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AXIAL VELOCITY DISTRIBUTION OF A TWO COMPONENT PLASMA IN A MAGNETISED TUBE OF SLOWLY VARYING SECTION

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ABSTRACT

In this study we investigate the combined effects of channel indentation and presence of neutral gas (impurities) on the flow of a two-component plasma gas through a magnetized cylinder with indentation. For small indentation, expressed in ε, analytic solutions are obtained for the axial velocities, induced magnetic fields, current densities and pressure gradients. The effects of the channel indentation and percentage impurities on the flow characteristics are shown graphically and discussed quantitatively.

Keywords: Indention, Neutral gas, Axial velocity, Magnetic field

INTRODUCTION

The growing need or requirement for confined plasmas is necessitated by its application in fusion reactions for energy production. Magnetic fields are used to confine plasma, and the desired configuration is such as to make the plasma follow a desired geometry. The maintenance of flows in uniform geometry is mainly achieved by a fair play between the plasma hydro magnetic pressure and the magnetic pressure (Priest et al, 1981). " An excess of the internal pressures over the external pressures results in flux tube bursting." Ordinarily, any sudden variations in the radius of a tube conveying a fluid results in pressure variations. Therefore, a study along this line, viz, effect of channel variations on the flow variables, is desirable and effort has been directed in this investigation to a two-component plasma flow in a cylindrical channel. In fact, the determination of flow through a tube of varying section is a fundamental one with obvious applications, not only in engineering but also in physiology. One of the initiators of the study, Manton(1971), considered the ax symmetric flow in tubes of varying section. He obtained an asymptotic series expansion in terms of a small parameter ε characteristic of the varying section, for the velocity, pressure, and shear stress and found that his solutions compared favorably with numerical results even for values of ε as large as 2. Other workers in this area include: Bestman (1983,1988), Burns and Parks (1967), McMichael et al (1983), Haldar (1994) and Deshikacha et al (1987). Some of the fluids investigated by these authors are blood and conducting non-gas fluids.

In the present study we adapt the methods devised by these authors (notably those of Bestman and Manton) to the problem of a two-component plasma flow in the presence of an external magnetic field. It must be noted, however, that no attempt is made to solve explicitly problems encountered in practically confined plasma geometry. The study is to theoretically show the existence of problems that may arise due to tube wall variation and partially ionized gasses in confined plasmas. The study of variation in magnetic flux lines in confined plasmas is in the regime of the study of stability theory. Instabilities in practically confined plasmas have been globally observed and, in certain cases, corrections based on theoretical models have been devised to contain these instabilities. The attempt here is, therefore, to show how the all important axial velocity and plasma current density are affected due to the two factors, namely, the presence of neutral gas (impurities) and unequal wall geometry. The plasma is considered only partially ionized. Two-component here refers to charged species (ions of both signs) and neutral non-ionized gas. The two species only interact through mutual collisions. A magnetic field applied to this binary mixture interacts only with the charged species and the collision of the ions with the neutral gas is responsible for indirect coupling of the magnetic with the bulk of the gas. This coupling of the two species to the to the magnetic field has been discussed by Spitzer (1962).

In section 2 the mathematical formulations are given while in section 3 analytic solutions are devised and in section 4 results are presented quantitatively. The solutions of the equations for the partially ionized gas flow variables are compared to those of the fully ionized gas and result presented for various sections of the tube wall.

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Problem Formulation

We consider the flow of a two-component (partially ionized) gas in a cylindrical channel of varying radius. The cylinder is magnetized such that as the gas flows the induced magnetic field component are given by (H'_r, 0, H'_z) and the velocity components by (u', 0, w'). r = 0 is the axis of symmetry of the channel while the tube wall is defined as:

$$r' = a_0(z',\varepsilon) = a_0 s(\varepsilon z'/a_0) \quad (1)$$

We introduce the following non-dimensional scaling:

$$z = \varepsilon z'/a_0; r = r'/a_0; (u_{i,n}, w_{i,n}) = (u'_{i,n}, \varepsilon w'_{i,n})/\varepsilon U \quad (2)$$

$$(H_r, H_z) = (H'_r, \varepsilon H'_z)/\varepsilon H_0; \quad p_{i,n} = p'_{i,n}/\rho_n U^2; \quad \beta = \rho_n/\rho_i \quad (3)$$

Here ε is a small parameter and a_0 is a suitable constant representing the tube wall.

The composite non-dimensional MHD equations for continuity, momentum and electromagnetic fields for steady flow in cylindrical polar coordinates (r, θ, z) are given as:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_{i,n}) + \frac{\partial w_{i,n}}{\partial z} = 0 \quad (4)$$

$$\varepsilon^2 R_e u_i \frac{\partial u_i}{\partial r} + \varepsilon^2 R_e w_i \frac{\partial u_i}{\partial z} = -\frac{\partial p}{\partial r} + \varepsilon\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_i)\right) + \varepsilon m^2(w_i H_r - u_i H_z)H_z + + \frac{\varepsilon a^2{}_0 \beta f_c}{\upsilon_i}(u_n - u_i) \quad (5)$$

$$\varepsilon R_e u_i \frac{\partial w_i}{\partial r} + \varepsilon R_e w_i \frac{\partial w_i}{\partial z} = -\varepsilon\frac{\partial p_i}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_i}{\partial z}\right) + \varepsilon^2\frac{\partial^2 w_i}{\partial z^2} + \varepsilon^2 m^2(w_i H_r - u_i H_z) + \frac{\beta a_0{}^2 f_{0c}}{\upsilon_i}(w_n - w_i) \quad (6)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_r) + \frac{\partial H_z}{\partial z} = 0 \quad (7)$$

$$\varepsilon^2\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = \varepsilon R_m(w_i H_r - u_i H_z) \quad (8)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_n) + \frac{\partial w_n}{\partial z} = 0 \quad (9)$$

$$\varepsilon^2 R_e u_n \frac{\partial u_n}{\partial r} + \varepsilon^2 R_e w_i \frac{\partial u_n}{\partial z} = -\frac{\partial p}{\partial r} + \varepsilon\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_n)\right) + \varepsilon^3\frac{\partial^2 u_n}{\partial z^2} - \frac{\varepsilon a^2{}_0\beta f_c}{\upsilon_i}(u_n - u_i) \quad (10)$$

$$\varepsilon R_e u_n \frac{\partial w_n}{\partial r} + \varepsilon R_e w_n \frac{\partial w_n}{\partial z} = -\varepsilon\frac{\partial p_n}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_n}{\partial z}\right) + \varepsilon^2\frac{\partial^2 w_n}{\partial z^2} - \frac{\beta a^2{}_0 f_c}{\upsilon_i}(w_n - w_i) \quad (11)$$

where, $R_e = (\rho U a_0/\mu_{i,n})$ is a Reynolds number and $R_m = \left(\frac{\rho_p \mu_m}{UH_0}\right)$ is a magnetic Reynolds number. U and H_0 represent characteristic velocity and magnetic fields, respectively.

Method of Solution

The concept of slowly varying radius enables us seek for series solutions about the small parameter ε, as follows:

$$(u_{i,n}, w_{i,n}) = (u^0{}_{i,n}, w^0{}_{i,n}) + \varepsilon(u^1{}_{i,n}, w^1{}_{i,n}) + \dots \quad (12)$$

$$(H_r, H_z) = (H^0{}_r, H^0{}_z) + \varepsilon(H^1{}_r, H^1{}_z) + \dots \quad (13)$$

$$p = \frac{1}{\varepsilon}p^0 + p^1 + \varepsilon p^{(2)} + \dots \quad (14)$$

and considering that $m^2 = 0(1/\varepsilon)$, with the result that

$$\varepsilon^2 m^2 = \varepsilon\chi \quad (15)$$

On substituting equations (12-14) in (4-11), we obtain the basic or zeroth order approximations as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru^0{}_{i,n}) + \frac{\partial w^0{}_{i,n}}{\partial z} = 0 \quad (16)$$

$$-k_i = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w^0_i}{\partial r}\right) + \sigma^2\beta(w^0_n - w^0_i) \quad (17)$$

$$-k_n = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w^0_n}{\partial r}\right) - (\sigma^2/\beta)(w^0_n - w^0_i) \quad (18)$$

where

$$k_{i,n} = -\frac{\partial p^0}{\partial z}; \quad \beta = \frac{\rho_n}{\rho_i} \quad (19)$$

and

$$\frac{1}{r}\frac{\partial}{\partial r}(rH^0_r) + \frac{\partial H^0_z}{\partial z} = 0 \quad (20)$$

$$\frac{\partial H^0_z}{\partial r} = 0 \quad (21)$$

$$\frac{\partial p^1_i}{\partial r} = \chi(w^0_i H^0_r - u^0_i H^0_z)H^0_z = 0 \quad (22)$$

$$\frac{\partial p^0}{\partial r} = 0 \quad (23)$$

with the result that H^0_z and p^0 are functions only of z. So writing $H^0_z = f(z)$, say, we obtain, from equation (20)

$$H^0_r = -\frac{1}{2r}\frac{df}{dz} = -\frac{1}{2r}f'(z) \quad (24)$$

where f(z) is a constant function of z which can be obtained from the conditions imposed by the problem. If we assume that the boundary wall is of low conductivity or under vacuum condition, then the boundary condition for the magnetic field is given as:

$(H_z, H_r) = 1$ $(=H_0)$ at $r = s(z)$ (25)

and yields

$$f(z) = 2\int^z \frac{dz}{s} \quad (26)$$

The velocity equations are subject to the no-slip conditions:

$(u^0_{i,n}, w^0_{i,n}) = 0$ on $r = s(z)$; $(u^0_{i,n}, w^0_{i,n}) < \infty$ on $r = 0$ (27)

Equations (16-19, 27) are solved to give:

$$w^0_{i,n} = \frac{k}{4}(r^2 - s^2) + A(z)(I_0(\alpha r) - I_0(\alpha s)) \quad (28)$$

where

$$k = (k_n\sigma_i^2\beta + k_i\sigma_n^2); \quad \alpha^2 = (\sigma_i^2\beta + \sigma_n^2/\beta) \quad (29)$$

It is evident that $w_i = w_n$ for the basic flow if $k_i = k_n$. In this situation the induced magnetic fields do not affect the flow velocities. The fluid is moved only by the pressure gradients, when $\varepsilon = 0$.

Substituting equation (28) in (16), after integrating and, applying the boundary conditions (27), we obtain:

$$u^0_{i,n} = \alpha A(z)\frac{r}{2}I_1(\alpha s)s_z + \frac{k}{4}rss_z + \frac{dA}{dz}\left(\frac{r}{2}I_0(\alpha s) + \frac{1}{\alpha}I_1(\alpha r)\right) \quad (30)$$

where

$$\frac{dA}{dz} + \frac{\alpha s I_1(\alpha s)s_z A(z)}{\alpha s I_0(\alpha s) - 2I_1(\alpha s)} - \frac{(k/4)\alpha s s_z}{\alpha s I_0(\alpha s) - 2I_1(\alpha s)} = 0 \quad (31)$$

Here, I_0 and I_1 are the modified Bessel's functions of the first kind of order zero and one, respectively, and $s_z = \frac{ds}{dz}$.

Equation (31) yields

$$A(z) = -\frac{(k/8)\alpha s^3}{(\alpha s I_0(\alpha s) - 2I_1(\alpha s))} \quad (32)$$

The solutions for the basic approximation are complete.

Also the first order approximate equations are given as:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru^1_{i,n})+\frac{\partial w^1_{i,n}}{\partial z}=0 \tag{33}$$

$$\frac{\partial p^{(2)}_i}{\partial r}=\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(ru^0_i))+\chi(H^0_rH^1_zw^0_i+H^1_rH^0_zw^0_i+H^0_rH^0_zw^1_i-2H^1_rH^0_zu^0_i) \tag{34}$$

$$R_eu^0_i\frac{\partial w^0_i}{\partial r}+R_ew^0_i\frac{\partial w^0_i}{\partial z}+\frac{\partial p^1_i}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial w^1_i}{\partial r})+\sigma^2_i\beta(w^1_n-w^1_i)+ \tag{35}$$

$$\chi(H^{(0)2}_rw^0_i-H^0_rH^0_zu^0_i)$$

$$\frac{\partial p^{(2)}_n}{\partial r}=\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(ru^0_n)) \tag{36}$$

$$R_eu^0_n\frac{\partial w^0_n}{\partial r}+R_ew^0_n\frac{\partial w^0_n}{\partial z}+\frac{\partial p^1_n}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial w^1_n}{\partial r})-\frac{\sigma^2_n}{\beta}(w^1_n-w^1_i) \tag{37}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH^1_r)+\frac{\partial H^1_r}{\partial z}=0 \tag{38}$$

$$\frac{\partial H^1_z}{\partial r}=R_m(u^0_iH^0_z-w^0_iH^0_r) \tag{39}$$

To solve for the velocities, $w^1_{i,n}$, the magnetic fields, $H^1_{r,z}$, and pressures $p^{(2)}_{i,n}$, we substitute the expressions for the basic approximations into equations (33-39).

To reduce the tedious algebra we expand the Bessel's functions about the arguments (α, r, s) to order $0(\alpha^2)$. With the result that equation (22) is integrated to yield:

$$p^1_i=(\frac{\chi k\alpha^2}{16})[\frac{r^6}{12}s-\frac{r^2}{4}s^3-\frac{r^4}{16}+\frac{r^2}{s}s_z+(r^2s^3-\frac{r^4}{16}s).s_z\int^z\frac{ds}{s}]\int^z\frac{ds}{s} \tag{40}$$

Similarly, the magnetic fields are given as:

$$H^1_z=(\frac{R_eR_m\alpha^2}{32})s_zs^5*$$

$$\{\int^z\frac{dz}{s}[\frac{1}{6}(\frac{r}{s})^4-(\frac{r}{s})^2+\frac{5}{6}]+s^5\{\frac{1}{24}(\frac{r}{s})^4-\frac{1}{8}(\frac{r}{s})^2+\frac{1}{12}\}+\{\frac{1}{6}(\frac{r}{s})^4-\frac{1}{12}(\frac{r}{s})^2-\frac{1}{12}\}\} \tag{41}$$

$$H^1_r=R_eR_mk\frac{\alpha^2}{16}[\frac{r^7s_z}{192s}-\frac{r^5}{72}(s_z+s^2_z\int^z\frac{ds}{s}+ss_{zz}\int^z\frac{dz}{s}+2\frac{s_z}{s^2})+$$

$$\frac{r^3}{8}(s_zs^2+3s^2_zs^2\int\frac{dz}{s}+s_{zz}s^3\int\frac{dz}{s}+s_{zz}s^2-\frac{1}{3}s_z)- \tag{42}$$

$$\frac{5}{24}(rs_zs^4+5rs^2_zs^4\int\frac{dz}{s}+rs_{zz}s^5\int\frac{dz}{s}+rs_zs^5)+\frac{rs_z}{12s^2}]$$

If we denote $f^{(1)}=\frac{\partial p^1}{\partial z}$, the velocities w^1_n and u^1_n can be written as:

$$w^1_n=B(z)(I_0(\alpha r)-I_1(\alpha s))-\frac{R_ef^1_n}{4}(r^2-s^2)-\frac{\alpha^2}{\beta}\{\frac{c_1}{64}(r^8-s^8)+$$

$$\frac{c_2}{36}(r^6-s^6)+\frac{c_3}{16}(r^4-s^4)\} \tag{43}$$

$$u_n^1 = B(z)\alpha I_1(\alpha r) r\frac{s_z}{2} - \frac{dB}{dz}\{\frac{1}{\alpha} I_1(\alpha r) - \frac{r}{2} I_0(\alpha s) - \frac{1}{4} R_e f_n^1 r s s_n\} +$$
$$\frac{R_e}{4}\frac{df_n^1}{dz}(\frac{1}{4}r^3 - \frac{1}{2}rs^2) + \frac{\alpha^2}{64\beta}\{\frac{dc_1}{dz}(\frac{1}{10}r^{10} - \frac{1}{12}rs^8) - \frac{c_1}{16}rs^7 s_z + \quad (44)$$
$$\frac{dc_2}{dz}\frac{1}{36}(\frac{1}{8}r^7 - \frac{1}{2}rs^6) - \frac{c_2}{12}rs^5 s_z + \frac{dc_3}{dz}\frac{1}{16}(\frac{1}{6}r^5 - \frac{1}{2}rs^4) - c_3\}$$

where

$$c_1 = (\frac{\chi R_e \alpha^2}{192})\frac{s}{s_z}\int\frac{dz}{s} - \frac{\chi R_e k\alpha^2}{64 s^2} \quad (45)$$

$$c_2 = \frac{\chi R_e k\alpha^2}{96}\{(\int^z\frac{dz}{s})^2 + s s_{zz}\int^z\frac{dz}{s} + 2s_z\int^z\frac{dz}{s}\} + \frac{\chi R_e k\alpha^2}{48}\{\frac{s_z}{s} - s_z\int^z\frac{dz}{s}\} \quad (46)$$

$$c_3 = \frac{\chi R_e k\alpha^2}{16}[\frac{3}{4}s_z s^3\int^z\frac{dz}{s} - \frac{1}{3}s_z\int^z\frac{dz}{s} - 4s^2 s_z^2\int^z\frac{dz}{s} - s_{zz}s^3(\int\frac{dz}{s})^2 + \frac{1}{2}s^2] \quad (47)$$

Here B (z) is a constant function of z only which, on applying the no slip condition, results in two separate equations: one relating the pressure $f^{(1)}{}_n$ and the other, the constant B (z).
After some tedious algebra we obtain:

$$B(z) = (\frac{1}{(\alpha s^2 I_0(\alpha s) - 2sI_1(\alpha s))})\alpha\sigma_n^2$$
$$.(\int s^{10}(\frac{dc_1}{dz})dz + \int\frac{c_1}{8}s_z s^9 dz + \frac{1}{4}\int s^8\frac{dc_2}{dz}dz + \frac{1}{6}\int c_2 s_z s^7 dz + \frac{1}{8}\int s^6\frac{dc_3}{dz}.dz + \frac{1}{4}\int c_3 s^5 s_z dz) \quad (48)$$

and

$$\frac{df_n^1}{dz}s^4 + 4f_n^{(1)}s_z s^3 = 0 \quad (49)$$

Equation (49) is reminiscent of the Reynolds equation for pressure in lubrication theory, and following procedure by Bestman (1983), the solution is given as:

$$w_n p_n^1 = -\int\frac{16}{s^4}dz \quad (50)$$

Following similar procedure as before we obtain results for the velocity of the ionized species as follows:

$$w_i^1 = D(z)(I_0(\alpha r) - I_0(\alpha s)) - a_1(r^2 - s^2) + a_2(r^4 - s^4) + a_3(r^6 - s^6) - a_4(r^8 - s^8) \quad (51)$$

where

$$D(z) = \frac{1}{(\alpha s^2 I_0(\alpha s) - 2sI_1(\alpha s))}.$$
$$\{\frac{1}{3}\int\alpha s^3\frac{da_1}{dz}dz + 4\int\alpha a_1 s_z s^2 dz + \frac{2}{3}\int a_2\alpha s^6\frac{da_2}{dz}dz + \quad (52)$$
$$8\int a_2\alpha s_z s^5 dz + \frac{3}{4}\int s^8\alpha\frac{da_3}{dz}dz + 12\int a_3\alpha s_z s^7 dz - \frac{4}{5}\int\alpha s^{10}\frac{da_4}{dz}dz - 16\int a_4 s_z s^9 dz\}$$

$$a_1 = c_3 - \sigma_i^2 \beta R_e \frac{f^1}{4}$$

$$a_2 = c_2 - \sigma_n^2 \frac{c_3}{16\beta}$$

$$a_3 = c_1 - \sigma_n^2 \frac{c_2}{36\beta} \qquad (53)$$

$$a_4 = \frac{c_1 \sigma_n^2}{64\beta}$$

The induced current density is given by:

$$J_\theta = \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial z}\right) \qquad (54)$$

DISCUSSION OF RESULTS

In the last two sections, we have formulated and solved for the flow velocity, induced magnetic fields and current densities of a partially ionized gas. We have also advanced approximate solutions for the pressures.

A primary observation is the dependence of the velocity components on the Reynolds parameter R_e, Hartmann parameter χ, the applied pressure gradients k and frequency parameter σ^2 . The induced magnetic field depends, in addition, on the magnetic Reynolds number R_m.

The choice of the small parameter ε (which is actually the ratio of radial to axial length scale) is found to linearise the very non-linear MHD equations, but has its consequences on the induced fields; for, in the limit $\varepsilon \to 0$, the flow is unaffected by the magnetic field and is given by the classical Hagen-Poiseuille equation. (see equations (28-30)). The azimuthal current densities, however, appear in first order approximations due to the fact that the basic flow crosses the applied magnetic field lines at an angle of the order ε. Thus an induced magnetic field appears at the higher approximations, but has no influence on the magnetic body force in the zeroth order approximation.

The coupling between the charged and neutral species through their frictional behaviour have been the reason for the appearance of the magnetic parameter M (χ) in the expression for w'^1_n .

To obtain a physical feel of how these parameters affect the flow pattern, numerical results are presented graphically for flow in locally dilated and constricted tubes defined as:

$$s(z) = \exp(\pm \varepsilon z) \qquad (55)$$

In the numerical calculations, ε is taken in the range (0.01-0.05). The axial velocities are obtained in equations (28, 43, 51) and the current densities are obtained from equation (54)

For purposes of comparison, we derive the expression for fully ionized plasma. This is achieved by setting the frequency parameter $\sigma^2 = 0$ in the relevant equations.

We examine the combined effects of channel geometry and the presence of neutral gas on the flow behaviour. For this we have taken $R_e = 5$; $R_m = 5$; $\chi = 5$; k = 2 while we vary ε from 0.01 to 0.1, β from 0.01 to 0.1; and σ^2 from 1 to 2. Results are displayed in figures 1 to 8.

Figures (1-3) show the axial velocity profile plotted against the radial distance for the ionic specie and display the effects of channel indentation and percentage composition of neutral gas in the mixture. The collision effect between the two gas components is also displayed. Figures (4-5) show, respectively, the magnitudes of the velocity profile for the composite gas and that for the fully ionized gas. We observe a tendency to reversal of velocity of the ionized gas in the mixture with the neutral gas. This velocity reversal is attributable to the collision frequency and the indentation of the channel. The behaviour of the gas velocity in the channel with indentation seems to differ between the two media, namely, the plasma with impurity and that without impurity. Whereas, in the fully ionized gas the magnitude of velocity decreases with ε in the composite mixture the velocity increases with ε. The only common feature is the tendency for higher velocity in a constricted channel section than that through a diverging section. In fact, the tendency to velocity reversal is seen to occur only at a diverging section of the channel.

Figures (6-7) show the plots of the current density $J_{i,n}$ for the composite gas versus the channel radius, while in figure 8, we show the current density for the fully ionized gas. We observe here that whereas the magnitudes of the current densities remain unaltered for channel indentation $\varepsilon = 0.01$, they vary considerably with large number of impurities (i.e. large ratio of β) and the collision frequency σ^2.

At and near the throat of any channel indentation (i.e. at z = 1, 0.5, 0) there is an observed variation of the current densities, especially at the higher value of the parameter ε.

What is clear, however, is the enhancement of the magnitude of the velocity of the plasma gas and the induced current density by the presence of the neutral gas (impurity) through their frictional behaviour.

The results of our analysis may be found useful in MHD flow problems, biomedicine and plasma confinement.

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