

# MAINTAINABILITY THROUGH RECRUITMENT IN MANPOWER SYSTEM OF CHANGING SIZE

P. I. UCHE and E. O. OSSAI

(Received 2 April 2008; Revision Accepted 15 May 2008)

## ABSTRACT

The maintainability conditions at three levels of manpower growth are established in Markov framework by re-defining the transition probability matrix. Useful relational results essential for planning and control as a system goes from one level of growth to another are proved.

**KEY WORDS:** Manpower structure, Maintainability, Markov model, Recruitment, growth

## 1. INTRODUCTION

Problems of maintainability in manpower control have been studied extensively in Markov models. As a result, several standard definitions and results have been presented. These can be found in many manpower planning works such as those of Bartholomew et al (1991), Bartholomew (1982), Vassiliou (1998), McClean (1991), Glen and Yang (1996) and Georgiou and Tsantas (2002). A manpower structure is simply maintainable (or one step maintainable) if it is possible to make the structure remain the same in the next accounting period, through the instrument of promotion or recruitment.

Three levels of growth or change in the size of a manpower system identified by authors are the contraction or negative growth, the zero or no growth and the expansion or positive growth. Each of these levels of growth has associated maintainability requirements. Yet, a given manpower system can experience change from one level of growth to another as a matter of policy needs of the management. When this happens, the problem that arises is the effect of this change on the maintainability requirements of the system.

In this study, we want to demonstrate some relationships existing in the maintainability requirements between the levels of manpower growth.

## 2. The Manpower Structure in Markov Model

The general Markov model for the current manpower structure as a function of the immediate past structure in a fixed size or zero growth system is given by

$$n(t) = n(t-1)P + n(t-1)w'r \tag{2.1}$$

(See for instance Bartholomew et al 1991)  $n(t)$  is the manpower structure at time  $t$ ,  $P$  is the  $k \times k$  transition matrix of the system,  $w$  is the wastage vector and  $r$  is the recruitment vector. We establish the terms of the relationships by working as follows.

Let the transition probability matrix  $P$  be broken into two as  $P = A + B$ , where  $B = \text{diag}\{P_{ii}\}$ ,  $i = 1, 2, \dots, k$ ; that is,  $B$  is a  $k \times k$  diagonal matrix whose diagonal entries are the corresponding diagonal entries of the matrix  $P$ . Therefore,  $A = P - B$  is the Matrix  $P$  with the diagonal entries all zero. Also, let  $b_1(t) = n(t)A'$  where  $A' = A - A'$ . Then, (2.1) can be written as

$$\begin{aligned} n(t+1) &= n(t)(A+B) + n(t)w'r \\ &= n(t)(A - A') + n(t)(A' + B) + n(t)w'r \\ &= n(t)A' + n(t)P' + n(t)w'r \\ &= b_1(t) + n(t)P' + n(t)w'r \end{aligned} \tag{2.2}$$

## 3. Maintainability Requirement in Fixed-Size (Zero Growth) Condition

By definition, (see for instance Georgiou and Tsantas 2002), for maintainability of a desired structure  $n$ , set  $n(t+1) = n(t) = n$ . Therefore, any structure  $n$  is maintainable in (2.2) (fixed-size or zero growth model) if it satisfies the steady-state condition given by

$$n = b_1^* + nP' + nw'r \quad \dots\dots\dots (3.1)$$

where  $b_1^* = nA^*$ .

In particular,  $n$  is maintainable through recruitment if there exists an  $r$  such that (3.1) is satisfied.

$$\text{That is if } r = \{n(I - P') - b_1^*\} \{nw'\}^{-1} \quad \dots\dots\dots (3.2)$$

where  $I$  is the identity matrix. But such an  $r$  must be a probability vector. Hence, the following two conditions must be satisfied. (i)  $\{n(I - P') - b_1^*\} I' = nw'$  and

(ii)  $b_1^* \leq n(I - P')$ .  $1 = [1, 1, \dots, 1]$ , that is a  $1 \times k$  vector of one's. However, condition (i) is always satisfied since by the stochastic condition of the manpower model  $(I - P')I' = w'$  and  $P' + A^* = P$ . This implies that a sufficient condition for the structure  $n$  to be maintainable through recruitment is that

$$b_1^* \leq n(I - P') \quad \dots\dots\dots (3.3)$$

#### 4. Maintainability Requirement in Positive Growth Condition, $\alpha > 0$

Let  $\alpha$  be a positive number representing the rate of change in the system. The change in the system can be written as  $\Delta(t) = n(t)I'\alpha$ . If we re-define the structure of the system (in stock proportions) as  $m(t) = n(t)\{n(t)I'\}^{-1}$ , a steady-state condition for the structure  $m(t)$  is reached (even though  $n(t)$  and  $n(t)I'$ , the total size, are changing in size) when the relative changes of  $n(t)$  with  $n(t)I'$  are such that the values of  $m(t)$  remain the same.

By the above definitions and from (2.2),

$$n(t+1) = n(t)A^* + n(t)P' + n(t)w'r + n(t)I'\alpha r \quad \dots\dots\dots (4.1)$$

With further manipulations and rearrangements due to the definitions, (4.1) results in

$$\begin{aligned} (1 + \alpha)m(t+1) &= m(t)A^* + m(t)P' + m(t)w'r + \alpha r \\ &= b_2(t) + m(t)\{P' + (w' + I'\alpha)r\} \quad \dots\dots\dots (4.2) \end{aligned}$$

Where,  $b_2(t) = m(t)A^*$ .

$$\text{From (4.2), a steady-state condition for } m \text{ implies that } (1 + \alpha)m = b_2^* + m\{P' + (w' + I'\alpha)r\} \dots\dots (4.3)$$

In particular,  $m$  is maintainable through recruitment if there exists  $r$  such that (4.3) is satisfied, that is, if

$$r = \{m(I - P') + \alpha m - b_2^*\} \{m(w' + I'\alpha)\}^{-1} \quad \dots\dots\dots (4.4)$$

Since  $\alpha$  is strictly positive, and since  $\{m(I - P') + \alpha m - b_2^*\} I' = m(w' + I'\alpha)$  is true ensuring that  $rI' = 1$ , the sufficient condition for the structure  $m$  to be maintainable through recruitment is that  $b_2^* \leq m(I - P') + \alpha m \dots\dots (4.5)$ .

#### 5. Maintainability Requirement in Negative Growth Condition, $\alpha < 0$

When  $\alpha < 0$ , the maintainability condition for the manpower structure  $m$  remains the same as in (4.5) except that the value of  $mw'$  is not equal to the absolute value of the contraction rate  $\alpha$ . This additional condition derives from the requirement that in (4.4)  $m(w' + I'\alpha) \neq 0$ , which, under contraction with  $\alpha < 0$ , is the same as  $mw' \neq |\alpha|$  since  $mw' > 0$ .

#### 6. Maintainability Relationships from the Foregoing Results.

Comparing (3.3) with (4.5) some important results are obtained.

(i) In recruitment control, any structure that can be maintained under fixed-size system can also be maintained under expanding system. This is because by (3.3),  $b_1^* \leq n(I - P')$ , which is the same condition as  $b_2^* \leq m(I - P')$ . Since  $\alpha > 0$ , we then have that

$$b_1^* \leq n(I - P') \Rightarrow b_2^* \leq m(I - P') \Rightarrow b_2^* \leq m(I - P') + \alpha m.$$

(ii) In recruitment control, any structure that can be maintained under a contracting (negative growth) system can also be maintained under non-contracting situation. This is because with  $\alpha < 0$ ,  $b_2^* \leq m(I - P') + \alpha m \Rightarrow b_2^* \leq m(I - P') \Rightarrow b_1^* \leq n(I - P')$ .

(iii) A wider set of maintainable structures exists when the system is expanding than when it is not. This is obvious by observing that it is easier to satisfy the condition for maintainability in (4.5) than that in (3.3) when  $\alpha > 0$ .

(iv) A smaller set of maintainable structures exists when the system is contracting than when it is not. This is obvious by observing that it is more difficult to satisfy the condition for maintainability in (4.5) than that in (3.3) when  $\alpha < 0$ .

In summary, we write  $M_{t,t-1}^c(q) \subseteq M_{t,t-1}^f(q) \subseteq M_{t,t-1}^E(q)$  where  $M_{t,t-1}^c(q)$  denotes maintainable set for a given structure  $q$  under contracting system  $c$  (That is  $\alpha < 0$ ).  $f$  and  $E$  denote fixed-size and expanding systems respectively under the same definition. We note that the results in (i) and (ii) are equivalent to saying that if  $n$  is recruitment maintainable for  $\alpha = 0$  then there exists  $\alpha$  in  $(0, \infty)$  such that  $m$  is recruitment maintainable, and if there exists  $\alpha$  in  $(-\infty, 0)$  such that  $m$  is recruitment maintainable then there exists  $\alpha$  in  $[0, \infty)$  such that  $n$  and  $m$  are recruitment maintainable. The converse is not in general true. The results in (iii) and (iv) agree with the results of Bartholomew (1982) who commented that the set of maintainable structures for good looking transition probability matrices is unexpectedly small and proposed that the idea of expansion and contraction in manpower systems can be viewed as another form of control factor through which the set of maintainable structures can be varied.

**7. Illustration of the Maintainability Relationships**

Let a 3 x 3 manpower structure  $n(t) = [20, 10, 10]$  be associated with

$$w = [0.20, 0.10, 0.4,] \text{ and } P = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.6 & 0.3 \\ 0 & 0.3 & 0.3 \end{bmatrix}$$

We show that if the structure is maintainable with  $\alpha \in (-\infty, 0)$  then there exists  $\alpha$  in  $[0, \infty)$  such that  $n$  and  $m$  are recruitment maintainable. With  $\alpha \in (-\infty, 0)$ , the condition for maintainability of  $m$  is that

$$b_2^* \leq m(I - P') + \alpha m. \text{ Now, from the definition in section 4, } m = [0.5, 0.25, 0.25]. \text{ Also } b_2^* = m(A - A') = [0, 0, 0] \text{ and } m(I - P') = [0.1, 0.025, 0.1]. \text{ For the } \alpha \in (-\infty, 0), \text{ let } \alpha' = |\alpha|. \text{ The problem becomes one of}$$

evaluating the  $\alpha$  in the question: Is  $b_2^* \leq [0.1, 0.025, 0.1] - [0.5\alpha', 0.25\alpha', 0.25\alpha']$ ? That is: is  $0 \leq 0.1 - 0.5\alpha', 0 \leq 0.025 - 0.25\alpha'$  and  $0 \leq 0.1 - 0.25\alpha'$ ? Since for  $\alpha \in (-\infty, 0)$  and  $\alpha' = |\alpha|$ ,  $0.025 - 0.25\alpha' < 0.1 - 0.5\alpha' < 0.1 - 0.25\alpha'$  any such  $\alpha'$  that satisfies  $0 \leq 0.025 - 0.25\alpha'$  satisfies the maintainability condition for  $m$ . This implies  $\alpha' \leq 0.1$  or  $-0.1 \leq \alpha \leq 0.1$ . That is  $\alpha \in (-0.1, 0.10)$ . Since  $(-\infty, 0) \cap (-0.1, 0.10) \neq \phi$  and  $[0, \infty) \cap (-0.1, 0.10) \neq \phi$  the relationship is established ( $\phi$  denotes the empty set). For this illustration, the value of  $\alpha$  for the maintainability of  $m$  can actually be any value in  $[-0.1, \infty)$ .

We can give an example to show that the converse is not in general true. Consider the 4 x 4 structure  $m = [0.40, 0.20, 0.20, 0.20]$  with  $w = [0.20, 0.20, 0.15, 0.20]$  and

$$P = \begin{bmatrix} 0.515 & 0.285 & 0 & 0 \\ 0 & 0.24 & 0.56 & 0 \\ 0 & 0 & 0.545 & 0.305 \\ 0 & 0 & 0 & 0.80 \end{bmatrix}$$

It is easy to verify that  $m$  is maintainable with  $\alpha = 0.4$  but not with  $\alpha = 0$  or  $\alpha \in (-\infty, 0]$ .

**8. CONCLUSION**

The maintainability conditions of the three levels of manpower growth structure in recruitment control have been studied. The establishment of these conditions in forms which yielded easily the desired structural comparison as a system goes from one level to another was aided by an appropriate re-definition of the transition probability matrix of the system. The results show relationship that assures that a manpower structure remains maintainable (in recruitment control) along an increasing rate of change ( $\alpha$ ). However, the converse is not in general true.

**REFERENCES**

- Bartholomew, D. J., 1982. *Stochastic Models for Social Processes* (3<sup>rd</sup> ed.) Wiley, Chichester.
- Bartholomew, D. J. et al., 1991. *Statistical Techniques for Manpower Planning*. John Wiley, New York.
- Georgiou, A. C. and Tsantas, N., 2002. Modelling Recruitment Training in Mathematical Human Resource planning. *Applied Stochastic Models in Business and Industry*, (18): 53 – 74.
- Glen, J. J. and Yang, C. L., 1996. A Model for Promotion Rate Control in Hierarchical Manpower Systems. *IMA Journal of Mathematics Applied in Business and Industry*, ( 7): (3):197-206.
- McClellan, S., 1991. Manpower Planning Models and their Estimation. *European Journal of Operational Research*, 51, 179 – 187.
- Vassiliou P-C. G., 1998. The Evolution of the Theory of Non-homogeneous Markov Systems. *Applied Stochastic Models and Data Analysis*, 13, 159-176.