DOI: http://dx.doi.org/10.4314/gjpas.v22i1.7

GLOBAL JOURNAL OF PURE AND APPLIED SCIENCES VOL. 22, 2016: 51-59
 ISSN 1118-0579
www.globaljournalseries.com, Email: info@globaljournalseries.com

VECTOR BILINEAR AUTOREGRESSIVE TIME SERIES MODEL AND ITS SUPERIORITY OVER ITS LINEAR AUTOREGRESSIVE COUNTERPART

I. A. IWOK

(Received 16 May 2015; Revision Accepted 16 June 2016)

ABSTRACT

In this research, a vector bilinear autoregressive time series model was proposed and used to model three revenue series (X_{1t}, X_{2t}, X_{3t}). The "orders" of the three series were identified on the basis of the distribution of autocorrelation and partial autocorrelation functions and were used to construct the vector bilinear models. The estimates obtained from the bilinear fits were compared graphically with those obtained from fitting linear (autoregressive) models. Residual variance and Box-Ljung Q statistic comparisons were also made. The result showed that vector bilinear autoregressive (BIVAR) models provide better estimates than the long embraced linear models.

KEYWORDS: Linear time series, Autoregressive process, Autocorrelation function, Partial autocorrelation function, Vector time series and bilinear vector process.

INRODUCTION

In the past seven decades, a time series was usually modeled as a linear function of its own past, using autoregressive (AR) or mixed autoregressive moving average (ARMA) frame work. This was because these models are easy to analyze and they provide fairly good approximations for the true underlying process.

However, the underlying structure of the series may not be linear and what is more, the series may not be Gaussian. In these situations, second order properties, such as co-variances and spectra, can no longer adequately characterize the properties of the series. This called for the emergence of non linear models in which bilinear forms a class.

Weiner (1958) considered a nonlinear relationship between an input u_t and an output X_t (both observable) using Volterra series expansion given by

$$X_t = \sum_{i=0}^{\infty} \sim_i u_{t-i} + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \sim_{ij} u_{t-i}u_{t-j} + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \sim_{ijk} u_{t-i}u_{t-j}u_{t-k} + \ldots \quad (1)$$

From a given finite realization of a process, one cannot estimate the parameters $\{\sim_i\}, \{\sim_{ij}\}, \{\sim_{ijk}\}, \ldots$ efficiently. To overcome this difficulty, Granger and Anderson (1978) introduced a class of nonlinear models called "*bilinear*" in the time series context [assuming $u_t = £_t$ (unobservable)] satisfying

$$¤(B)X_t = {}^{-}(B)£_t + \sum_{i=1}^{r}\sum_{j=1}^{s} {}_{ij} X_{t-i}£_{t-j} \quad (2)$$

where $¤(B)$ and ${}^{-}(B)$ are p^{th} order AR and q^{th} order moving average (MA) polynomials on backward shift operator B and ${}_{ij}$ are constants.

Maravall (1983) used a bilinear model to forecast Spanish monetary data and reported a near 10% improvement in one-step ahead mean square forecast errors over several autoregressive moving average (ARMA) alternatives. There is no doubt that most of the economic or financial data assume fluctuations due to certain factors.

James (2014) used a bilinear model to forecast South Africa's gross domestic product (GDP). Comparison was made with the forecasts generated by vector autoregressive (VAR) models. The result showed that bilinear forecasts were better than the VAR forecasts.

The general form of the bilinear model according to Rao (1980) is given by the difference equation:

I. A. Iwok, Department of Mathematics/Statistics, University of Port-Harcourt, P.M.B.5323, Port-Harcourt, Rivers State, Nigeria.

$$X(t)+\sum_{j=1}^{p}a_j X(t-j)=\sum_{j=1}^{r}c_j e(t-j)+\sum_{l=1}^{m}\sum_{l^1=1}^{k}b_{ll^1}X(t-l)e(t-l^1) \qquad (3)$$

Where $\{e(t)\}$ is an independent white noise process and $c_0 = 1$. $\{X(t)\}$ is termed the bilinear process. The autoregressive moving average model $ARMA(p,r)$ is obtained from (2) by setting $b_{ll^1}=0 \;\; \forall \;\; l$ and l^1.

Iwueze (2002) studied the existence and computation of all second order moments of the vector valued time series of the form

$$X_t = Ce_t + AX_{t-1} + \sum_{j=1}^{q} b_j e_{t-j} + \sum_{j=1}^{q} B_j X_{t-j}e_{t-j}$$

where $X_t = \left(X_t, X_{t-1}, X_{t-2}, ..., X_{t-p+1}\right)^T$, C and bj are given $p \times 1$ matrices with real entries, A and B_j are given matrices with real entries; and $p = \max(r,m), g = \min(m,s), q = \max(h,g)$. He found that the vectorial representation leads to an important result on matrix algebra with respect to the spectral radius of Kronecker product of matrices.

Boonchai and Eivind (2005) gave the general form of multivariate bilinear time series models as:

$$X(t)=\sum A_i.X(t-i)+\sum M_j.e(t-j)+\sum\sum\sum B_{dij}.X(t-i).e_d(t-j)+e(t) \qquad (4)$$

Here the state $X(t)$ and noise $e(t)$ are n-vectors and the coefficients A_i, M_j and B_{dij} are n by n matrices. If all $B_{dij}=0$, we have the class of well-known vector $ARMA$-models.

Iwok and Etuk (2009) established the vector form of bilinear autoregressive moving average time series model as:

$$X_{it} = \sum_{r=1}^{n}\sum_{k=1}^{\max p} ¥_{k.ir} X_{rt-k} + \sum_{s=1}^{n}\sum_{l=1}^{\max q} a_{l.it} £_{st-l} + \sum_{r=1}^{n}\sum_{k=1}^{\max p}\sum_{l=0}^{\max q} {}_{kl.ir} X_{it-k} £_{rt-l}$$
$$+ \sum_{\substack{s=1\\ s\neq i}}^{n}\sum_{l=1}^{\max q}\sum_{k=0}^{\max p} {}_{kl.is} X_{st-k} £_{it-l} + £_{it} \qquad (5)$$

and recorded its advantages over the pure vector autoregressive moving average models.

We have noted here that except for Boonchai and Eivind (2005) who gave a theoretical form in population dynamics, other works in bilinear time series were based either on mixed ARMA univariate cases or vector of lagged variables of the same time series.

In this research, however, an AR bilinear process is isolated from the vector framework of Iwok and Etuk (2009). That is, a multivariate bilinear AR case where each element of the vector is being explained by the lag values of itself and other time series variables in both the linear and non linear components of the model. This differs from the work of Iwueze (2002) where only one time series was involved and the vector form referred to lagged variables of the same series. Our objectives extend to comparing the performance of our new vector concept with the long embraced linear model.

METHODS OF ESTIMATION

Let $\boldsymbol{X_{it}^{\prime}} = \left[X_{1t}, X_{2t}, ..., X_{nt}\right]$ be a vector of n-dimensional time series.

Linear AR Model

(i) Univariate case:

This is a model in which the current value of the process is expressed as a finite, linear aggregate of previous values of the process and a shock $£_{it}$.

For the n-series, $\{X_{it}\}$ is called a p^{th} order autoregressive process [denoted by $AR(p)$] if it satisfies the difference equation,

$$X_{it} = ¤_1 X_{it-1} + ¤_2 X_{it-2} + ... + ¤_p X_{it-p} + £_{it} \qquad (6)$$

where

$¤_1, ¤_2, ..., ¤_p$ are constants and $\{£_{it}\}$ are a purely random processes.

(i) Vector Case:

The general vector analogue to the univariate Autoregressive time series models for the n- series is:

$$X_{it} = \sum_{r=1}^{n} \sum_{k=1}^{\max p_i} ¥_{k.ir} X_{rt-k} + £_{it} \quad \text{------------} \quad (7)$$

where $¥_{k.ir}$ are the Autoregressive (AR) parameters. p_i are the AR orders. $£_{it}^{1} = [£_{1t}, £_{2t}, ..., £_{nt}]$ is a vector of white noise.

Vector Non Linear Models

Given the vector elements $X_{1t}, X_{2t}, ..., X_{nt}$, the non linear model for a pure AR process is:

$$X_{it} = \sum_{r=1}^{n} \sum_{k=1}^{\max p} \sum_{l=0}^{\max q} {}_{kl.ir} X_{it-k} £_{rt-l} + £_{it} \quad \text{------------} \quad (8)$$

where ${}_{kl.ir}$ are the bilinear parameters of the product series and $l = 0 \ \forall \ q$.

Bilinear Vector Autoregressive Model (BIVAR)

Combining equations (7) and (8)**,** the BIVAR model emerges:

$$X_{it} = \sum_{r=1}^{n} \sum_{k=1}^{\max p} ¥_{k.ir} X_{rt-k} + \sum_{r=1}^{n} \sum_{k=1}^{\max p} \sum_{l=0}^{\max q} {}_{kl.ir} X_{it-k} £_{rt-l} + £_{it} \quad \text{------------} \quad (9)$$

Unlike (6), Equation (9) comprises both the linear and non linear components. This study seeks to compare the performances of the two models (Linear and Bilinear). The parameters of the different models are estimated using linear and intrinsic linear regression techniques.

RESULTS

Estimates for the linear models:

The distribution of autocorrelation and partial autocorrelation functions of the non stationary series suggested pure AR process of order 3 for X_{1t} , AR process of order 2 for X_{2t} and AR of order 1 for X_{3t}. The regression estimates obtained provide the following Autoregressive models for the three series of the vector:

(i) $X_{1t} = 0.309589 X_{1t-1} + 0.354064 X_{1t-2} + 0.321507 X_{1t-3}$ ------------ (10)

(ii) $X_{2t} = 0.497391 X_{2t-1} + 0.436982 X_{2t-2}$ ------------ (11)

(iii) $X_{3t} = 0.817610 X_{3t-1}$ ------------ (12)

Table1: Three sources of internal generated revenue (X_{1t},X_{2t},X_{3t})

S/N	X_{1t}	X_{2t}	X_{3t}	S/N	X_{1t}	X_{2t}	X_{3t}	S/N	X_{1t}	X_{2t}	X_{3t}
1.	30.87	17.01	13.86	41.	186.82	139.41	47.41	81.	164.91	145.21	19.70
2.	31.26	17.31	13.95	42.	169.89	137.98	31.91	82.	215.65	139.52	76.13
3.	29.35	16.10	13.25	43.	176.91	147.73	29.18	83.	167.03	151.33	15.70
4.	30.05	18.68	11.37	44.	256.21	238.38	17.83	84.	219.36	160.19	59.17
5.	25.96	17.46	8.50	45.	260.00	169.12	90.88	85.	176.06	129.01	47.05
6.	30.31	20.55	9.76	46.	434.75	308.15	126.60	86.	251.51	70.66	180.85
7.	31.54	17.04	14.50	47.	258.23	207.11	51.12	87.	325.11	207.01	118.10
8.	45.20	23.85	21.35	48.	169.79	143.58	26.21	88.	257.86	192.54	65.32
9.	41.07	20.57	20.50	49.	358.15	328.97	29.18	89.	195.03	162.92	32.11
10.	45.46	24.86	20.60	50.	397.26	383.01	14.25	90.	220.52	165.52	55.00
11.	48.17	29.65	19.03	51.	279.01	152.71	126.30	91.	225.77	107.42	118.35
12.	40.17	28.67	11.50	52.	220.75	157.39	63.36	92.	167.89	120.52	47.37
13.	45.79	29.76	16.03	53.	178.99	149.68	29.31	93.	198.30	112.85	85.45
14.	32.76	22.89	9.87	54.	164.50	105.69	58.81	94.	257.08	115.70	141.38
15.	30.77	23.25	7.52	55.	192.33	138.53	53.80	95.	183.01	110.86	72.15
16.	32.07	21.97	10.10	56.	198.54	100.29	98.25	96.	106.12	76.75	29.37
17.	37.83	19.64	18.19	57.	143.54	86.21	57.33	97.	207.17	156.60	50.57
18.	43.85	22.60	21.25	58.	155.90	124.20	31.70	98.	209.36	179.21	30.15
19.	30.77	12.60	18.17	59.	198.51	120.68	77.83	99.	309.66	191.79	117.87
20.	37.06	14.53	22.53	60.	260.93	175.79	85.14	100.	391.27	258.99	135.28
21.	31.96	10.61	21.35	61.	299.44	270.84	28.60	101.	388.93	232.97	155.96
22.	29.00	10.30	18.70	62.	211.02	185.08	25.94	102.	250.32	198.14	52.18
23.	30.36	15.04	15.32	63.	188.06	158.68	29.38	103.	328.70	289.35	39.15
24.	36.63	16.90	19.73	64.	252.71	247.66	5.05	104.	475.41	285.73	189.68
25.	45.77.	30.45	15.32	65.	185.72	160.47	25.25	105.	396.98	241.31	155.67
26.	50.00	31.50	18.50	66.	101.75	77.25	24.50	106.	461.13	317.68	143.45
27.	72.50	55.20	17.30	67.	145.56	118.56	27.00	107.	331.10	138.69	192.41
28.	77.18	51.73	25.45	68.	184.41	156.59	27.83	108.	363.17	263.85	99.32
29.	104.08	67.58	36.50	69.	184.41	156.59	27.82	109.	248.5	202.20	46.30
30.	120.70	80.90	39.80	70.	149.33	73.20	76.13	110.	339.98	224.38	115.60
31.	157.31	111.47	45.87	71.	153.39	138.19	15.70	111.	377.75	245.45	132.30
32.	220.45	164.79	55.66	72.	171.38	115.46	55.92	112.	300.42	244.67	55.75
33.	198.76	132.35	66.41	73.	180.48	96.33	84.15	113.	366.28	303.08	63.20
34.	171.03	156.70	14..33	74.	170.13	135.03	35.10	114.	441.37	270.02	171.35
35.	231.97	205.76	26.21	75.	184.16	145.96	38.20	115.	246.69	151.69	95.00
36.	343.58	321.12	22.46	76.	124.36	81.71	42.65	116.	416.48	327.73	88.75
37.	143.73	132.88	10.85	77.	222.96	194.45	28.51	117.	320.97	213.59	107.35
38.	126.16	91.21	34.95	78.	175.75	151.25	24.50	118.	347.35	272.14	75.21
39.	107.93	74.75	33.18	79.	614.93	587.93	27.00	119.	422.91	291.66	131.25
40.	162.04	139.41	22.63	80.	142.32	114.50	27.82	120.	641.23	485.56	155.67

Source: Monthly generated revenue (for a period of ten years) from Ikot Ekpene L.G.A. of Nigeria.

Estimates for the BIVAR models:

The bilinear vector autoregressive model consists of two parts. The first part is the linear vector AR process, while the second part is the product of lagged vector elements and white noise. Estimates of the BIVAR parameters and fits were obtained by treating equation (9) as an intrinsically linear model. The following parameter estimates were obtained: $X_{1t} = 0.266X_{1t-1} + 0.0610X_{2t-1} + 0.0322X_{3t-1} + 0.397X_{1t-2} + 0.163X_{2t-2} + 0.178X_{1t-3}$

$$+0.00158£_{1t-0}X_{1t-1} - 0.000724£_{2t-0}X_{1t-1} - 0.000253£_{3t-0}X_{1t-1}$$

$$+0.00196£_{1t-0}X_{1t-2} + 0.000291£_{2t-0}X_{1t-2} - 0.00132£_{3t-0}X_{1t-2} \quad \text{--} \quad (13)$$

$$+0.000432£_{1t-0}X_{1t-3} + 0.000918£_{2t-0}X_{1t-3} + 0.00135£_{3t-0}X_{1t-3}$$

$$X_{2t} = 0.0870X_{1t-1} + 0.433X_{2t-1} - 0.177X_{3t-1} + 0.0903X_{1t-2} + 0.497X_{2t-2} - 0.0609X_{1t-3}$$

$$+0.000036£_{1t-0}X_{2t-1}+0.00149£_{2t-0}X_{2t-1}+0.000549£_{3t-0}X_{2t-1}$$
$$+0.000304£_{1t-0}X_{2t-2}+0.00260£_{2t-0}X_{2t-2}-0.00114£_{3t-0}X_{2t-2} \quad \text{---} \quad (14)$$

$$X_{3t}=0.0922X_{1t-1}-0.0544X_{2t-1}+0.651X_{3t-1}+0.0073X_{1t-2}+0.0079X_{2t-2}+0.0435X_{1t-3}$$
$$+0.000064£_{1t-0}X_{3t-1}-0.000374£_{2t-0}X_{3t-1}+0.0104£_{3t-0}X_{3t-1} \quad \text{--} \quad (15)$$

As could be seen above, these models are linear in states X_{it-k} but non-linear jointly with $£_{it-l}$ as the name ‘bilinear’ implies.

A Comparison of the Linear Model and Vector Bilinear Autoregressive Time Series Models

(i) *Residual Variances*

After fitting the models, the calculated residual variances from the estimated linear equations (10)-(12) are 88.23 for X_{1t}, 76.25 for X_{2t} and 40.28 for X_{3t} . Similarly, the residual variances for the bilinear vector models in equations (13)-(15) are 16.36 for X_{1t}, 22.90 for X_{2t} and 21.96 for X_{3t}.Comparatively, the residual variances of the bilinear models are smaller than the variances obtained from the linear models. This makes bilinear vector AR models superior to the linear AR counter part.

(ii) *A Portmanteau Lack of Fit Test*

A variant of the Box and Pierce Q statistic is the Box-Ljung statistic, define as

$$Q^{*}=N(N+2)\sum_{k=1}^{mN}\left(\frac{r_k^2}{N-k}\right)\sim ¡^{2}_{(m)}$$

where the first m autocorrelation functions of the residuals $r_k(\hat{£})$’s are examined for evidence of adequacy of the model. If the model is inappropriate, the average values of Q^{*} will be inflated.

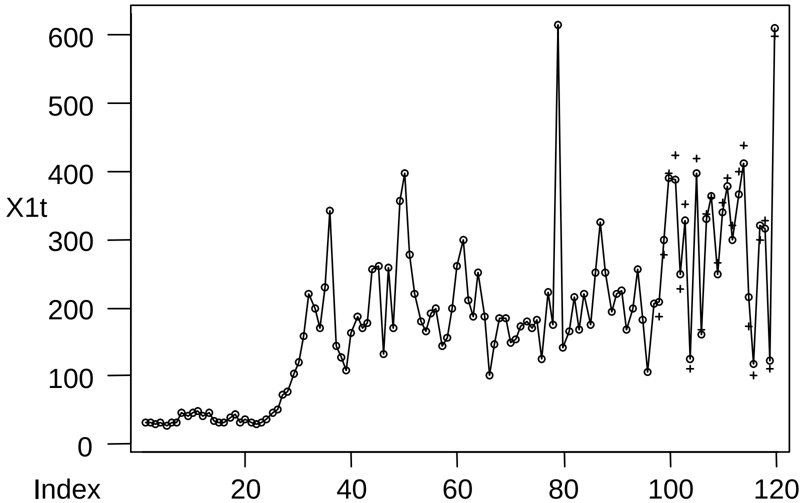
First, let $Q^{*}_{LIN.AR}(X_{it})$ be the Box-Ljung Q statistic obtained from fitting AR model to X_{it} and let $Q^{*}_{V.B.AR}(X_{it})$ be the Box-Ljung Q statistic obtained from fitting BIVAR model to X_{it}.

This test considered the first 20 autocorrelations of the $\hat{£}_t$’s and the calculated Q^{*}’s were as follows: $Q^{*}_{V.B.AR}(X_{1t})=27.61$, $Q^{*}_{V.B.AR}(X_{2t})=44.49$, $Q^{*}_{V.B.AR}(X_{3t})=26.25$ and $Q^{*}_{LIN.AR}(X_{1t})=36.18$, $Q^{*}_{LIN.AR}(X_{2t})=44.72$, $Q^{*}_{LIN.AR}(X_{3t})=48.74$.

Since the Q^{*} values for the AR fitted models are greater than the values of the BIVAR Q^{*} statistics, we conclude that the BIVAR models are the most appropriate.

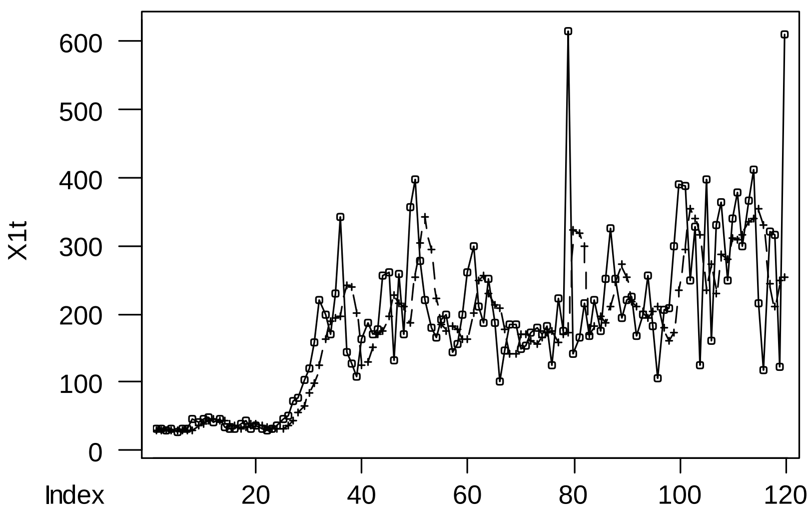
(iii) Plots of Actual and Estimate Values

The actual and estimated values of the models are plotted in figures 1(a and b), 2(a and b) and 3(a and b) below. Each figure displayed contains two plots (the actual marked by ‘o’ and the estimate marked by ‘+’)



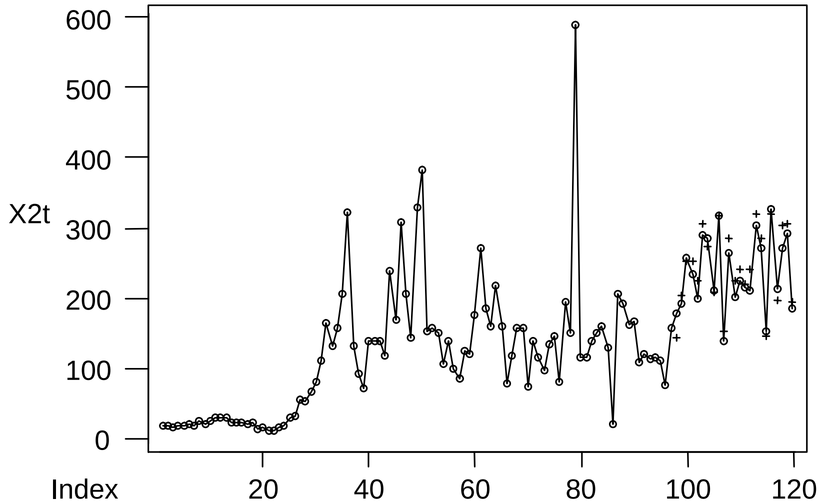
Key: o (actual plot) and + (estimates plot)

Figure 1a: Vector BILINEAR-AR Plots of actual and estimates of X_{1t}



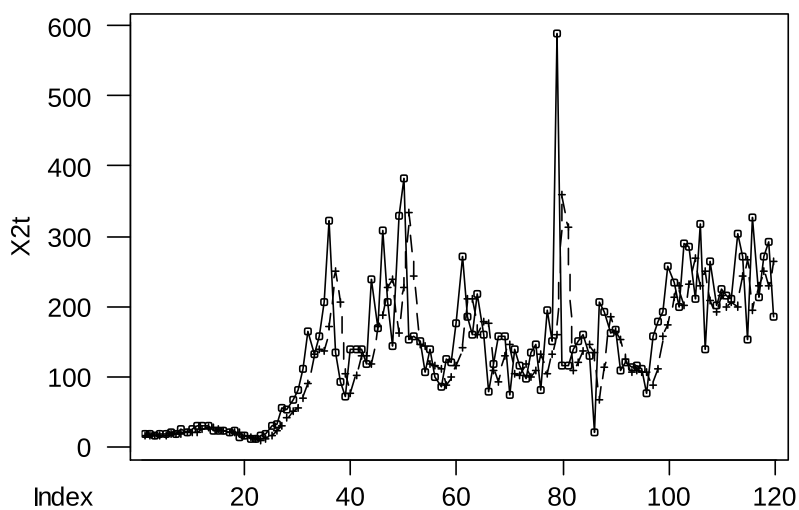
Key: o (actual plot) and + (estimates plot)

Figure 1b: Linear -AR Plots of actual and estimates of X_{1t}



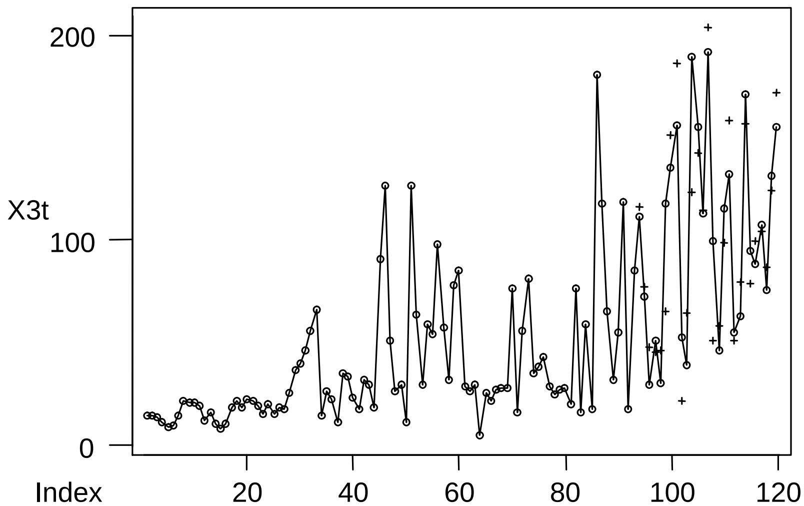
Key: o (actual plot) and + (estimates plot)

Figure 2a: Vector BILINEAR-AR Plots of actual and estimates of X_{2t}

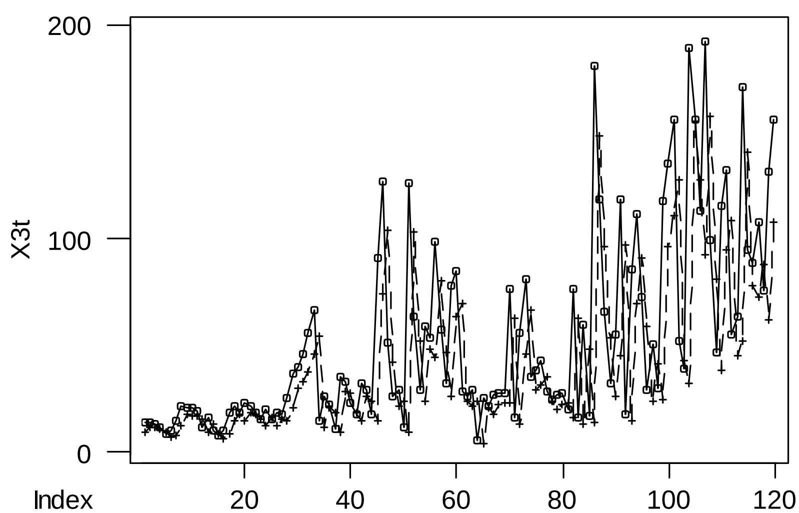


Key: o (actual plot) and + (estimates plot)

Figure 2b: Linear-AR Plots of actual and estimates of X_{2t}



Key: o (actual plot) and + (estimates plot)
Figure 3a: Vector BILINEAR-AR Plots of actual and estimates of X_{3t} .



Key: o (actual plot) and + (estimates plot)
Figure 3b: Linear -AR Plots of actual and estimates of X_{3t} .

Examination of the actual and estimates plots above show that AR model estimates exhibit less interwoven behaviour with the real data than BIVAR estimates plots. This is an indication of a high degree performance of the bilinear models.

DISCUSSION

As noted in the literature review, most works assume that a bilinear model is a function of lag variables of the dependent variables. This is a situation where the same series is modeled using the lag values of itself. This work, however, differs in this approach. In this work, each element of the vector is being explained by the lag values of itself and other time series variables in both the linear and non linear components of the model. It is believe that this research has provided another approach to bilinear time series modeling.

CONCLUSIONS

As mentioned earlier, a linear time series model such as AR expresses itself as a linear combination of its past and has been widely used in diverse fields. Due to non stationarity of most series, however, bilinear models have replaced linear models by offering better analytical tools for analyzing several time series data. From the minimum variance property, Q statistic and graphical verdict shown in this work, there is no gain saying the fact that some series especially, revenue series assume not only linear component but also non linear part. This is so because of the random nature of observations assume by certain processes. The result of this work confirms that non linear models such as 'bilinear vector AR' are superior to pure linear AR models.

REFERENCES

Boonchai, K. S and Eivind, S., 2005. Multivariate bilinear time series; a stochastic alternative in population dynamics. Geophysical research Abstracts,Vol.7,02219.SRef-ID:1607-7962/gra/EGU05-A-02219. Available from: © European Geosciences Union 2005.http://www.cosis.net/abstract.

Granger, C. W and Anderson, A., 1978. Non linear Time Series Modeling. Applied Time Series Analysis. Academic Press, New York.

Iwok, I. A and Etuk, E. H., 2009. On the comparative performance of pure vector Autoregressive moving average and vector bilinear autoregressive Moving average time series models. Asian Journal of Mathematics and Statistics.ISSN 1994-5419. © 2009 Asian Network for Scientific Information.

Iwueze, S., 2002. Vectorial Representation and its application to covariance analysis of super-diagonal bilinear time series models. The physical scientist. 1: 85-96.

James, T. S., 2014. Forecasting with bilinear models. South African Journal of Applied Statistics; 3, (8): 52-57. ISBN: 2-0345-4215-7.

Maravall, A., 1983. An application of non linear time series forecasting. Journal of Business and Economic Statistics. 4:66-74. Available from: http://www.tijsat.tu.ac.th/toc.id=2_diretory/vip.pdf
.

Rao, T. S., 1980. On the Theory of Bilinear Time Series Models. Journal of Royal Statistical Society Vol. 43, 2:244-255. Available from: http://www.3.stat.sinica.edu.tw/statistica.DOI:10.1057/jors.1980.78

Weiner, N., 1958. Nonlinear Problems in Random Theory, John Wiley, New York.