}(R.Ray)=(1-g)\overline{y}+g\overline{Y}(R)$$

The optimal value of the design parameter in the above and such similar estimators is chosen per their first order approximate MSEs. Quenouille(1956)'s technique and other such techniques have led to proposition of unbiased/almost-unbiased product and ratio

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estimators in the literature, e.g., Robson(1957) and Shukla(1976). Same techniques would lead to almost unbiased ratio-cum-product estimators, as well.

The purpose of this paper is to propose a technique using $g$ only to lead ourselves to an almost unbiased ratio-cum-product estimator making first order approximation to its sampling BIAS almost zero without sacrificing in terms of its MSE to the same order of approximation. This is achieved by considering a two-parameter variant of the relevant ratio-cum-product estimation method.

## PROPOSED FAMILY OF RATIO-CUM-PRODUCT ESTIMATORS

In the literature various ratio-cum-product estimators have already been proposed. However, in all these estimators only one parameter has been used for the mixing. This parameter is assigned an optimal value, its optimality being in reference to the minimization of the large sample approximation to the MSE of the estimator.

Presently, we have tried a mixing of the mean per unit estimator: $\bar{y}$, ratio estimator : $\bar{Y}(R)$ and the product estimator: $\bar{Y}(P)$ using two design-parameters rather than one. The motivation behind being apparently that of having two degrees of freedom for manipulation. The additional degree of freedom is used for controlling the sampling BIAS for the estimator without having to pay the cost for the same in terms of the increment in its MSE which is to be there otherwise. Thus, we propose the generalized ratio-cum-product method of estimation using the following estimator,

$$\bar{Y}(R*P)=(1+a+b)\bar{y}-a\bar{Y}(R)-b\bar{Y}(P) \qquad .......(1)$$

where, $a$ and $b$ are the two non-stochastic design-parameters for the proposed family. Here, we need two degrees of freedom in order to obtain the optimal values of the non-stochastic parameters $a$ and $b$. We obtain these values by (i) considering the first order large sample approximation to sampling BIAS of the estimator and equating it to zero; (ii) by minimizing the first order MSE of the estimator subject to the implication of (i). Thus, we can claim that our method of estimation is almost unbiased.

## OPTIMAL CHOICE OF DESIGN PARAMETERS' VALUES

In the sequel we consider the choice of the values for the two design-parameters in the proposed estimator which is optimal in the sense of controlling its sampling BIAS and MSE, as described in the preceding section. Let us introduce,

$$e_1=(\bar{y}-\bar{Y})/\bar{Y} \text{ and } e_2=(\bar{x}-\bar{X})/\bar{X} \qquad .......(2)$$

Then

$$E(e_1)=0, E(e_2)=0, E(e_1^2)=C^2(Y)/n, E(e_2^2)=C^2(X)/n \text{ and } E(e_1e_2)=GC^2(X)/n \qquad .......(3)$$

So, both $e_1$ and $e_2$ are of the order of $1/\sqrt{n}$. Let us denote the first order BIAS and MSE of $\bar{Y}(R*P)$ by B(.) and M(.), respectively. Using (2) and (3), the expressions for B(.) and M(.) come out to be :

$$B(.) = (G(a-b)-a)\bar{Y}C^2(X)/n \qquad .......(4)$$

$$M(.) = (C^2(Y)+(a-b)^2C^2(X)+2(a-b)GC^2(X)\bar{Y}^2/n \qquad .......(5)$$

Now, using the two degrees of freedom, mentioned above, the optimal values of the two design-parameters happen to be :

$$a_0 = -G^2 \text{ and } b_0 = G - G^2 \quad ......(6)$$

Again, using these optimal values of $a$ and $b$, $\bar{Y}(R*P)$ and first order MSE of $\bar{Y}(R*P)$ become :

$$\bar{Y}(R*P) = \bar{y} + G(\bar{y} - \bar{Y}(P)) + G^2(\bar{Y}(R) + \bar{Y}(P) - 2\bar{y}) \quad .......(7)$$

$$M(.) = (C^2(Y) - G^2C^2(X))\bar{Y}^2/n \quad .......(8)$$

Thus the performance of the proposed estimator $\bar{Y}(R*P)$ depends upon $G$ and it is not always practically possible to guess $G$ 100% accurately. So, we have considered an implicit use of a good guess, say $g$, of $G$ by letting $REG(G) = (g - G)/G$, $REG(G)$ designates the relative error in guessing $G$.

## NUMERICAL COMPARISONS

We have compared the proposed estimator with the usual unbiased estimator($\bar{y}$) and ratio/product estimator in terms of their MSEs and BIASes. The algebraic comparisons of MSEs and BIASes of these estimators being very complex, we have carried out the comparisons numerically. We have used the stagger of $G$-values as follows: We have considered two values each of $\sigma_Y$(2,4), $\sigma_X$(1,2), $\bar{Y}$(2,4) and $\bar{X}$(1,2) ; whereas ten values of $\rho$(±0.2, ±0.4, ±0.6, ±0.8 and ±0.9) have been considered. As mentioned above, the performance of $\bar{Y}(R*P)$ depends upon the relative error in guessing $G$, five levels(0.2, 0.1, 0.0, -0.1, -0.2) of REG(G) have been considered for the purpose of numerical comparisons. For each value-combination of $G$, we have generated 100 random samples of sizes 10, 20 and 50 each using Box-Muller(1958)'s approach. Over this replication of 100 samples and for different value-combinations, actual value of the estimators and thus their MSEs and BIASes have been calculated. To make the study more comprehensive the cases of positive and negative correlation have been considered separately. In order to show that the proposed estimator is a better choice than $\bar{y}$ or $\bar{Y}(R)/\bar{Y}(P)$, we have defined relative efficiency (RE(.)) for the estimator (.) as,

$$RE(.) = (MSE(\bar{y})/MSE(.)).100\% \quad .......(9)$$

and bias-wise relative betterment (RB(.)) as,

$$RB(.) = (Bias(.)/BIAS(*)).100\% \quad ......(10)$$

where, (*) is $\bar{Y}(R)$ when $\rho > 0$ and $\bar{Y}(P)$ when $\rho < 0$. Now, noting the number of times a particular estimator has the maximum $RE(.)/RB(.)$, we have calculated relative frequency($RF(.)$) of its being winner among the other estimators in the competition. Table 4.1 and Table 4.2 give the results of this study.

TABLE : 4.1 ($\rho > 0$)

$RF(.)$ for $\bar{y}$, $\bar{Y}(R)$ and $\bar{Y}(R*P)$

| Estimators → / Sample Size ↓ | | $\bar{y}$ | $\bar{Y}(R)$ | $\bar{Y}(R*P)$ |
|---|---|---|---|---|
| MSE | 10 | 0.1850 | 0.1700 | 0.6450 |
| | 20 | 0.1250 | 0.1200 | 0.7550 |
| | 50 | 0.0600 | 0.0750 | 0.8650 |
| BIAS | 10 | ---- | 0.2075 | 0.7925 |
| | 20 | ---- | 0.1625 | 0.8375 |
| | 50 | ---- | 0.1175 | 0.8825 |

TABLE : 4.2 ($\rho < 0$)

$RF(.)$ for $\bar{y}, \bar{Y}(P)$ and $\bar{Y}(R^*P)$

| Estimators → Sample Size ↓ | | $\bar{y}$ | $\bar{Y}(P)$ | $\bar{Y}(R^*P)$ |
|---|---|---|---|---|
| | 10 | 0.2950 | 0.3600 | 0.3450 |
| MSE | 20 | 0.1925 | 0.2700 | 0.5375 |
| | 50 | 0.0575 | 0.1475 | 0.7950 |
| | 10 | ---- | 0.4675 | 0.5325 |
| BIAS | 20 | ---- | 0.3125 | 0.6875 |
| | 50 | ---- | 0.1800 | 0.8200 |

## ILLUSTRATIONS

We now illustrate the results by considering two small populations, one of which (Population I) was considered by Goodman and Hartley(1950) and the other (Population II) was considered by Sukhatme(1954). The two populations are as:

Population I :

| | | | | |
|---|---|---|---|---|
| Unit | 1 | 2 | 3 | 4 |
| x-values | 2 | 2 | 4 | 6 |
| y-values | 2 | 6 | 6 | 10 |

Population II :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Unit | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| x-values | 63.7 | 155.3 | 245.7 | 344.4 | 491.6 | 767.5 | 1604.0 |
| y-values | 25.4 | 50.1 | 76.0 | 99.2 | 150.8 | 244.4 | 425.1 |

We have considered all the possible samples of sizes 2 units and 3 units from these populations. Now, using (9) and (10), we have calculated (in %) the mean relative efficiency ($MRE(.)$) and mean bias-wise relative betterment ($MRB(.)$) of the estimators studied by us over these samples. Table 5.1 and Table 5.2 give the results of this study.

Table 5.1 : Population I

$MRE(.)$ and $MRB(.)$ for $\bar{y}, \bar{Y}(R), \bar{Y}(P)$ and $\bar{Y}(R^*P)$

| Estimators → Sample Size ↓ | | $\bar{y}$ | $\bar{Y}(R)$ | $\bar{Y}(P)$ | $\bar{Y}(R^*P)$ |
|---|---|---|---|---|---|
| MRE(.) | 2 | 100.00 | 290.89 | 26.13 | 343.25 |
| | 3 | 100.00 | 422.41 | 28.53 | 446.38 |
| MRB(.) | 2 | ---- | 100.00 | 33.09 | 118.98 |
| | 3 | ---- | 100.00 | 26.67 | 100.60 |

Table 5.2 : Population II

$MRE(.)$ and $MRB(.)$ for $\bar{y}, \bar{Y}(R), \bar{Y}(P)$ and $\bar{Y}(R^*P)$.

| Estimators → Sample Size ↓ | | $\bar{y}$ | $\bar{Y}(R)$ | $\bar{Y}(P)$ | $\bar{Y}(R^*P)$ |
|---|---|---|---|---|---|
| MRE(.) | 2 | 100.00 | 4580.96 | 15.12 | 10880.90 |
| | 3 | 100.00 | 3831.30 | 19.68 | 10943.25 |
| MRB(.) | 2 | ---- | 100.00 | 6.90 | 159.41 |
| | 3 | ---- | 100.00 | 7.99 | 190.98 |

In these tables, the last column gives the relative efficiency/bias-wise relative betterment corresponding to the proposed estimator. It clearly indicates a gain in the relative efficiency/bias-wise relative betterment over the other estimators.

## CONCLUSIONS

Tables 4.1 and 4.2 show the performance of the proposed estimator vis-a-vis $\bar{y}, \bar{Y}(R)/\bar{Y}(P)$. When $\rho > 0$, it is clear from Table 4.1 that $\bar{Y}(R^*P)$ performs consistently better than $\bar{y}$ and $\bar{Y}(R)$ and when $\rho < 0$, it is not so good but it is still a better choice in comparison to $\bar{y}$ and $\bar{Y}(P)$.

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