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A COMPARATIVE ANALYSIS OF NUMEROV AND NYSTROM METHODS

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ABSTRACT

In this paper, two test problems were used to study the behaviour and performance of two numerical methods (Numerov and Nystrom) of solving second order initial value problem (IVP) of ordinary differential equation without the first order term. A test implementation of each of the methods and a comparative analysis of the two methods was done. The result showed that only Nystrom method is suitable for solving problems with negative exponential solution as Numerov method provided no meaningful approximation to it. Also both Numerov and Nystrom methods provided reasonable approximation for problems with cosine solution, but with Nystrom consistently providing a better approximation.

KEY WORDS: Numerov Method, Nystrom Method, Initial Value Problem, Test Problem

INTRODUCTION

As a result of the contribution of numerical solution of differential equations to Engineers and physical scientists much research work has been done in creating new numerical methods, adapting and improving existing ones and investigating the stability and accuracy of the methods Iserles (1996), Kreyszig (1999), Olayi (2000). In this paper, two numerical methods, Numerov and Nystrom for solving second order ordinary differential equation problems without first order term were considered. A test implementation of each and a comparative analysis of the two methods were done to determine the accuracy of each of the methods.

Two test problems were used for the analysis. The use of test problems is informed by its efficiency in studying the behaviour and performance of numerical methods of solution to differential equations Dahlquist and Bjorck (1974),Olayi and Wajiga (1999). Also, one can get a good insight into how a numerical method function by considering test problems which are simple enough to analyze theoretically but still so general that they can present difficulties for prospective numerical methods.

The test problems used for this study are:

(i) $y^{//} = y, y(0) = 1, y^{/}(0) = -1$ (1.1)

With exact solution $y(x) = e^{-x}, x \geq 0$. The exponential function e^{x}, is a decreasing function if $x < 0$ and an increasing function if $x > 0$ Kreyszig (1999).

(ii) $y^{//} = -y, y(0) = 1, y^{/}(0) = 0$ (1.2)

with exact solution $y(x) = \cos$ x. The cosine function is a periodic function with period $2\prod$. Boyce and Diprima (1997). The two problems will hereafter be referred to as test problem I and test problem II respectively. Here, the solution of the test problems were considered on [0,1] using the classical Numerov and Nystrom algorithm.

THEORY

Numerov and Nystrom methods are numerical methods for solving second order ordinary differential equation without the first order term. That is equations of the form:

$$y^{//} = f(x, y) \quad (2.1)$$

This type of differential equations has applications in celestial mechanics, mechanical system and heat flow in a homogenous medium Gear (1971), Stackgold (1979),Fatunla (1988). When initial conditions are assigned to equation (1.3), it becomes:

$$y^{//} = f(x, y), y(x_0) = y \ , y^{/}(x_0) = y_1 \quad (2.2)$$

Equation (2.2) is called initial value problem for second order ordinary differential equation Iserles (1996), Kreyszig (1999), Ogbu (2008).

Numerov method is a member of the family of linear multi-step method while Nystrom method is a single step method for solving equation (2.2) Isaac and Unanam (2010).

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ALGORITHM FOR NUMEROV AND NYSTROM METHODS

The classical Numerov formula for equation (2.2) is given as

$$y_{n+1} - 2y_n + y_{n-1} = \frac{h^2}{12}(f_{n+1} + 10f_n + f_{n-1}) \quad (3.1)$$

Where $y_k \cong y(x_k), f_k \cong f(x_{k,} y_k), k = 0,1,2...,$ with the truncation error $\frac{h^2}{20} y_t^{6}, y^{6}_t$ denoting the sixth derivative of some point in the interval of integration Olayi (1998).
Also, the Nystrom algorithm for equation (2.2) is given as

$$y_{n+1} = y_n + hy^1{}_n + ½(k_1 + k_2)$$
$$y^1{}_{n+1} = y^1{}_n + \frac{k_1 + 3k_2}{2h} \quad (3.2)$$

where
$$k_1 = \frac{h^2 f(x_{n,} y_n)}{2}$$
$$k_2 = \frac{h^2 f(y_n + \frac{2}{3} hy^1{}_n + \frac{4}{9} k_1)}{2}$$

where $y_m \cong y(x_m), y^1{}_m \cong y^1(x_m)$ and $f_m \cong f(x_{m,} y_m), m = 0,1,2,.....$, Gear (1971) Equation (3.2) has a Local truncation error of $0(h^4)$ in both y and y^1, giving a global error of $0(h^3)$ in each component Gear (1971).

IMPLEMENTATION/RESULT

The classical Numerov and Nystrom formulae (3.1) and (3.2) respectively, were used to solve the two test problems (1.1) and (1.2) at the designated step-sizes, h = 0.2, 0.1 and 0.05 and the result of the exercise are shown in table1 to table 5 for selected points of the solution space [0,1]. The error column in the tables are each define by Error $= y_{computed} - y_{exact}$

TABLE 1: Errors of Numerov method with various step-sizes for test problem I

	y_{exact}	$h = 0.2$ **error**	$h = 0.1$ **error**	$h = 0.05$ **error**
0.2	0.81873	-0.77860	-1.79861	-3.81869
0.4	0.67032	-1.58845	-3.66940	-7.78015
0.6	0.54881	-2.46205	-5.68774	-12.05597
0.8	0.44933	-3.43447	-7.93372	-16.81564
1.0	0.36788	-4.54471	-10.49844	-22.25012

TABLE 2 : Errors of Nystrom method with various step sizes for test problem I

x_n	y_{exact}	$h = 0.2$ **error**	$h = 0.1$ **error**	$h = 0.05$ **error**
0.2	0.81873	0.00003	0.00001	0.0000008
0.4	0.67032	0.00006	0.00001	0.0000014
0.6	0.54881	0.00011	0.00002	0.0000016
0.8	0.44933	-0.00184	-0.00005	0.0000390
1.0	0.36788	-0.00182	-0.00339	0.0005394

TABLE 3: Errors of Numerov method with various step sizes for test problem II

x	y_{exact}	$h=0.2$ **error**	$h=0.1$ **error**	$h=0.05$ **error**
0.2	0.999994	-0.03986	-0.029864	-0.024914
0.4	0.999976	-0.11800	-0.098399	-0.088666
0.6	0.999945	-0.23128	-0.202885	-0.188675
0.8	0.999903	-0.37331	-0.342783	-0.321062
1.0	0.999848	-0.54032	-0.508858	0.480448

TABLE 4: Errors of Nystrom method with different step sizes for test problem II

x	y_{exact}	$h=0.2$ **error**	$h=0.1$ **error**	$h=0.05$ **error**
0.2	0.999994	-002008	-0.015004	-0.012554
0.4	0.999976	-0.08029	-0.06729	-0.074885
0.6	0.999945	-0.16958	-0.160465	-0.162385
0.8	0.999903	-0.29575	-0.291593	-0.316883
1.0	0.999848	-0.45524	-0.453798	-0.468027

TABLE 5: Comparison of errors of Numerov and Nystrom methods with various step sizes for test problem II

		Numerov			Nystrom		
x_n	y_{exact}	$h=0.2error$	$h=0.1error$	$h=0.05error$	$h=0.2error$	$h=0.1error$	$h=0.05error$
0.2	**0.999994**	**-0.03986**	**-0.029864**	**-0.024914**	**-0.02008**	**-0.015004**	**-0.012554**
0.4	**0.999976**	**-0.11800**	**-0.098399**	**-0.088666**	**-0.08029**	**-0.06729**	**-0.074885**
0.6	**0.999945**	**-0.23128**	**-0.202885**	**-0.188675**	**-0.16958**	**-0.160455**	**-0.162385**
0.8	**0.999903**	**-0.37331**	**-0.342783**	**-0.321062**	**-0.29575**	**-0.291593**	**-0.316883**
1.0	**0.999848**	**-0.54032**	**-0.508858**	**-0.480448**	**-0.45524**	**-0.453798**	**-0.468027**

DISCUSSION

After numerical computation, certain common points (x=0.2, 0.4, 0.6, 0.8, and 1.0) of the solution space [0,1] were selected. The estimates at each of these points were compared with the exact solution and the error was calculated as a parameter of comparison. The result shows that errors of Numerov method increased as the step size decreased (Table 1) whereas the errors of Nystrom solutions decreased with decrement in step-size (Table 2) when the two methods were applied to test problem I (eqn. 1.1). This means that Numerov method cannot be used to solve test problem I while Nystrom method provided reasonable approximation to the solution of test problem 1.

For test problem II (eqn. 1,2), the errors of both methods decreased as the step-size is decreased (Table 3 and 4) signifying that both methods provided similar reasonable approximations, but with Nystrom consistently providing better approximation than Numerov (Table 5)

CONCLUSION

From the result obtained, it is evident that Numerov method cannot be used to solve problems with negative exponential solution. Also, both methods provided similar reasonable approximation for problems with cosine solution, but with Nystrom consistently providing better approximation than Numerov.

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