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PURE DIAGONAL BILINEAR MOVING AVERAGE VECTOR MODELS

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ABSRACT

The purpose of this work was to develop special models from the general Bilinear Moving Average Vector (BMAV) Models. These models were established with only pure diagonal coefficients of the vectors of nonlinear components. The autocorrelation and partial autocorrelation functions for the vector series obtained suggested a pure moving average process for the vector models. A Pure Diagonal Bilinear Moving Average Vector (PDBMAV) Models were obtained. This is a special case of the general Bilinear Moving Average (BMAV) Models. This is so because of the associative products of vectors and their respective white noise forming the principal diagonal of the vector matrices. The estimates obtained validate PDBMAV models established. These are shown in figures '1', '2', and '3'.

KEY WORDS: Pure Diagonal Vectors, Bilinear Vector Models, Moving Average Vector Models, Bilinear Moving Average Vector Models.

INTRODUCTION

Most time series analysts assume linearity and stationarity, for technical convenience, when analyzing macroeconomic and financial time series data, (Franses, 1998). However, some of the microeconomic and financial data are not linear, due to its dynamic behaviour. Classical linear models are not appropriate for modelling such nonlinear series, (Subba Rao and Gabr, 1984). In most cases, nonlinear forecast is superior to linear forecast. Maravall (1983) used a bilinear model to forecast Spanish monetary data and reported a near 10% improvement in one-step ahead mean square forecast errors over several ARMA alternatives. There is no-gainsaying the fact that most of the economic or financial data assume fluctuations due to certain factors. That is why the use of nonlinear models in forecast gives higher precision than linear models.

Let e_t be a sequence of independently and identically distributed random variables defined on a probability space (Ω,B,P) with $E(e_t) = 0$ and $E(e^2_t) = \sigma^2 < \infty$. The general superdiagonal bilinear model X_t with respect to e_t is

$$X_t = e_t + \sum_{i=1}^{r} a_i X_{t-i} + \sum_{j=1}^{h} b_j e_{t-j} + \sum_{i=1}^{m} \sum_{j=1}^{s} c_{ij} X_{t-i} e_{t-j} \qquad 1.1$$

where a_i, b_j, c_{ij} are fixed time independent parameters, (Akamanam, Bhaskara Rao and Subramanyam, 1986).

Oyet (1991) defined a process $(X_t)_{t\in z}$ on a probability space (Ω,ξ,P) as a time varying bilinear process of order (p,q,P,Q) and denoted by BL(p,q,P,Q), if it satisfies the following stochastic difference equation:

$$X_t = \sum_{i=1}^{P} a_{i,t}(a)X_{t-i} + \sum_{j=1}^{q} c_{j,t}(c)\varepsilon_{t-j} + \sum_{i=1}^{P} \sum_{j=1}^{Q} b_{ij,t}(b)X_{t-i}\varepsilon_{t-j} + \varepsilon_t$$

where $(a_{i,t}(a))_{1\le i\le p}$, $(c_{j,t}(c))_{1\le j\le q}$, $(b_{ij,t}(b))_{1\le i\le P,\ 1\le j\le Q}$ are time-varying coefficients which depend on finite dimensional unknown parameter vectors a, c and b respectively. The sequence $(\varepsilon_t)_{t\in z}$ is a heteroscedastic white noise process. That is, $(\varepsilon_t)_{t\in z}$ is a sequence of independent random variables, not necessarily identically distributed, with mean zero and variance σ_t^2. Moreover ε_t is independent of past X_t. The initial values X_t, $t < 1$, and $\varepsilon_{t,\ t<1}$
are assumed to be equal to zero.

Boonchai and Eivind (2005) stated the general form of a multivariate bilinear time series model as

$$X_t = \sum A_i X_{t-i} + \sum M_j e_{t-j} + \sum\sum\sum B_{dij} X_{t-i} e_{dt-j} + e_t.$$

Here the state X_t and noise e_t are n-vectors and the coefficients A_i, M_j, and $B_{dij} = 0$, and we have the class of well-known vector ARMA models. The bilinear models include additional product terms $B_{dij}X_{t-i}e_{dt-j}$, as the

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theoretical point of view, it is therefore natural to consider bilinear models in the process of extending linear theory to non-linear cases. According to Boonchai and Eivind (2005), a particular reason for introducing bilinear time series in population dynamics is that they are suitable for modelling environmental noise. One may start with a deterministic system with (constant) parameters that describe conditions that depend on a fluctuating environment. Boonchai and Eivind (2005) made extension first to univariate and then to multivariate bilinear models. The main results give conditions for stationarity, ergodicity, invertibility, and consistency of least square estimates.

Usoro and Omekara (2007) established Bilinear Moving Average Vector (BMAV) Models. Their models involved interactive products of vectors and their white noise. In their work, the parameters of the non linear parts were the coefficients of the interactive products. Their models were used for estimation and forecast of values of each vector.

In this paper, the interest is in the pure diagonal parameters forming the vector matrices. This calls for a special case of the General BMAV models. The parameters of the pure diagonal matrices are the coefficients of only the associative products of the vectors and their respective white noise.

2. ESTIMATION OF PARAMETERS OF THE VECTOR MODEL

A. LINEAR MOVING AVERAGE VECTOR MODELS

The general Vector Moving Average, VMA(q) process is given

$$\begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ \cdot \\ \cdot \\ X_{vt} \end{pmatrix} = \begin{pmatrix} \epsilon_{1t} \\ C_{2t} \\ \epsilon_{1t} \\ \cdot \\ \cdot \\ C_{vt} \end{pmatrix} + \begin{pmatrix} \lambda_{1.11} & \lambda_{1.12} & \lambda_{1.13} \cdots & \lambda_{1.1m} \\ \lambda_{1.21} & \lambda_{1.22} & \lambda_{1.23} \cdots & \lambda_{1.2m} \\ \lambda_{1.31} & \lambda_{1.32} & \lambda_{1.33} \cdots & \lambda_{1.3m} \\ \cdot & \cdot & \cdot \cdots & \cdot \\ \cdot & \cdot & \cdot \cdots & \cdot \\ \lambda_{1.s1} & \lambda_{1.s2} & \lambda_{1.s3} \cdots & \lambda_{1.sm} \end{pmatrix} \begin{pmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \\ \epsilon_{3t-1} \\ \cdot \\ \cdot \\ C_{vt-1} \end{pmatrix} + \begin{pmatrix} \lambda_{2.11} & \lambda_{2.12} & \lambda_{2.13} \cdots & \lambda_{2.1m} \\ \lambda_{2.21} & \lambda_{2.22} & \lambda_{2.23} \cdots & \lambda_{2.2m} \\ \lambda_{2.31} & \lambda_{2.32} & \lambda_{2.33} \cdots & \lambda_{2.3m} \\ \cdot & \cdot & \cdot \cdots & \cdot \\ \cdot & \cdot & \cdot \cdots & \cdot \\ \lambda_{2.s1} & \lambda_{2.s2} & \lambda_{2.s3} \cdots & \lambda_{2.sm} \end{pmatrix} \begin{pmatrix} \epsilon_{1t-2} \\ \epsilon_{t-2} \\ \epsilon_{3t-2} \\ \cdot \\ \cdot \\ C_{vt-2} \end{pmatrix}$$

$$+ \begin{pmatrix} \lambda_{3.11} & \lambda_{3.12} & \lambda_{3.13} \cdots & \lambda_{3.1m} \\ \lambda_{3.21} & \lambda_{3.22} & \lambda_{3.23} \cdots & \lambda_{3.2m} \\ \lambda_{3.31} & \lambda_{3.32} & \lambda_{3.33} \cdots & \lambda_{3.3m} \\ \cdot & \cdot & \cdot \cdots & \cdot \\ \cdot & \cdot & \cdot \cdots & \cdot \\ \lambda_{3.s1} & \lambda_{3.s2} & \lambda_{3.s3} \cdots & \lambda_{3.sm} \end{pmatrix} \begin{pmatrix} C_{1t-3} \\ C_{2t-3} \\ C_{3t-3} \\ \cdot \\ \cdot \\ \epsilon_{vt-3} \end{pmatrix} + \cdots + \begin{pmatrix} \lambda_{q.11} & \lambda_{q.12} & \lambda_{q.13} \cdots & \lambda_{q.1m} \\ \lambda_{q.21} & \lambda_{q.22} & \lambda_{q.23} \cdots & \lambda_{q.2m} \\ \lambda_{q.31} & \lambda_{q.32} & \lambda_{q.33} \cdots & \lambda_{q.3m} \\ \cdot & \cdot & \cdot \cdots & \cdot \\ \cdot & \cdot & \cdot \cdots & \cdot \\ \lambda_{q.s1} & \lambda_{q.s2} & \lambda_{q.s3} \cdots & \lambda_{q.sm} \end{pmatrix} \begin{pmatrix} C_{1t-q} \\ C_{2t-q} \\ C_{3t-q} \\ \cdot \\ \cdot \\ \epsilon_{vt-q} \end{pmatrix}$$

The expansion of the above matrices gives the following models,

$$X_{1t} = \epsilon_{1t} + \sum_{b=1}^{q} \sum_{j=1}^{v} \sum_{h=1}^{m} \lambda_{b\,1h}\epsilon_{\cdot h} \quad \ldots \ldots \ldots \quad (2.1)$$

$$X_{2t} = \epsilon_{2t} + \sum_{b=1}^{q} \sum_{j=1}^{v} \sum_{h=1}^{m} \lambda_{b\,2h}\epsilon_{jt-b} \quad \ldots \ldots \ldots$$

$$X_{3t} = \epsilon_{3t} + \sum_{b=1}^{q} \sum_{j=1}^{v} \sum_{h=1}^{m} \lambda_{b\,3h}\epsilon_{jt-b} \qquad (2.3)$$

$$\vdots$$

$$X_{nt} = \epsilon_{vt} + \sum_{b=1}^{q} \sum_{j=1}^{v} \sum_{h=1}^{m} \lambda_{b.vh}\epsilon_{jt-b} \qquad (2.4)$$

Therefore, models '2.1', '2.2','2.3' and '2.4' express linear moving average relationships of $X_{1t}, X_{2t}, X_{3t}, \ldots X_{nt}$ vectors with distributed lags of $\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}, \ldots, \epsilon_{nt}$. $\lambda_{b\,1h}, \lambda_{b\,2h}, \lambda_{b\,3h}, \ldots, \lambda_{b\,vh}$ are the matrices of coefficients of the moving average vector series. The above models can further be written as

$$X_{jt} = \epsilon_{vt} + \sum_{b=1}^{q} \sum_{j=1}^{v} \sum_{h=1}^{m} \lambda_{b\,vh}\epsilon_{jt-b} \qquad 2.5$$

$$j = 1, \ldots, n,\ n = v,$$

where, X_{it} i^{th} vector series, $\lambda_{b\,1}, \lambda_{b\,2}, \lambda_{b\,3}, \ldots, \lambda_{b.q}$ are the matrices of coefficients of the moving average vector serie.

B. NONLINEAR PURE DIAGONAL MOVING AVERAGE VECTOR MODELS

The nonlinear vector models involving pure diagonal coefficients is given as,

$$\begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ \vdots \\ X_{nt} \end{pmatrix} \quad \begin{pmatrix} \beta_{01\,11} & 0 & 0 & \ldots & 0 \\ 0 & \beta_{01\,22} & 0 & \ldots & 0 \\ 0 & 0 & \beta_{01\,33} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \beta_{01\,nv} \end{pmatrix} \begin{pmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \\ \epsilon_{3t-1} \\ \vdots \\ \epsilon_{vt-1} \end{pmatrix} (X_{1t-0}, X_{2t-0}, X_{3t-0}, \ldots, X_{nt-0})$$

$$\begin{pmatrix} \beta_{02\,11} & 0 & 0 & \ldots & 0 \\ 0 & \beta_{02\,22} & 0 & \ldots & 0 \\ 0 & 0 & \beta_{02\,33} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \beta_{02\,nv} \end{pmatrix} \begin{pmatrix} \epsilon_{1t-2} \\ \epsilon_{2t-2} \\ \epsilon_{3t-2} \\ \vdots \\ \epsilon_{vt-2} \end{pmatrix} (X_{1t-0}, X_{2t-0}, X_{3t-0}, \ldots, X_{nt-0})$$

$$\begin{pmatrix} \beta_{03\,11} & 0 & 0 & \ldots & 0 \\ 0 & \beta_{03\,22} & 0 & \ldots & 0 \\ 0 & 0 & \beta_{03\,33} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \beta_{03\,nv} \\ \vdots & \vdots & \vdots & \ldots & \vdots \end{pmatrix} \begin{pmatrix} \epsilon_{1t-3} \\ \epsilon_{2t-3} \\ \epsilon_{3t-3} \\ \vdots \\ \epsilon_{vt-3} \\ \vdots \end{pmatrix} (X_{1t-0}, X_{2t-0}, X_{3t-0}, \ldots, X_{nt-0})$$

$$\begin{pmatrix} \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \beta_{0q\,11} & 0 & 0 & \cdots & 0 \\ 0 & \beta_{0q\,22} & 0 & \cdot & 0 \\ 0 & 0 & \beta_{0q\,33} & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & \beta_{0q\,nv} \end{pmatrix} \begin{bmatrix} \cdot \\ \cdot \\ \epsilon_{1t-q} \\ \epsilon_{2t-q} \\ \epsilon_{3t-q} \\ \cdot \\ \cdot \\ \epsilon_{vt-q} \end{bmatrix} (X_{1t-0}, X_{2t-0}, X_{3t-0}, \ldots, X_{nt-0})$$

The above matrices remain principal diagonal for i=j, and b=1. The model becomes,

$$X_{it} = \sum_{i=j}^{n} \sum_{a=b}^{v} \sum^{p} \sum^{q} \beta_{0b.ij} X_{it} \epsilon_{jt-b} \qquad 2.6$$

(C) PURE DIAGONAL BILINEAR MOVING AVERAGE VECTOR MODEL

The above model is the combination of the linear and nonlinear pure diagonal parts. This combination produces the following model:

$$X_{jt} = \epsilon_{vt} + \sum_{b=1}^{q} \sum_{j=1}^{v} \sum_{h=1}^{m} \lambda_{b.vh} \epsilon_{jt-b} + \sum_{i=j}^{n} \sum_{b=1}^{v} \sum^{q} \beta_{0b.ij} X_{it} \epsilon_{jt-b} \qquad 2.7$$

$j = 1, \ldots, n,\ n = v,$

3 ESTIMATION OF PARAMETERS OF THE VECTOR MODELS

$$\begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{bmatrix} = \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{bmatrix} + \begin{pmatrix} \lambda_{1\,11} & \lambda_{1\,12} & \lambda_{1\,13} \\ \lambda_{1\,21} & \lambda_{1\,22} & \lambda_{1\,23} \\ \lambda_{1\,31} & \lambda_{1\,32} & \lambda_{1\,33} \end{pmatrix} \begin{bmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \\ \epsilon_{3t-1} \end{bmatrix} + \begin{pmatrix} 0 & 0 & \lambda_{2\,13} \\ 0 & 0 & \lambda_{2\,23} \\ 0 & 0 & \lambda_{2\,33} \end{pmatrix} \begin{bmatrix} \epsilon_{1t-2} \\ \epsilon_{2t-2} \\ \epsilon_{3t-2} \end{bmatrix}$$

$$+ \begin{pmatrix} b_{01\,11} & 0 & 0 \\ 0 & b_{01\,22} & 0 \\ 0 & 0 & b_{01\,33} \end{pmatrix} \begin{bmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \\ \epsilon_{3t-1} \end{bmatrix} (X_{1t-0}, X_{2t-0}, X_{3t-0}) + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_{02\,33} \end{pmatrix} \begin{bmatrix} \epsilon_{1t-2} \\ \epsilon_{2t-2} \\ \epsilon_{3t-2} \end{bmatrix} (X_{1t-0}, X_{2t-0}, X_{3t-0})$$

Expansion of the matrices provides the following models.

$$X_{1t} = \epsilon_{1t} + \lambda_{1\,11}\epsilon_{1t-1} + \lambda_{1\,12}\epsilon_{2t-1} + \lambda_{1\,13}\epsilon_{3t-1} + \lambda_{2\,13}\epsilon_{3t-2} + b_{01\,11}X_{1t-0}\epsilon_{1t-1} + \beta_{01\,22}X_{2t-0}\epsilon_{2t-1} + \beta_{01\,33}X_{3t-0}\epsilon_{3t-1} + \beta_{02\,33}X_{3t-0}\epsilon_{3t-2} \qquad 3.1$$

$$X_{2t} = \epsilon_{2t} + \lambda_{1.21}\epsilon_{1t-1} + \lambda_{1.22}\epsilon_{2t-1} + \lambda_{1\,13}\epsilon_{3t-1} + \lambda_{2\,23}\epsilon_{3t-2} + b_{01\,11}X_{1t-0}\epsilon_{1t-1} + \beta_{01\,22}X_{2t-0}\epsilon_{2t-1} + \beta_{01\,33}X_{3t-0}\epsilon_{3t-1} + \beta_{02.33}X_{3t-0}\epsilon_{3t-2} \qquad 3.2$$

$$X_{3t} = \epsilon_{3t} + \lambda_{1\,31}\epsilon_{1t-1} + \lambda_{1\,32}\epsilon_{2t-1} + \lambda_{1\,13}\epsilon_{3t-1} + \lambda_{2\,33}\epsilon_{3t-2} + b_{01\,11}X_{1t-0}\epsilon_{1t-1} + \beta_{01\,22}X_{2t-0}\epsilon_{2t-1} + \beta_{01\,33}X_{3t-0}\epsilon_{3t-1} + \beta_{02\,33}X_{3t-0}\epsilon_{3t-2} \qquad 3.3$$

The above models are called Pure Bilinear Moving Average Vector Diagonal (PDBMAV) models.

According to Lewis and Stevens (1991), the multiple partitions and predictor vector interactions calls for multivariate adaptive regression method for the estimation of the above models. X_{1t}, X_{2t} and X_{3t} are functions of linear vectors on one part and nonlinear interactions of vectors and their respective white noise on the other part. The least squares regression, using 'minitab' give the following estimated models for X_{1t}, X_{2t} and X_{3t}:

$$X_{1t} = \epsilon_{1t} - 0.496\epsilon_{1t-1} + 0.178\epsilon_{2t-1} - 0.375\epsilon_{3t-2} + 0.00227X_{1t-0}\epsilon_{1t-1} - 0.00044b_{01\,22}X_{2t-0}\epsilon_{2t-1} + 0.00111X_{3t-0}\epsilon_{3t-1} - 0.00208\,b_{02\,33}X_{3t-0}\epsilon_{3t-2} \quad \ldots 3.4$$

$$X_{2t} = \epsilon_{2t} + 0.004\epsilon_{1t-1} - 0.407\epsilon_{2t-1} + 0.056\,\epsilon_{3t-2} + 0.00070X_{1t-0}\epsilon_{1t-1} + 0.00091X_{2t-0}\epsilon_{2t-1} + 0.00178X_{3t-0}\epsilon_{3t-1} - 0.00174b_{02\,33}X_{3t-0}\epsilon_{3t-2} \quad \ldots 3.5$$

$$X_{3t} = \epsilon_{3t} - 500\epsilon_{1t-1} + 0.585\epsilon_{2t-1} - 0.431\epsilon_{3t-2} + 0.00157X_{1t-0}\epsilon_{1t-1} - 0.00135X_{2t-0}\epsilon_{2t-1} - 0.00067\,b_{01\,33}X_{3t-0}\epsilon_{3t-1} - 0.00034b_{02\,33}X_{3t-0}\epsilon_{3t-2} \quad . 3.6$$

CONCLUSION

A Pure Diagonal Bilinear Moving Average (PDBMAV) model obtained for each vector series made the model a special case of Bilinear Moving Average Vector (BMAV) models. This is because the coefficients of the nonlinear part were restricted to only the principal diagonal of the coefficient matrices. This has significantly reduced the number of multiple coefficients in the General BMAV models to smaller number of coefficients, whose estimates are slightly better than that of the BMAV models. Appendices '1' and'2' are the original and estimated values of the vector series. The values in appendix '2' show slight deviations from actual values in appendix '1'. This further proves fitness of the models established.

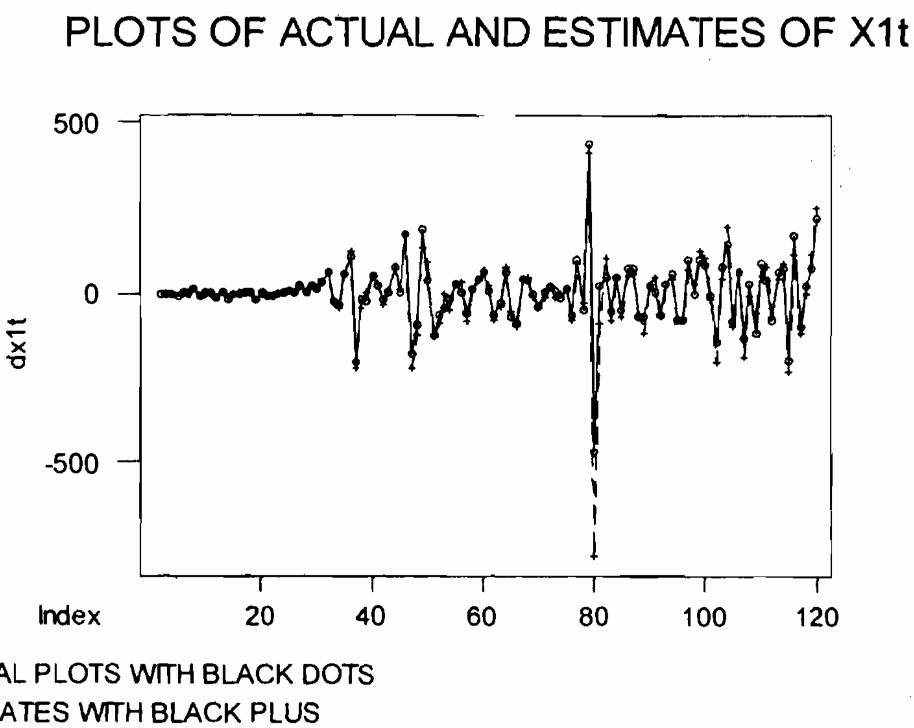
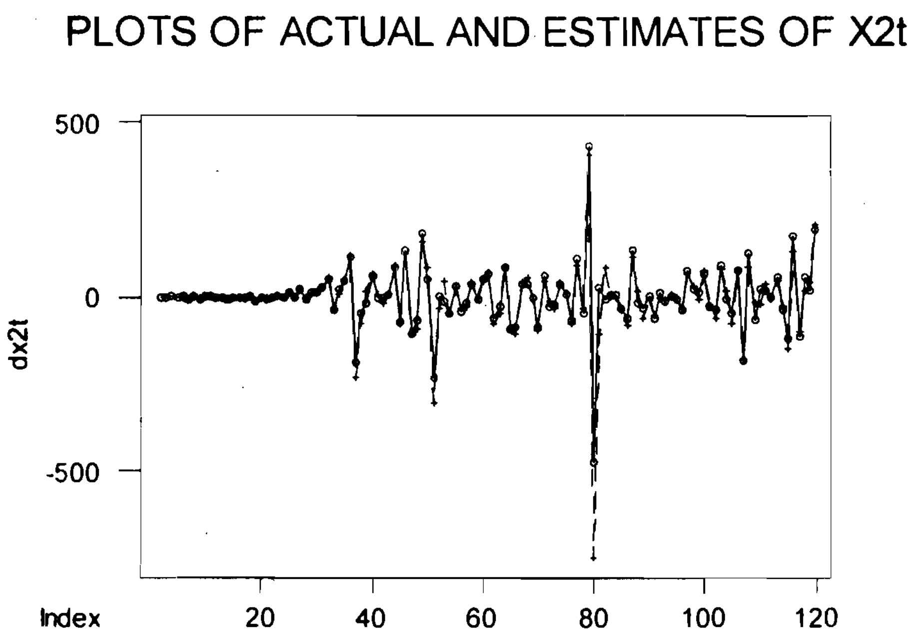
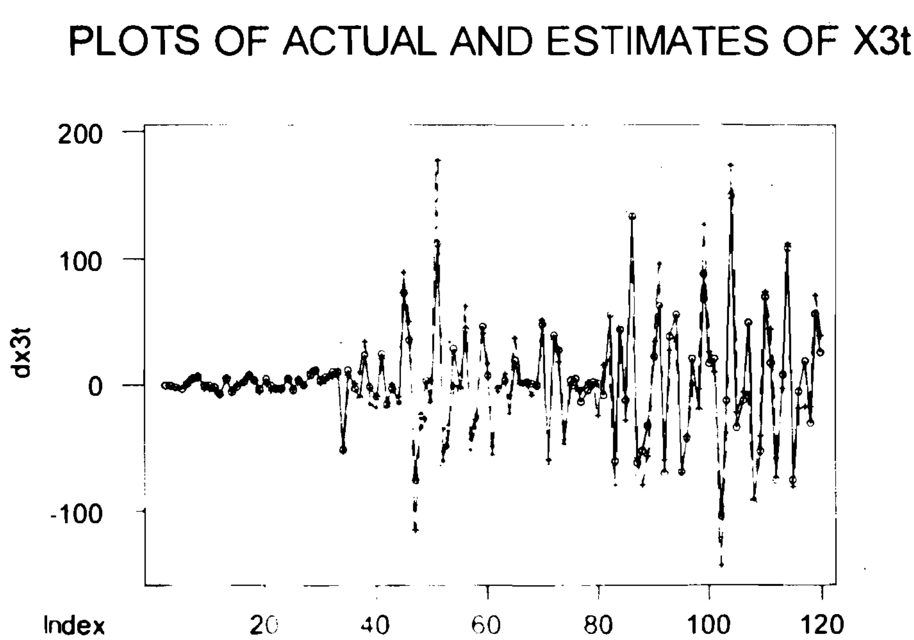


FIGURE 1: Plots of Actual and Estimates of X1t



ACTUAL PLOTS WITH BLACK DOTS
ESTIMATES WITH BLACK PLUS

FIGURE 2: Plots Actual and Estimates of X2t



ACTUAL PLOTS WITH BLACK DOTS
ESTIMATES WITH BLACK PLUS

FIGURE 3: Plots of Actual and Estimates of X3t

APPENDIX 1: STATIONARY VECTOR SERIES OF INTERNALLY GENERATED REVENUE SERIES

s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}	s/n	X_{1t}	X_{2t}	X_{3t}
1				41	24.78	0.00	24.78	81	22.59	30.71	-8.12
2	0.39	0.30	0.09	42	-16.93	-1.43	-15.50	82	50.74	-5.69	56.43
3	-1.91	-1.21	-0.70	43	7.02	9.75	-2.73	83	-48.62	11.81	-60.43
4	0.70	2.58	-1.88	44	79.30	90.65	-11.35	84	52.33	8.86	43.47
5	-4.09	-1.22	-2.87	45	3.79	-69.26	73.05	85	-43.30	-31.18	-12.12
6	4.35	3.09	1.26	46	174.75	139.03	35.72	86	75.45	-58.35	133.80
7	1.23	-3.51	4.74	47	-176.52	-101.04	-75.48	87	73.60	136.35	-62.75
8	13.66	6.81	6.85	48	-88.44	-63.53	-24.91	88	-67.25	-14.47	-52.78
9	-4.13	-3.28	-0..85	49	188.36	185.39	2.97	89	-62.83	-29.62	-33.21
10	4.39	4.29	0.10	50	39.11	54.04	-14.93	90	25.49	2.60	22.89
11	3.22	4.79	-1.57	51	-118.25	-230.30	112.05	91	5.25	-58.10	63.35
12	-8.51	-0.98	-7.53	52	-58.26	4.68	-62.94	92	-57.88	13.10	-70.98
13	5.62	1.09	4.53	53	-41.76	-7.71	-34.05	93	30.41	-7.67	38.08
14	-13.03	-6.87	-6.16	54	-14.49	-43.99	29.50	94	58.78	2.85	55.93
15	-1.99	0.36	-2.35	55	27.83	32.84	-5.01	95	-74.07	-4.84	-69.23
16	1.30	-1.28	2.58	56	6.21	-38.24	44.45	96	-76.89	-34.11	-42.78
17	5.76	-2.33	8.09	57	-55.00	-14.08	-40.92	97	101.05	79.85	21.20
18	6.02	2.96	3.06	58	12.36	37.99	-25.63	98	2.19	22.61	-20.42
19	-13.08	-10.00	-3.08	59	42.61	-3.52	46.13	99	100.30	12.58	87.72
20	6.29	1.93	4.36	60	62.42	55.11	7.31	100	84.61	67.20	17.41
21	-5.10	-3.92	-1.18	61	9.91	66.45	-56.54	101	-5.34	-26.02	20.68
22	-2.96	-0.31	-2.65	62	-59.82	-57.16	-2.66	102	-138.61	-34.83	-103.78
23	1.36	4.74	-3.38	63	-22.96	-26.40	3.44	103	78.38	91.41	-13.03
24	6.27	1.86	4.41	64	64.65	88.98	-24.33	104	146.71	-3.82	150.53
25	9.14	13.55	-4.41	65	-66.99	-87.19	20.20	105	-78.43	-44.42	-34.01
26	4.23	1.05	3.18	66	-83.97	-83.22	-0.75	106	64.15	76.37	-12.22
27	22.50	23.70	-1.20	67	43.81	41.31	2.50	107	-130.03	-178.99	48.96
28	4.68	-3.47	8.15	68	38.85	38.03	0.82	108	32.07	125.16	-93.09
29	26.90	15.85	11.05	69	0.00	0.00	0.00	109	-114.67	-61.65	-53.02
30	16.62	13.32	3.30	70	-35.08	-83.39	48.31	110	91.48	22.18	69.30
31	36.64	30.57	6.07	71	4.56	64.99	-60.43	111	37.77	21.07	16.70
32	63.11	53.32	9.79	72	17.49	-22.73	40.22	112	-77.33	-0.78	-76.55
33	-21.69	-32.44	10.75	73	9.10	-19.13	28.23	113	65.86	58.41	7.45
34	-27.73	24.35	-52.08	74	-10.35	38.70	-49.05	114	75.09	-33.06	108.15
35	60.94	49.06	11.88	75	14.03	10.93	3.10	115	-194.68	-118.33	-76.35
36	111.61	115.36	-3.75	76	-59.80	-64.25	4.45	116	169.79	176.04	-6.25
37	-199.85	-188.24	-11.61	77	98.60	112.74	-14.14	117	-95.54	-114.14	18.60
38	-17.57	-41.67	24.10	78	-47.21	-43.20	-4.01	118	26.41	58.55	-32.14
39	-18.23	-16.46	-1.77	79	439.18	436.68	2.50	119	75.56	19.52	56.04
40	54.11	64.66	-10.55	80	-472.61	-473.43	0.82	120	218.32	193.90	24.42

Maravall, A., 1983. An application of nonlinear time series forecasting. Journal of Business and Economic Statistics, 1: 66-74

Subba Rao, T. and Gabr, M. M., 1984. An introduction to bispectral analysis and bilinear Time series models. Lecture Notes in Statistics No 24. Springer Verlag.

Usoro, A. E. and Omekara, C. O., 2007. Global Journal of Pure and Applied Sciences, University of Calabar, Nigeria. 13 (3): 437-441.