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MODELLING AND CHARACTERISATION OF TRANSISTORS

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ABSTRACT

Models and characterisation of active devices that control the flow of energy operating within and outside the active region of the operating domain are presented. Specifically, the incremental charge carrier and Ebers Moll models of the bipolar junction transistor are presented and the parameters of electrical behaviour of the device at the terminals were measured and analysed. The parameters determined could be employed to evaluate input and output impedances, current, voltage and power gains of the device at appropriate operating points. The measured parameters compare favourably with published manufacturers' data to within 5%.

Keywords: Modelling Bipolar Junction Transistor

INTRODUCTION

Modelling here relates to the electrical description of the components used in circuits. Such device modelling starts with the description of a component in physical and structural terms and arrives by a process of approximation and interpretation of measurements at a description of electrical behaviour of the component as viewed at the terminals (Gray and Searle, 1969). These models are usually expressed in terms of simple elements.

In this paper, we are concerned with modelling of active devices and circuit components, which can control the flow of energy in circuits. Such requirement as in amplification cannot bemet by circuits with passive components whose power dissipation results in loss of signal power. A class of components known as active devices meets this requirement, namely transistors and vacuum tubes. Such active devices can in addition endow a circuit with the capability of power gain.

To intelligently use a component or device in a circuit, be it active or passive, it is essential to have some characterization or description of the electrical behaviour of such a device. The description of the electrical behaviour of a device as viewed at its terminals or points of connection to a circuit constitutes a circuit model for that device.

A simple approach to the modelling involves the measurement of the electrical behaviour of the device at the terminals and the direct use of these measured parameters as a model for the device. However, sole reliance on measurement leads to a number of difficulties (Gray, 1967). First, measurements can only be made under fixed appropriate environmental and operating set of conditions and secondly such method generally produces a large compendium of information, which may be difficult to analyse. In order to circumvent these difficulties, modelling should be guided by the understanding of the internal physical behaviour of the device. As such, a better selection of terminal measurements will yield parameters over wide ranges of operation and a theoretical framework can be developed. Such can provide basis for extrapolation of the measured parameters.

The thrust of this paper is to present the processes of modelling bipolar junction transistor (BJT). Both methods - measurement and theoretical evaluation would be presented and analysed. Specifically, the incremental, Ebers-Moll and hybrid-pi models would be presented and the circuits analysed for the active linear portion of operation of the device. The Ebers-Moll model would be employed to also consider the saturated and cutoff regions of the device when it operates as a switch.

THEORY

Modelling and Analysis of Linear Active Circuits

Amplifiers that employ active devices are often operated in such a way that linear input-output

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relationships are obtained. Here, the range of variation of the currents and voltages at the terminals is restricted such that non-linearities are not perceived. This is the basis for linear incremental models, which offer important advantages when circuits contain energy-storage elements and when the dynamics of the active devices are considered (Gray et al, 1964). Such incremental analysis: (i) is dependent on the operating point. (ii) Suppresses the evidence of the inherently non-linear nature of the device. (iii) applies only to the signal components of the variables.

Incremental models for BJT

The incremental model starts with a mathematical description of physical laws that govern the device behaviour followed by the linearisation of the incremental variable. The final stage is the analysis of the internal behaviour to yield relationships among the incremental terminal variables.

Models for incremental behaviour of BJT are presented when they operate in the linear active region. Considering a pnp structure.

$$v_{EB} \rightarrow V_{EB} + \Delta v_{EB} \quad \ldots (1)$$

and

$$v_{CB} \rightarrow V_{CB} + \Delta v_{CB} \quad \ldots (2)$$

where Δv_{EB} and Δv_{CB} represent the incremental values.

The minority carrier concentration at the space-charge layer edge is given as (Gray and Searle, 1969)

$$P_b(0) = P_{bo} \exp(qv_{EB} / kT) = P_{bo} \exp(qV_{EB} / kT) \exp(q\Delta v_{EB} / kT)$$

And for small enough values of Δv_{EB},

$$P_b(0) = P_{bo} \exp(qV_{EB} / kT) \{ 1 + (q\Delta v_{EB} / kT) \} \quad \ldots (3)$$

Where P_{bo} is the equilibrium minority carrier concentration. The incremental component $P_b(0)$ can be expressed as $\Delta P_b(o) = P_B(0)\, q\, \Delta v_{EB} / kT$ where $P_B(o)$ is the d.c component of $P_b(o)$.

The incremental longitudinal diffusion current is given as (Gray and Searle, 1969)

$$\Delta i_c = qAD_b\, \Delta P_b(o) / W = \{ qAD_b / W \times P_B(o) \}\, q\, \Delta v_{EB} / kT$$

where A= Area, D_b = base diffusion coefficient and W = junction width.

The expression in the braces bracket represents the quiescent d.c current $|I_c|$ at the operating point. Rewriting;

$$\Delta i_c = -g_m \Delta v_{EB} \quad \ldots (4)$$

then, $g_m = (q / kT) \times I_c$ where g_m is termed the incremental transconductance.

The incremental charge Δq_B is given as (Gray and Searle, 1969)

$$\Delta q_B = qA [\tfrac{1}{2} \Delta P_b(0) W] \quad \ldots (5)$$

$$= C_b \Delta v_{EB} \quad \ldots (6)$$

where $C_b = (W^2 / 2D_b)\, g_m$, is the base charging capacity. This expression in equation (6) provides a measure of the charge that must be supplied at the base to accommodate a change in emitter-to-base voltage with a range of value 5 - 200pF

$$\Delta i_{BQ} = (d\Delta q_B / dt) = - C_b\, d(\Delta v_{EB}) / dt$$

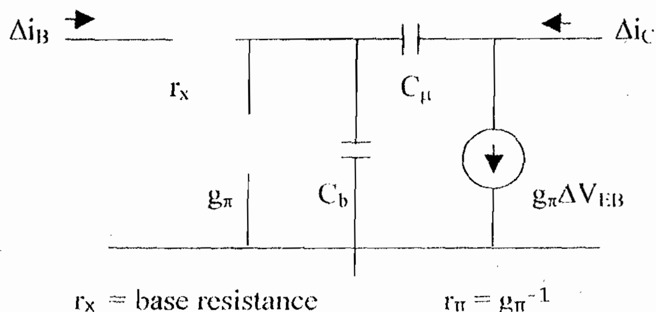
For the increased recombination in base and emitter, we have

$$\Delta i_{BR} = g_\pi \Delta v_{EB}$$

where g_π is the incremental input conductance whose value is less than g_m by a factor of (10-500). Therefore, the total incremental base current is

$$\Delta i_{BR} + \Delta i_{BQ} = \Delta i_B = g_\pi \Delta v_{EB} - C_b\, d(\Delta v_{EB})/dt \quad \ldots (7)$$

The network model comprised of linear circuit elements shown in Fig. 1.



r_X = base resistance $\qquad r_\pi = g_\pi^{-1}$

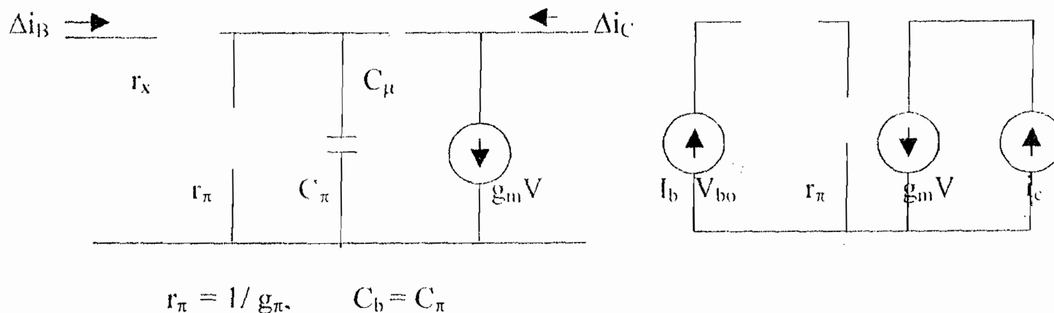
Fig. 1 BJT model including external capacitors

This incremental model is valid under dynamic conditions where $d(\Delta v_{CB})/dt$ is small enough and $|i_b| < |i_c|$ with $g_\pi << g_m$ for frequencies much less than the inverse of base region life time, τ_B. If the model is extended to account for capacitors that straddle the metallurgical junction; $\Delta i_c = -g_m\, \Delta v_{EB} + C_{jc}\, d(\Delta v_{cB})/dt$

$$\Delta i_B = -g_\pi \Delta v_{cB} - C_{je}\, d(\Delta v_{EB})/dt - C_{jc}\, d(\Delta v_{cB})/dt$$

Hybrid – Pi Parameter.

The hybrid-pi parameters based on the incremental model of section 2.1 may be determined either by direct measurement on the BJT at the desired operating point or by calculation from measured data supplied by a transistor manufacturer. The hybrid-pi model is illustrated in Fig 2.



$r_\pi = 1/g_\pi, \qquad C_b = C_\pi$

Fig. 2 (a) Hybrid-pi model (b) low frequency version of (a).

Provided I_c is much greater than I_{co},

$$g_m = (q/KT)\,|I_c| \quad \ldots (8)$$

and from Fig. 2 (b), $I_c / I_b = \beta_o = g_m r_\pi$. For silicon, h_{fe} (dc) $= \beta_o$ and so,

$$r_\pi = \beta_o / g_\pi \quad \ldots (9)$$

Similarly, $\beta_o = g_m / \{g_\pi + s(C_\pi + C_\mu)\}$ $\ldots$ (10)

If w_B is the 3dB point frequency,

$$w_B = g_\pi / (C_\pi + C_\mu) \quad \ldots (11)$$

and the gain bandwidth, $w_T = \beta_o w_B$ $\ldots$ (12)

EBERS MOLL (E-M) MODELS FOR A BJT.

The description in section 2 is based on the explicit conditions that transistors operate in active region, but in some applications, particularly those in which the transistor is used as a switch rather than a controllable power modulator, Ebers-Moll model is more appropriate. This model is based on physical reasoning, and is of considerable value in the prediction of large signal switching operation (Ebers and Moll, 1954)

Analysis of E-M model

Each terminal current comprised of the sum of forward and reverse components. The forward component of the emitter current is:

$$I_{EF} = I_{ES} \exp [(qV_{EB}/kT) - 1]$$

where I_{ES} is the reverse saturation current with $V_{CB} = 0$.

$$I_{CF} = - \alpha_F I_{EF} = -\alpha_F I_{ES} \exp [(qV_{EB}/kT) - 1]$$

Where $\alpha_F = I_{CF} / I_{EF}$ is the common base circuit forward current gain.

$$I_{CF} = - (1-\alpha_F) I_{ES} \exp [(qV_{EB}/kT) - 1]$$

The reverse components of the terminal currents are governed by similar relationships except that the roles of collector and emitter are interchanged.

$$I_{ER} = - \alpha_R I_{CS} \exp [(qV_{CB} /kT) -1]$$

$$I_{CR} = I_{CS} \exp [(qV_{CB} /kT) -1]$$

$$I_{ER} = -(1- \alpha_R)I_{CS} \exp [(qV_{CB} /kT) -1]$$

where I_{CS} denotes the saturation characteristic of the reverse component of carrier distribution and α_R denotes the reverse short circuit amplification factor.

The total currents are given by the superposition of the independent component currents. And thus, for a p-n-p transistor;

$$I_E = I_{EF} + I_{ER} = I_{ES} \exp [(qV_{EB} /kT) -1] - \alpha_R I_{CS} \exp [(qV_{CB} /kT) -1]$$

$$I_C = I_{CF}+ I_{CR} = - \alpha_F I_{ES} \exp [(qV_{EB} /kT) -1] + I_{CS} \exp [(qV_{EB} /kT) -1]$$

$$I_B = I_{BF} +I_{BR} = -(1-\alpha_F) I_{ES} \exp [(qV_{EB} /kT) -1] - (1-\alpha_R)I_{CS} \exp [(qV_{CB} /kT) -1]... \quad (13)$$

I_C is the summation of forward carrier transport across the base controlled by V_{EB} plus diode current controlled by V_{CB}.

Equations (13) express the terminal currents as a function of the junction voltages - for arbitrary values (forward and reverse) of those voltages. These equations are the E-M relationships and Fig (3) shows the circuit models interpreting the E-M equations.

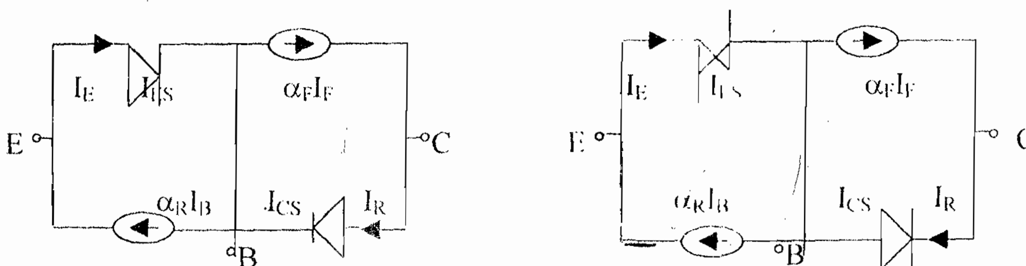


Fig (3) (a) p-n-p. Transistor (complete) (b) n-p-n transistor.

For n-p-n transistors, equation (13) holds but with sign reversed. The parameters α_F and α_R are short circuit forward and reverse common-base current gains respectively.

Since current generators depend directly on the terminal currents rather than junction voltages, equation (13) can be re-written as (Ryder, 1974):

$$I_E = -\alpha_R I_C + I_{EO} \exp[(qV_{EB}/kT) - 1]$$
$$I_C = -\alpha_F I_E + I_{CO} \exp[(qV_{EB}/kT) - 1] \quad \ldots\ldots(14)$$
$$I_B = -I_{EO} \exp[(qV_{EB}/kT) - 1] - I_{CO} \exp[(qV_{CB}/kT) - 1]$$

where $I_{EO} = (1 - \alpha_F)I_{ES}$ and $I_{CO} = (1 - \alpha_R)I_{CS}$. I_{EO} and I_{CO} are the emitter and collector open circuit saturation currents.

Charge carriers or dynamic models bear explicit relationships with the hybrid-pi and E-M models. This is because the charge control model reduces to the E-M model for static situations and also reduces to hybrid-pi model for small signal dynamic operation in the forward active region with the collector reverse-biased.

The method developed in section 3.1 applies for both forward and reverse voltages in four regions of operation. These are;

(i) the forward active region with emitter junction forward biased and collector junction reverse biased for which the model in section 2 specifically applies;
(ii) the reverse active region in which the collector junction is forward biased and the emitter junction reverse biased;
(iii) the cut-off region in which both junctions are reverse biased; and
(iv) the saturation region in which both junctions are forward biased.

EXPERIMENTS

Determination of Incremental Parameters

From Section (2.2), it is observed that the hybrid-pi parameters bear a direct relationship to the incremental parameters. So, it suffices to determine the hybrid-pi parameters, which can either be by direct measurement or by calculation from measured data supplied by the manufacturer. In the latter case, the measurements are made at some standard current, often I_C =10mA (Gray and Searle, 1969) so that the parameter values are subsequently converted to values appropriate for the desired operating point.

The following hybrid-pi parameters were obtained by carrying out measurements at the device (test transistor) terminals.

(i) the incremental low frequency CE short circuit current gain β_o (h_{fe}).
(ii) the CE unity gain frequency w_T, and the gain bandwidth.
(iii) the dc collector current, I_c.
(iv) the common base output capacitance C_{ob} (= C_μ)

To determine the transconductance, g_m - heavy reliance is placed on the relation between g_m and I_c given by:

$$g_m = q / kT \; |I_C|, \quad \text{for which } I_C = 5\text{mA} \quad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$
$$= 5/26 \text{ S}$$

This is assumed accurate, since $I_C >> I_{CO}$ (I_{CO} 10μA at 300K (Ryder, 1974).

By operating the transistor in the CE mode and measuring the low frequency short circuit current gain, at frequencies (< 2000Hz) when the reactances are ignored,

$$hfe = \beta_o = g_m r_\pi \quad \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

The value of the capacitor C_μ was determined by measuring the capacitance between base and collector leads of the transistor with the emitter incrementally open circuited as indicated in Fig. 4.

V_{CB}

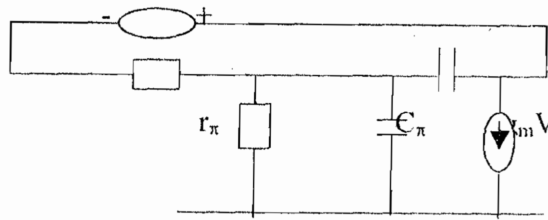


Fig. 4 Circuit for measuring C_μ

In effect, the output capacitance, C_{ob} was measured in the common base connection. It is seen that g_m, C_π and r_π do not enter into this measurement since V=0. The network reduced to series combination of r_x and C_μ. Since this was determined at 1000Hz and $1/wC_\mu >> r_x$

$$C_\mu = C_{ob} = 4.6 \text{ pF}$$

To determine C_π, we invoke the expression in equation (10)

$$\beta = g_m / [g_\pi + s (C_\pi + C_\mu)]$$

Here, β, the short circuit current gain at high frequency where g_π is negligible is:

$$\beta = g_m / [s (C_\pi + C_\mu)] \quad \text{.......................................(17)}$$

And $C_\pi = (g_m / w_T) - C_\mu$, from (11).

From the graph of magnitude of $|\beta|$ as a function of frequency (Gray and Searle, 1969), there was a break downward occurring at a frequency $w_\beta \cong 40$ MHz $= g_\pi / (C_\pi + C_\mu)$ which is the frequency at which the gain is 0.707 of the mid-band gain. From the graph also, $w_T = 20 \times 10^8$

$$w_T = \beta_o w_\beta = 49 \times 10^6 \times 40 = 1.96 \times 10^8$$

Substituting for g_m and w_T, we obtain $C_\pi = 96$ pF

E-M Circuit Models : Parameter Evaluation.

These models contain four parameters - two saturation currents and two current gains, only three of which are independent. The key to evaluation of these parameters lies in the measurement of the terminal I-V characteristics under appropriately chosen special conditions. With the collector junction short circuited i.e. $V_{CB} = 0$, the model equations of Fig. 3 show that I_R or (I_{CR}) is zero and thus I_{CS} and α_R do not appear in the models and then;

$$\alpha_F = -I_C / I_E \quad | V_{CB} = 0 \quad \text{.......................... (18)}$$

or equivalently; $\beta_F = I_C / I_B \quad | V_{CB} = 0$ .... (19)

Also, $I_E = I_{ES} \exp [(q V_{EB} / KT) - 1] \quad | V_{CS} = 0$ .... (20)

Equation (20) is simply an expression for the I-V relationship of a junction diode and I_{ES} was determined by plotting ln I_E against $q V_{EB} / KT$.

At T = 300K (27^0C), $q / KT = 38.66$,

And $I_{ES} = (1 / 38.66) X (\Delta \ln I_E / \Delta V_{CB})$.

The reverse parameters I_{CS} and α_R were measured by interchanging the emitter and collector leads. For measuring the parameters I_{EO} and I_{CO}, measurements were made on one junction with the opposite terminal open-circuited.

For purposes of comparison, typical values of the E-M parameters were determined from manufacturer's data sheets which gave values of h_{FE} (d.c. forward gain) = $\Delta I_C / \Delta I_B$ and since for silicon transistors $I_{CO} (\beta + 1) << I_C$,

$$h_{FE} = \beta_F \text{ and hence } \alpha_F = h_{FE} / (1 + h_{FE})$$

I_{cs} is also provided, and where graphs were provided, appropriate interpolation were carried out. α_R is between 0.5 and 0.9 corresponding to β_R between 1and 10.

So, $I_{ES} = (\alpha_R / \alpha_F) I_{CS} \approx I_{CS} / 2$

Since the E-M circuit model is based on the analysis in which terminal currents arise from the flow and recombination of excess carriers in the neutral regions and neglects currents associated with generation and recombination in space charge layers (Searle et al, 1966) which may be dominant when junction is reversed but negligible in forward mode. Thus, in forward active and saturation regions where the emitter junction is forward biased I_{ES} was measured with V_{EB} positive. On the other hand, if it is to operate in the cut-off region where junctions are reverse biased, I_{ES} was measured with V_{EB} negative.

RESULTS AND DISCUSSION

For the incremental or hybrid pi model Fig 1 the intrinsic capacitances are included. This permits the evaluation of the parameters at all frequencies of interest (low, mid band and high) by making necessary assumptions.

Table 1

Parameter	Calculated/Measured value (BC 108)	2N3564 Evaluated from Data sheet
g_m	0.196 S @ Ic = 5mA	0.2 @ Ic = 5 mA
β_o	49	80
r_π	250Ω	400Ω
r_x	50Ω	30Ω
$C\pi$	96 pF	42 pF
w_T	20 × 10^8	--
C_μ	4.6pF	5.6pF
g_π	0.004 S	-

For the E-M model from equation (22), since

$\beta_o = \beta_F = 49$, $\alpha_F = 0.98$ $I_{ES} = 6.2 \times 10^{-9}$ A

$I_{CS} = 1.8 \times 10^{-8}$ A $\beta_R = 8.42$ $\alpha_R = 0.57$

In evaluating the parameters for the hybrid pi model, assumption is made that the transistor has uniform base region abrupt junction and low level injection. The advantage of this model is that it takes account of device performance in all regions of frequency spectrum.

Here g_m increases linearly with Ic and at high frequencies r_π of only extrinsic base resistance and hence independent of Ic. The change with voltage of the parameters C_π, C_μ and $g_m = 1/r_\pi$ arises due to the base width W in equation (6) which decreases with increasing v_{EB} which is a constant about 0 6v for silicon.

E-M circuit model is based on an analysis in which terminal currents arise only from the flow and recombination of excess carriers in neutral regions. This model also permitted aside from active region parameter evaluation for saturated and cut off regions.

The values obtained compared favourably with specifications from data sheet. These parameters when employed at appropriate points give the characterization of a transistor the essentials being (i) the voltage gain (ii) current gain (iii) power gain (iv) the frequency response (v) the input and output impedances.

In general the choice of model depends on the requirement of the design engineer – at what frequencies the devices are to operate and in what region (active, cutoff or saturated). The parameters measured and calculated would characterise devices operating as power modulators and switches.

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