

ON INCOMPRESSIBLE FLUID FLOW IN PRESENCE OF SLIP PARAMETER

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ABSTRACT

This paper studies the effect of slip in the velocity profile of an incompressible fluid flow. Incompressible fluid flow with slip is characterised by low pressures and high temperature generated as a result of a reduction in fluid density. It was discovered that the presence of slip reduces the flow velocity.

KEY WORDS: Incompressibility, Slip flow, transience, velocity Profile, Fluid density

1. INTRODUCTION

Incompressible fluid in presence of slip has wide applications in Science and Engineering. Some of these application areas of this subject matter are enumerated in street (1960). It is to be noted here that incompressible fluid flow with slip is generated as a result of a reduction in fluid density. A lot of work has been done on incompressible low density fluid flow in the presence of a slip and at average temperatures (see for example Ram (1990)). Also Bestman and Mbeledogu (1991) studied incompressible slip flow at very high temperature in the presence of thermal radiation.

It is interesting to note that none of these works considered transient slip flow. In fact, generally, little information exist in the literature concerning transient slip flows. In this paper therefore, we shall consider the Rayleigh slip flow problem as a basis for the future study of more difficult fluid flows encountered practically in Sciences and Engineering.

2. BASIC EQUATIONS

The Rayleigh problem is a very simple example of impulsive motion displaying most characteristic features of classical impulsive flows. For example an impulsive motion of a flat plate is governed by the equations

$$\frac{\delta u}{\delta t} = \nu \frac{\delta^2 u}{\delta y^2} \dots\dots\dots(1)$$

subject to the slip conditions

$$t > 0: u = u_0 + \lambda \frac{\delta u}{\delta y} \text{ on } y = 0, u \rightarrow 0 \text{ as } y \rightarrow \infty$$
$$t > 0: u = 0 \text{ for } 0 < y < \infty \dots\dots\dots(2)$$

where λ is the slip parameter; $\lambda > 0$ for $\lambda = 0$ coincides with the no slip condition which applies for the normal high density problem; u is the fluid velocity and ν is the Kinetic Viscosity coefficient.

3. METHOD OF SOLUTION

The momentum equation (1) and the associated initial boundary condition (2) can best be solved using the Laplace transform method.

So that the La place transform of equations (1) and (2) with respect to t . is given by the equations

$$p\bar{u} = \frac{d^2\bar{u}}{dy^2}$$

$$\bar{u} = \frac{u_0}{p} + \lambda \frac{d\bar{u}}{dy} \text{ on } y = 0, \bar{u} \rightarrow 0 \text{ as } y \rightarrow \infty \dots\dots\dots (3)$$

where p is the transformed variable, with a bar placed over the transformed function as shown above.

The solution that meets the condition stated in equation (3) is

$$\frac{\bar{u}}{u_0} = \frac{1}{p} \left[\frac{e^{-\sqrt{\frac{p}{\nu}}y}}{1 + \lambda \sqrt{\frac{p}{\nu}}} \right] \dots\dots\dots (4)$$

We now find the inverse La place transform of equation (4) by referring to Standard Mathematical tables (see for example Abramowitz and Stegun (1964) to obtain

$$\frac{\bar{u}}{u_0} = - \exp \left[\sqrt{\frac{\nu}{\lambda}} \frac{y}{\sqrt{vt}} + \frac{vt}{\lambda^2} \right] \operatorname{erfc} \left[\frac{\sqrt{\nu}\sqrt{t}}{\lambda} + \frac{y}{2\sqrt{vt}} \right] + \operatorname{erfc} \left[\frac{y}{2\sqrt{vt}} \right] \dots\dots\dots (5)$$

Alternatively, equation (5) can be written in a more simpler form as

$$\frac{\bar{u}}{u_0} = - \exp \left[2\xi\eta + \xi^2 \right] \operatorname{erfc} \left[\xi + \eta \right] + \operatorname{erfc} (\eta)$$

where $\xi = \frac{\sqrt{\nu t}}{\lambda}$ and $\eta = \frac{y}{2\sqrt{vt}}$

Practically ξ is the non dimensional slip coefficient.

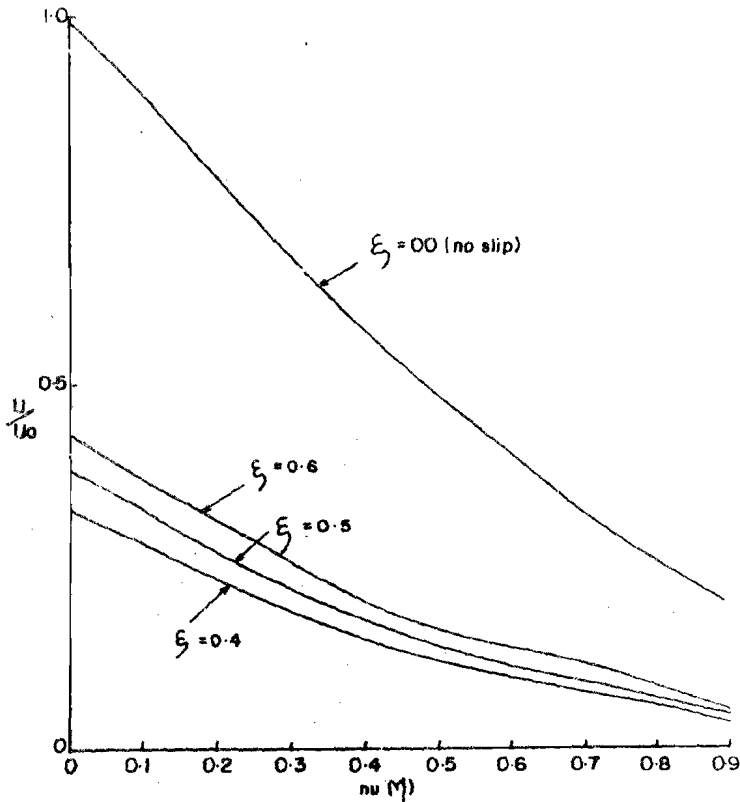


Figure 1: Velocity Distribution

4. INTERPRETATION OF RESULT

We observe from equation (6) that local similarity exists in the velocity profile of incompressible fluid flow with slip, given, appropriate values of ξ . This fact is clear as shown by the graph in figure (1) showing the velocity distribution of the fluid flow for various values of ξ (say $\xi = 0.4, 0.5$ and 0.6). From this graph it is concluded that the presence of slip reduces the velocity of an incompressible fluid flow.

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