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SOME STIFFLY STABLE SECOND DERIVATIVE CONTINUOUS LINEAR MULTISTEP METHODS WITH A HYBRID POINT FOR STIFF IVPs IN ODEs.

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ABSTRACT

Based on Gear's fixed step size backward differentiation methods, Gear (1968), second derivative continuous linear multistep methods with an off-step point are presented. This type of methods provides a means of bypassing the order barriers imposed by Dahlquist (1963) and also provides continuous solutions of IVPs in ODEs. The stiff stability of these methods is determined by using the boundary locus. Instability sets in at $k = 10$. Numerical results of the methods solving a non-linear and a linear stiff initial value problems in ordinary differential equations are compared to that of the state -of -the-art code, ODE 15s in MATLAB.

KEY WORDS: Continuous Linear Multistep Methods, Hybrid Method, Off-Step Point, Stiff Stability, Boundary Locus.

INTRODUCTION

For the numerical solution of the initial value problem

$$y' = f(x, y(x)), \; y(a) = y_0, \; x \in (a, b), y \in R^m, f \in R^{m+1} \tag{1.1}$$

whose solution is stiff, let us consider the family of hybrid second derivative continuous linear multistep methods

$$y_{n+k} = \sum_{j=0}^{k-1} \lambda_j(t) y_{n+j} + \lambda_v(t) y_{n+v} + h\theta_{1,v}(t) f_{n+v} + h\theta_{1,k}(t) f_{n+k} + h^2 \theta_{2,k}(t) f'_{n+k} \tag{1.2}$$

And the hybrid predictor

$$y_{n+v} = \sum_{j=0}^{k} \lambda_j^*(t) y_{n+j} + h\theta_{3,k}(t) f_{n+k} \tag{1.3}$$

where t is the scaled variable given as $t = (x - x_{n+1})/h$. The value of v is taken to be $k - \frac{1}{2}$ for a fixed k. These methods will be derived for the case where the IVP (1.1) is a scalar, that is $m = 1$. However our examples in section 5 will show how it can be implemented to the case where the IVP (1.1) is a vector, that is $m \geq 2$. The extension in (1.2) consists of the addition of the terms $\lambda_v(t) y_{n+v}$ to the left hand side and $h\theta_{1,v}(t) f_{n+v}$ to the right hand side of the Gear's fixed step size backward differentiation methods, Gear (1968).

Similar methods for the solution of (1.1) were introduced by Otunta et al (2007), Ikhile and Okuonghae (2007), Okuonghae (2008). Discrete formulation of the same sort are found in Butcher (1965, 1987, 2001), Enright (1974), Gragg and Stetter (1964), Kohfeld and Thompson (1968) and Gear (1964, 1968).

The idea behind the proposed methods is the use of collocation and interpolation procedure discussed in Onumanyi et al (1996), Arevalo et al (2002), Sirisena et al (2001).

The proposed methods are found to be highly stable and give accurate result.

Sections 2 and 3 contain the derivation of methods (1.2) and (1.3) in continuous form respectively. In section 4 the stability of the methods is obtained using the boundary locus. In sections 5 and 6 we have numerical experiment and conclusion respectively.

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2.0 Derivation of the Second Derivative Continuous Linear Multistep Methods (CLMM)

Let the numerical solution of (1.1) be in the form of the polynomial interpolant

$$y(x) = \sum_{j=0}^{k+3} a_j x^j \tag{2.1}$$

where $a_j's$ are the real parameter constants to be determined. Collocating and interpolating (2.1) as appropriate in (1.2) yield the linear system of equations

$$\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{k+3} \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & \cdots & x_{n+1}^{k+3} \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & \cdots & x_{n+2}^{k+3} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{n+k-1} & x_{n+k-1}^2 & x_{n+k-1}^3 & \cdots & x_{n+k-1}^{k+3} \\ 1 & x_{n+v} & x_{n+v}^2 & x_{n+v}^3 & \cdots & x_{n+v}^{k+3} \\ 0 & 1 & 2x_{n+k} & 3x_{n+k}^2 & \cdots & (k+3)x_{n+k}^{k+2} \\ 0 & 1 & 2x_{n+v} & 3x_{n+v}^2 & \cdots & (k+3)x_{n+v}^{k+2} \\ 0 & 0 & 2 & 6x_{n+k} & \cdots & (k+3)(k+2)x_{n+k}^{k+1} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ . \\ . \\ a_{k+3} \end{bmatrix} = \begin{bmatrix} y_n \\ y_{n+1} \\ y_{n+2} \\ \vdots \\ y_{n+k-1} \\ y_{n+v} \\ f_{n+k} \\ f_{n+v} \\ f'_{n+k} \end{bmatrix} \tag{2.2}$$

Adopting the approach in Nwachukwu (2010), we obtain the continuous coefficients $\lambda_0(t), \lambda_1(t), \lambda_2(t), \cdots, \lambda_{k-1}(t), \lambda_k(t) = 1, \lambda_v(t), \theta_{1,k}(t), \theta_{2,k}(t), \theta_{1,v}(t)$. For $k = 1, 2, 3$, these coefficients are given in Table 2.1. In Table 2.2, we have the discrete coefficients, the error constant and the order of the Scheme (1.2).

Table (2.1): Continuous Coefficients of the Scheme (1.2).

k	t	j	$\lambda_j(t)$	$\lambda_j(k-1)$	$\theta_{1,k}(t)$	$\theta_{1,k}(k-1)$	$\theta_{2,k}(t)$	$\theta_{2,k}(k-1)$
1	0	0	$\frac{1}{5}-\frac{12t^2}{5}-\frac{16t^3}{5}$	$\frac{1}{17}$	0	0	0	
		$\frac{1}{2}$	$\frac{16}{17}-\frac{32t^3}{17}-\frac{48t^4}{17}$	$\frac{16}{17}$	$\frac{4}{17}+\frac{60t^3}{17}+\frac{60t^4}{17}$	$\frac{4}{17}$	0	**0**
		1	1	1	$\frac{5}{17}+t-\frac{44t^3}{17}-\frac{32t^4}{17}$	$\frac{5}{17}$	$\frac{-1}{34}+\frac{t^2}{2}+\frac{18t^3}{17}+\frac{10t^4}{17}$	$-\frac{1}{34}$
2	1	0	$\frac{-23t}{711}+\frac{131t^2}{711}-\frac{265t^3}{711}+\frac{224t^4}{711}-\frac{68t^5}{711}$	$-\frac{1}{711}$	0	0	0	0
		1	$1-\frac{369t}{79}+\frac{453t^2}{79}+\frac{145t^3}{79}-\frac{58t^4}{79}+\frac{228t^5}{79}$	$\frac{8}{79}$	0	0	0	0
		$\frac{3}{2}$	$\frac{3344t}{711}-\frac{4208t^2}{711}-\frac{1040t^3}{711}+\frac{4528t^4}{711}-\frac{1984t^5}{711}$	$\frac{640}{711}$	$-\frac{424t}{237}+\frac{1096t^2}{237}-\frac{104t^3}{237}-\frac{1064t^4}{237}+\frac{5607t^5}{237}$	$\frac{6}{2\text{[illegible]}}$	0	0
		2	1	1	$\frac{32t}{79}-\frac{117t^2}{79}+\frac{63t^3}{79}+\frac{128t^4}{79}-\frac{84t^5}{79}$	$\frac{22}{79}$	$-\frac{13t}{158}+\frac{25t^2}{79}-\frac{33t^3}{158}-\frac{26t^4}{79}+\frac{22t^5}{79}$	$\frac{-2}{79}$

3	2	0	$-\frac{1638t}{19525}+\frac{5343t^2}{19525}-\frac{13633t^3}{39050}+\frac{8523t^4}{39050}-\frac{1308t^5}{19525}+\frac{158t^6}{19525}$	$\frac{4}{19525}$	0	0	0	0
		1	$1-\frac{5296t}{2343}+\frac{6304t^2}{7029}+\frac{10964t^3}{7029}-\frac{4219t^4}{2343}+\frac{4924t^5}{7029}-\frac{676t^6}{7029}$	$-\frac{3}{781}$	0	0	0	0
		2	$\frac{12006t}{781}-\frac{15063t^2}{781}-\frac{8831t^3}{1562}+\frac{28623t^4}{1562}-\frac{7200t^5}{781}+\frac{1142t^5}{781}$	$\frac{108}{781}$	0	0	0	0
		$\frac{5}{2}$	$-\frac{69376t}{5325}+\frac{289408t^2}{15975}+\frac{70976t^3}{15975}-\frac{89152t^4}{5325}+\frac{137152t^5}{15975}-\frac{21952t^5}{15975}$	$\frac{108}{781}$	$\frac{28096t}{3905}-\frac{132128t^2}{11715}-\frac{16336t^3}{11715}+\frac{40192t^4}{3905}-\frac{67952t^5}{11715}+\frac{11552t^6}{11715}$	$\frac{1152}{3905}$	0	0
		3	1	1	$-\frac{1647t}{781}+\frac{2862t^2}{781}+\frac{27t^3}{781}-\frac{2566t^4}{781}+\frac{1620t^5}{781}-\frac{296t^5}{781}$	$\frac{210}{781}$	$\frac{342t}{781}-\frac{1227t^2}{1562}+\frac{20t^3}{781}+\frac{1087t^4}{1562}-\frac{362t^5}{781}+\frac{70t^6}{781}$	$\frac{-18}{781}$

Table (2.2): The Discrete Coefficients, the Error Constant (EC) and Order (P) of the Scheme (1.2).

k	t	v	λ_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
1	0	$\frac{1}{2}$	$\frac{1}{17}$	0	0	0	0	0	0	0
2	1	$\frac{3}{2}$	$-\frac{1}{711}$	$\frac{8}{79}$	0	0	0	0	0	0
3	2	$\frac{5}{2}$	$\frac{4}{195252}$	$-\frac{3}{781}$	$\frac{108}{781}$	0	0	0	0	0
4	3	$\frac{7}{2}$	$-\frac{9}{163807}$	$\frac{64}{83575}$	$-\frac{24}{3343}$	$\frac{576}{3343}$	0	0	0	0
5	4	$\frac{9}{2}$	$\frac{16}{792891}$	$-\frac{1125}{4316851}$	$\frac{160}{88099}$	$-\frac{1000}{88099}$	$\frac{18000}{88099}$	0	0	0
6	5	$\frac{11}{2}$	$-\frac{100}{11128249}$	$\frac{32}{275907}$	$-\frac{3375}{4506481}$	$\frac{320}{91969}$	$-\frac{1500}{91969}$	$\frac{21600}{91969}$	0	0
7	6	$\frac{13}{2}$	$\frac{3600}{789672949}$	$\frac{32}{275907}$	$\frac{5488}{14017863}$	$-\frac{7875}{4672621}$	$\frac{27440}{4672621}$	$-\frac{102900}{4672621}$	$\frac{1234800}{4672621}$	0
8	7	$\frac{15}{2}$	$-\frac{49}{19282999}$	$\frac{115200}{3258826831}$	$\frac{548800}{2333242879}$	$\frac{175616}{173546991}$	$-\frac{63000}{19282999}$	$\frac{175616}{19282999}$	$\frac{548800}{19282999}$	$\frac{5644800}{19282999}$
9	8	$\frac{17}{2}$	$\frac{78400}{5154763267\ 9}$	$-\frac{3969}{178365511}$	$\frac{4665600}{3014377135\ 9}$	$-\frac{14817600}{2158222683\ 1}$	$\frac{395136}{178365511}$	$-\frac{1020600}{178365511}$	$\frac{2370816}{178365511}$	$-\frac{6350400}{178365511}$
10	9	$\frac{19}{2}$	$-\frac{63504}{6597841156\ 3}$	$\frac{784000}{5281928238\ 7}$	$-\frac{19845}{182765683}$	$\frac{15552000}{30887400427}$	$-\frac{37044000}{2211464764\ 3}$	$\frac{790272}{182765683}$	$-\frac{1701000}{182765683}$	$\frac{3386880}{182765683}$

k	t	v	λ_8	λ_9	λ_v	$\theta_{1',v}$	$\theta_{1,k}$	$\theta_{2,k}$	EC	P
1	0	$\frac{1}{2}$	0	0	$\frac{16}{17}$	$\frac{4}{17}$	$\frac{5}{17}$	$-\frac{1}{34}$	$\frac{1}{8160}$	4
2	1	$\frac{3}{2}$	0	0	$\frac{640}{711}$	$\frac{64}{237}$	$\frac{22}{79}$	$-\frac{2}{79}$	$\frac{1}{28440}$	5
3	2	$\frac{5}{2}$	0	0	$\frac{1536}{1775}$	$\frac{1152}{3905}$	$\frac{210}{781}$	$-\frac{18}{781}$	$\frac{3}{218680}$	6

4	3	$\frac{7}{2}$	0	0	$\frac{3416064}{4095175}$	$\frac{36864}{117005}$	$\frac{876}{3343}$	$-\frac{72}{3343}$	$\frac{3}{468020}$	7
5	4	$\frac{9}{2}$	0	0	$\frac{31293440}{38851659}$	$\frac{204800}{616693}$	$\frac{22620}{88099}$	$-\frac{1800}{88099}$	$\frac{25}{7400316}$	8
6	5	$\frac{11}{2}$	0	0	$\frac{1273692160}{1635852603}$	$\frac{2457600}{7081613}$	$\frac{23220}{91969}$	$-\frac{1800}{91969}$	$\frac{5}{2575132}$	9
7	6	$\frac{13}{2}$	0	0	$\frac{2159162163\ 20}{2866512804\ 87}$	$\frac{240844800}{668184803}$	$\frac{1162980}{4672621}$	$-\frac{88200}{4672621}$	$\frac{245}{205595324}$	10
8	7	$\frac{15}{2}$	0	0	$\frac{2587390640\ 128}{3548862418\ 959}$	$\frac{1027604480}{2757468857}$	$\frac{4740120}{19282999}$	$-\frac{352800}{19282999}$	$\frac{490}{636338967}$	11
9	8	$\frac{17}{2}$	$\frac{57153600}{178365511}$	0	$\frac{7441305651\ 11808}{1054097540\ 652871}$	$\frac{1664719257\ 60}{4336065572\ 41}$	$\frac{43366680}{178365511}$	$-\frac{3175200}{178365511}$	$\frac{13230}{2550626807\ 3}$	12
10	9	$\frac{19}{2}$	$-\frac{7938000}{182765683}$	$\frac{63504000}{182765683}$	$\frac{2665818846\ 81584640}{3899166434\ 96966443}$	$\frac{3329438515\ 200}{8441764132\ 087}$	$\frac{44001720}{182765683}$	$-\frac{3175200}{182765683}$	$\frac{9450}{2613549266\ 9}$	13

3.0 Derivation of the Second Derivative Hybrid Predictor CLMM.

The coefficients of the hybrid predictor (1.3)

$\lambda_0^*(t),\ \lambda_1^*(t),\ \lambda_2^*(t),\cdots,\ \lambda_k^*(t),\ \lambda_v^*(t)=1\ \textit{and}\ \theta_{3,k}(t)$ are obtained using the interpolant

$$y(x) = \sum_{j=0}^{k+2} b_j x^j \quad (3.1)$$

where b_j's are now the real parameter constants to be determined to obtain a particular method of (1.3) for a fixed k. Following the same procedure as in section 2.0 we have a class of continuous hybrid predictor for (1.2). Table 3.1 gives the continuous coefficients of the methods (1.3) for $k=1,\ 2,3$. Table 3.2 contains the discrete coefficients, the error constant and the order of the hybrid.

Table (3.1): Continuous Coefficients of the Hybrid Predictor (1.3)

k	t	j	$\lambda_j^*(t)$	$\lambda_j^*(k-\frac{3}{2})$	$\theta_{3,k}(t)$	$\theta_{3,k}(k-\frac{3}{2})$
1	$\frac{-1}{2}$	0	t^2	$\frac{1}{4}$	0	0
		$\frac{1}{2}$	1	1	0	0
		1	$1-t^2$	$\frac{3}{4}$	$t+t^2$	$\frac{-1}{4}$
2	$\frac{1}{2}$	0	$-\frac{t}{4}+\frac{t^2}{2}-\frac{t^3}{4}$	$\frac{-1}{32}$	0	0
		1	$1-t-t^2+t^3$	$\frac{3}{8}$	0	0
		$\frac{3}{2}$	1	1	0	0
		2	$\frac{5t}{4}+\frac{t^2}{2}-\frac{3t^3}{4}$	$\frac{21}{32}$	$-\frac{t}{2}+\frac{t^3}{2}$	$\frac{-3}{16}$
3	$\frac{3}{2}$	0	$\frac{-2t}{9}+\frac{4t^2}{9}-\frac{5t^3}{18}+\frac{t^4}{18}$	$\frac{1}{96}$	0	0
		1	$1-t-\frac{3t^2}{4}+t^3-\frac{t^4}{4}$	$\frac{-5}{64}$	0	0
		2	$2t-\frac{3}{2}t^3+\frac{t^4}{2}$	$\frac{15}{32}$	0	0
		$\frac{5}{2}$	1	1	0	0
		3	$\frac{-7t}{9}+\frac{11t^2}{36}+\frac{7t^3}{9}-\frac{11t^4}{36}$	$\frac{115}{192}$	$\frac{t}{3}-\frac{t^2}{6}-\frac{t^3}{3}+\frac{t^4}{6}$	$\frac{-5}{32}$

Table (3.2): The Discrete Coefficients, the Error Constant (EC) and Order (P) of the Hybrid Predictor (1.3)

k	**t**	**v**	λ_0^*	λ_1^*	λ_2^*	λ_3^*	λ_4^*	λ_5^*	λ_6^*	λ_7^*
1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	0	0	0	0	0	0
2	$\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{32}$	$\frac{3}{8}$	$\frac{21}{32}$	0	0	0	0	0
3	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{1}{96}$	$-\frac{5}{64}$	$\frac{15}{32}$	$\frac{115}{192}$	0	0	0	0
4	$\frac{5}{2}$	$\frac{7}{2}$	$-\frac{5}{1024}$	$\frac{7}{192}$	$-\frac{35}{256}$	$\frac{35}{64}$	$\frac{1715}{3072}$	0	0	0
5	$\frac{7}{2}$	$\frac{9}{2}$	$\frac{7}{2560}$	$-\frac{45}{2048}$	$\frac{21}{256}$	$-\frac{105}{512}$	$\frac{315}{512}$	$\frac{5397}{10240}$	0	0
6	$\frac{9}{2}$	$\frac{11}{2}$	$-\frac{7}{4096}$	$\frac{77}{5120}$	$-\frac{495}{8192}$	$\frac{77}{512}$	$-\frac{1155}{4096}$	$\frac{693}{1024}$	$\frac{20559}{40960}$	0
7	$\frac{11}{2}$	$\frac{13}{2}$	$\frac{33}{28672}$	$-\frac{91}{8192}$	$\frac{1001}{20480}$	$-\frac{2145}{16384}$	$\frac{1001}{4096}$	$-\frac{3003}{8192}$	$\frac{3003}{4096}$	$\frac{275847}{573440}$
8	$\frac{13}{2}$	$\frac{15}{2}$	$-\frac{429}{524288}$	$\frac{495}{57344}$	$-\frac{1365}{32768}$	$\frac{1001}{8192}$	$-\frac{32175}{131072}$	$\frac{3003}{8192}$	$-\frac{15015}{32768}$	$\frac{6435}{8192}$
9	$\frac{15}{2}$	$\frac{17}{2}$	$\frac{715}{1179648}$	$-\frac{7293}{1048576}$	$\frac{8415}{229376}$	$-\frac{7735}{65536}$	$\frac{17017}{65536}$	$-\frac{109395}{262144}$	$\frac{17017}{32768}$	$-\frac{36465}{65536}$
10	$\frac{17}{2}$	$\frac{19}{2}$	$-\frac{2431}{5242880}$	$\frac{13585}{2359296}$	$-\frac{138567}{4194304}$	$\frac{53295}{458752}$	$-\frac{146965}{524288}$	$\frac{323323}{655360}$	$-\frac{692835}{1048576}$	$\frac{46189}{65536}$

	T	v	λ_8^*	λ_9^*	λ_{10}^*	λ_v^*	$\theta_{3,k}$	EC	P
1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	1	$-\frac{1}{4}$	$\frac{1}{48}$	2
2	$\frac{1}{2}$	$\frac{3}{2}$	0	0	0	1	$-\frac{3}{16}$	$\frac{1}{128}$	3
3	$\frac{3}{2}$	$\frac{5}{2}$	0	0	0	1	$-\frac{5}{32}$	$\frac{1}{256}$	4
4	$\frac{5}{2}$	$\frac{7}{2}$	0	0	0	1	$-\frac{35}{256}$	$\frac{7}{3072}$	5
5	$\frac{7}{2}$	$\frac{9}{2}$	0	0	0	1	$-\frac{63}{512}$	$\frac{3}{2048}$	6
6	$\frac{9}{2}$	$\frac{11}{2}$	0	0	0	1	$-\frac{231}{2048}$	$\frac{33}{32768}$	7
7	$\frac{11}{2}$	$\frac{13}{2}$	0	0	0	1	$-\frac{429}{4096}$	$\frac{143}{196608}$	8
8	$\frac{13}{2}$	$\frac{15}{2}$	$\frac{1700127}{3670016}$	0	0	1	$-\frac{6435}{65536}$	$\frac{143}{262144}$	9
9	$\frac{15}{2}$	$\frac{17}{2}$	$\frac{109395}{131072}$	$\frac{29582839}{66060288}$	0	1	$-\frac{12155}{131072}$	$\frac{221}{524288}$	10
10	$\frac{17}{2}$	$\frac{19}{2}$	$-\frac{692835}{1048576}$	$\frac{230945}{262144}$	$\frac{573713569}{1321205760}$	1	$-\frac{46189}{524288}$	$\frac{4199}{12582912}$	11

4.0 The Stability of the Methods

According to Gear (1968), a numerical method is said to be stiffly-stable if (i) its region of absolute stability contains R_1 and R_2 and (ii) it is accurate for all $h\lambda \in R_2$ when applied to the scalar test equation $y' = \lambda y$, λ a complex constant with $\operatorname{Re}\lambda < 0$, where

$$R_1 = \{h\lambda / \operatorname{Re} h\lambda < -a\}$$

$$R_2 = \{h\lambda / -a \le \operatorname{Re} h\lambda \le b,\ -c \le \operatorname{Im} h\lambda \le c\}$$

and a, b and c are positive constants.

The regions R_1 and R_2 of the complex $h\lambda$ -plane are shown in figure 4.1.

The diagram in figure 4.1 is the boundary locus form of the region of stiff stability of any given numerical integrator.

Putting (1.3) into (1.2) for a corresponding value of k and applying the resultant method to the scalar test equation $y' = \lambda y,\ \operatorname{Re}(\lambda h) < 0,\ z = \lambda h$ we obtain the stability polynomial

$$\pi(r, z) = r^k - \sum_{j=0}^{k-1} \lambda_j r^j - \lambda_v \left(\sum_{j=0}^{k} \lambda_j^* r^j + z\theta_{3,k} r^k \right) - z\theta_v \left(\sum_{j=0}^{k} \lambda_j^* r^j + z\theta_{3,k} r^k \right) - z\theta_{1,k} r^k - z^2 \theta_{2,k} r^k$$

The application of boundary Locus requires the plotting of $\operatorname{Re}(z)$ against $\operatorname{Im}(z)$ in order to reveal the interval of absolute stability of the methods. For the proposed methods, the boundary Loci are shown in Figure 4.2. The methods are stiffly stable for $k \le 9$. But instability sets in at $k = 10$ since the stability region is not connected.

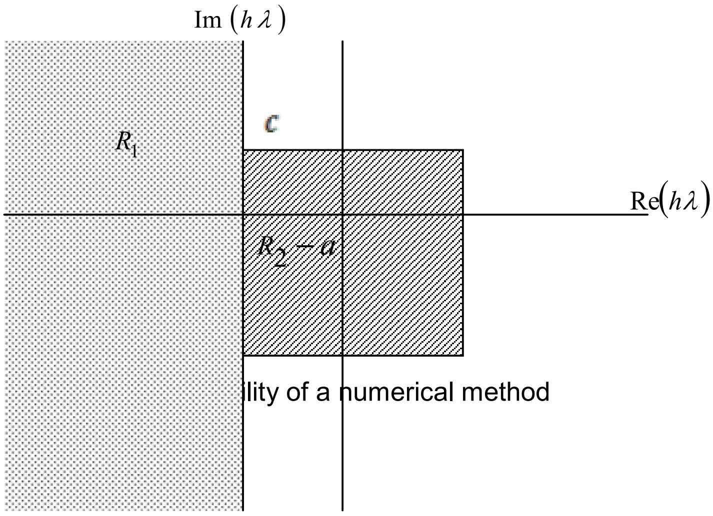

Figure 4.1 lity of a numerical method

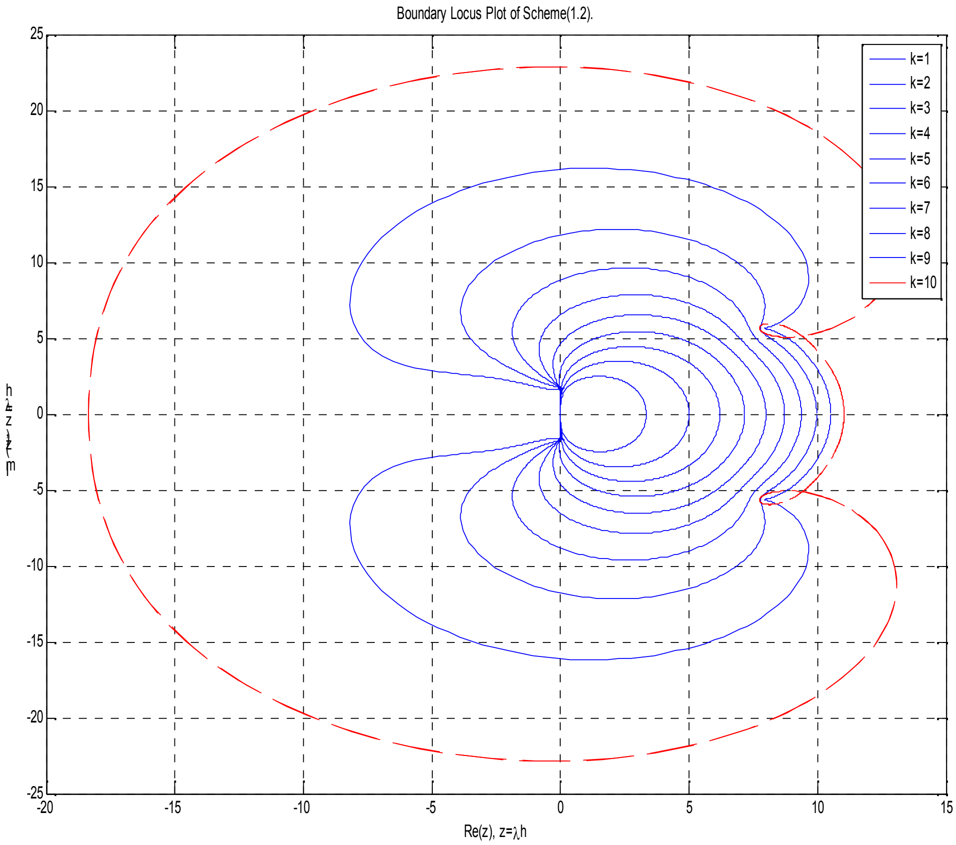

Figure 4.2

5.0 Numerical Experiment

To demonstrate the application of the new Schemes (1.2) and (1.3)

$$y_{n+1} = \frac{1}{34}\left(8hf_{n+\frac{1}{2}} + 10hf_{n+1} - h^2 f'_{n+1} + 2y_n + 32y_{n+\frac{1}{2}}\right) \quad (5.1)$$

$$y_{n+\frac{1}{2}} = \frac{1}{4}\left(-hf_{n+1} + y_n + 3y_{n+1}\right) \quad (5.2)$$

for $k = 1$, we consider the numerical solution of the nonlinear stiff IVP in Higham and Higham (2000), Enright (1974) with $x \in 0(0.0001)\,3$

$$y_1' = -0.04y_1 + 10^4 y_2 y_3$$

$$y_2' = 0.04\,y_1 - 10^4 y_2 y_3 - 3\times 10^7 y_2^2 \;,\quad y(0) = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad (5.3)$$

$$y_3' = 3\times 10^7 y_2^2$$

and the linear problem in Enright (1974)

$$y' = \begin{bmatrix} -0.1 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & -100 & 0 \\ 0 & 0 & 0 & -1000 \end{bmatrix} y, \quad y(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad y(x) = \begin{pmatrix} e^{-0.1x} \\ e^{-10x} \\ e^{-100x} \\ e^{-1000x} \end{pmatrix} \tag{5.4}$$

with x in the range [0, 50] and h = 0.0001

Applying method (5.1) with the corresponding hybrid method (5.2) to the initial value problems above leads to solving implicit set of equations which demands the use of the Newton Raphson iterative scheme

$$y_{n+k}^{[s+1]} = y_{n+k}^{[s]} - F'\left(y_{n+k}^{[s]}\right)^{-1} F\left(y_{n+k}^{[s]}\right), \; s = 0, 1, 2, \cdots \tag{5.5}$$

as reported in Enright (1974), Fatunla (1988), Lambert (1991). The starting values for the iterative scheme (5.5) are generated by using the inverse Euler's method.

$$y_{n+1}^{(0)} = y_n + \frac{h y_n y_n'}{y_n - h y_n'} \tag{5.6}$$

The plots of the numerical solutions of $y_2(x)$ of the nonlinear stiff problem and $y_1(x)$ of the linear stiff problem are given in figures (5.1) and (5.2) respectively.

CONCLUSION

We have considered a class of second derivative continuous linear multistep methods of order $k+3$. The methods are stiffly stable for step number $k \leq 9$. In figure (5.1), the numerical result of the class of methods compares favourably with the state-of-the art code, ODE 15s in MATLAB. The graphs of the numerical result in Figure (5.2) show that the methods perform better than Enright's method when applied to the linear problem.

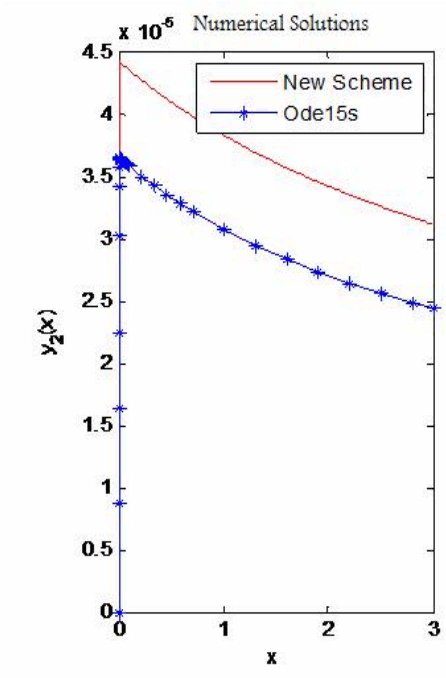

Figure 5.1

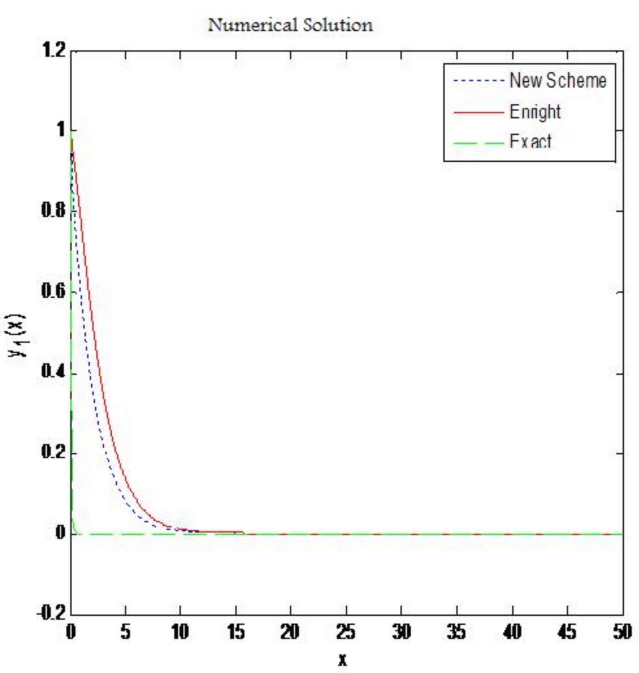

Figure 5.2

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