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ESTIMATION OF IMPULSE RESPONSE FUNCTION USING THE LEFT AND RIGHT PARAMETERS

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ABSTRACT

Another method of identification of impulse response function in a transfer function is proposed. This method parameterised the transfer function model into a dynamic system model involving rignt and left model parameters. The right and left parameters of the models are estimated by applying the iterative multiple regression approach on the input and output processes respectively. Results obtained are comparable to those obtained from direct application of the iterative multiple regression on the input process.

1. INTRODUCTION

The problem of identifying the impulse response function $\{H_K\}_{K \in N}$ of the system defined by the input process$\{X_t\}_{t \in T}$ and output process $\{Y_T\}_{T \in T}$ described by the equation (1) :

$$Y_t = \sum_{k=0}^{\infty} h_k X_{t-k} + N_t \qquad (1)$$

where N_t , the noise process is uncorrelated with X_s for all s≤t has been investigated by many authors.

Box & Jenkins (1968, 1970) proposed a method for identifying a physical realizable linear system in the time domain, in the presence of added noise. The impulse response function is obtained by solving the least square equations for the impulse response function. They, however observe that these equations, which do not generally provide efficient estimates, are cumbersome to solve and also require the knowledge of the truncation point.

Newland (1975) proposed a revisit of Yule-Walker equation and solving it directly by taking advantage of increased computing power of modern computers to overcome Box & Jenkins shortcomings.

Chatfield (1979) proposed a method for system identification using the inverse autocorrelation functions and with it, he reanalysed Box and Jenkins (1970) gas furnace data to obtain comparable results by a much quicker computational method.

Enang (2004) proposed the estimation of the impulse response function by iteratively applying the least square approach. The orthogonal projections of y_t onto the space spanned by $\{x_{t-s}\}_{0 \le s \le k}$ for $0 \le k \le \infty$ was taken after which the backward elimination procedure proposed by Draper & Smith (1968) was applied to the result of the iterative multiple regression procedure to eliminate insignificant lags.

Using Enang (2004) as a base, this work proposed an alternative method of estimating impulse response function by parametrising the transfer function model in such a way that the behavior of the input and output processes are taken into consideration in the identification procedure

2.1 Model Identification Procedure

In this paper a linear system with input $\{X_t\}_{t \in T}$ and output $\{Y_t\}_{t \in T}$ where the output is corrupted by a noise process $\{N_t\}_{t \in T}$ is considered. The impulse response function of the system $\{h_k\}_{k \ge 0}$ is such that

$$Y_t = h(B)X_t + N_t \qquad (2)$$

where,

$$h(B) = h_0 + h_1B + h_2B^2 + \ldots$$

Assuming that both $\{X_t\}_{t \in T}$ and $\{Y_t\}_{t \in T}$ are stationary (possibly after differencing) and mean corrected, and that $\{N_t\}_{t \in T}$ is stationary with mean zero and uncorrelated with $\{X_t\}_{t \in T}$

The equation (2) may be parsimoniously parameterised to form

$$Y_t = \delta^{-1}(B)\omega(B)X_{t-b} + N_t \qquad (3)$$

based on the system model

$$\delta(B)\, Y_t = \omega(B)X_{t-b} + N_t^{*}, \qquad (4)$$

where

$$\delta(B) = \sum_{i=0}^{r} \delta_i B^i$$

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is the left operator, $\delta_0 = 1$, and $\delta_1, \delta_2, \dots \delta_r$ are the left parameters, moreover,

$$\omega(B) = \sum_{i=0}^{s} \omega_i B^i$$

is the right operator with $\omega_0, \omega_1, \dots, \omega_s$ as the right parameters.

It is clear from algebra (Yosida etal 1971) that the product of $\delta(B)$ operator and its inverse, $\delta^{-1}(B)$ satisfies the equation

$$\delta^{-1}(B)\delta(B) = \delta_0\lambda_0 B^0 + (\delta_0\lambda_1 + \delta_1\lambda_0)B^1 + \dots = 1$$

for coefficients $1 = \delta_0, \delta_1, \delta_2 \dots \delta_r$ of $\delta(B)$ and coefficients $\lambda_0, \lambda_1, \lambda_2 \dots \lambda_k, \dots$ of $\delta^{-1}(B)$. This can be expanded into a system of linear equations as illustrated below

$$\delta_0\lambda_0 = 1$$
$$\delta_0\lambda_1 + \delta_1\lambda_0 = 0$$
$$\delta_0\lambda_2 + \delta_1\lambda_1 + \delta_2\lambda_0 = 0$$

$$\delta_0\lambda_p + \delta_1\lambda_{p-1} + \delta_2\lambda_{p-2} + \dots + \delta_{p-1}\lambda_1 + \delta_p\lambda_0 = 0$$

or in matrix form

$$\begin{pmatrix} \delta_0 & 0 & 0\cdots & 0\cdots \\ \delta_1 & \delta_0 & 0\cdots & 0\cdots \\ \delta_{12} & \delta_1 & \delta_0\cdots & 0\cdots \\ \vdots & \vdots & \vdots\ddots & \vdots\cdots \\ \delta_k & \delta_{k-1} & \delta_{k-2}\cdots & \delta_0 \\ \vdots & \vdots & \vdots\cdots & \vdots \end{pmatrix} \begin{pmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_k \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \end{pmatrix}$$

The solution of this system gives the needed coefficients of the inverse of the left operator which is multiplied by the right operator to obtain the impulse response function.

The details of the identification procedure is described below: From (4) we expressed Y_t as

$$Y_t = (1 - \delta(B))Y_t + \omega(B)X_{t-b} + N_t,$$

$$\text{where} \quad 1 - \delta(B) = \sum_{j=1}^{r} \delta_j^* B^j,$$

and we solve it by iterative multiple regression, essentially as described by Enang (2004), except the selection here is for two sets of variables. The algorithm used to selects the variables is as follows:

$$Y_t = \omega_0 X_t$$
$$Y_t = \delta_1 Y_{t-1} + \omega_0 X_t$$
$$Y_t = \delta_1 Y_{t-1} + \omega_0 X_t - \omega_1 X_{t-1}$$
$$Y_t = \delta_1 Y_{t-1} + \delta_2 Y_{t-2} + \omega_0 X_t - \omega_1 X_{t-1}$$
$$Y_t = \delta_1 Y_{t-1} + \delta_2 Y_{t-2} + \omega_0 X_t - \omega_1 X_{t-1} - \omega_2 X_{t-2}$$

. . . .

In terms of program organisation, we use the scheme

Y_t	Y_{t-1}	Y_{t-2}	Y_{t-3}	Y_{t-4}	…	Y_{t-N+1}	Y_{t-N}	X_t	X_{t-1}	X_{t-2}	X_{t-3}	X_{t-4}	…	X_{t-N+2}	X_{t-N+1}
Sel. Ord:	2	4	6	8	..	2N-2	2N	1	3	5	7	9	..	2N-3	2N-1

which was done by the program (See appendix 6.2 for the computer algorithm). After solving each multiple regression problem generated, and analysing the error squares, we could select the suitable model

3. APPLICATION

Both the input and output data were extracted from the yearly annual reports of the Calabar port of the Nigerian Ports Authority covering the period from April, 1978 to December, 1997 (monthly data).

The input series $\{X_t\}_{t \in T}$ is the total inward and outward traffic handled in the port known as throughput measured in metric tones.

The output series $\{Y_t\}_{t \in T}$ comes from two components:

(a) Revenue collected in US Dollars

(b) Revenue in Naira

Since the analysis is done in Nigeria, we analyse the total revenue in Naira. Considering the fact that the real value of Naira has changed enormously from 1978 till 1997 and interest is to study the revenue generated and not the inflation. There was need to collect another data namely the composite consumer's price index (combined urban and rural extracted from the Central Bank of Nigeria Statistical bulletin) with 1985 base year for the period under consideration. These were then used respectively to divide the revenue (output) to obtain values that are comparable among the years (Olisaobe,1991).

4. RESULTS AND DISCUSSION

The left parameters (white) with their estimated error square contributions (black) are shown in FIG 1 From here, it can be observed that the contribution to the total error by lag 4 is negligible and can thus be ignored. The order of the left operator r is 3. The coefficients of the inverse of the left operator shown in FIG. 2 damped out exponentially which signifies that the system is stable. FIG. 3 displays the coefficients of the right operator (white) and their error square contributions (black). It shows that lag 3 does not contribute to the total error square and as such should be ignored. The order of the operator s is 4. The parameterized form of the identified system model is thus given as

$$(1 - 0.744053B - 0.519434B^2 - 0.332078B^3)y_t = (0.434775 - 0.288193B - 0.307386B^2 - 0.340154B^4)x_t + n_t$$

where,

$$\delta(B) = 1 - 0.7441B - 0.5194B^2 - 0.3320B^3$$

is the left operator and

$$\omega(B) = 0.4348 - 0.2882B - 0.3079B^2 - 0.3402B^4$$

is the right operator

The impulse response function by this method obtained from the product of the inverse of the left operator and the right operator is shown in FIG. 4.

By the manual backward elimination procedure, it can be observed that lag 1 inclusion will tend to increase the total error, and as such should be dropped, also, lag 2 contribution is negative and again, should be dropped. Lags 3, 4, 5 and 6 should be included while lag 7 and above should be ignored The transfer function model is given as

$$y_t = 0.43477500x_t + 0.05362779\, x_{t-3} - 0.13636299\, x_{t-4} + 0.09953854\, x_{t-5} - 0.02103891\, x_{t-6} + n_t$$

Comparing this model to Enang (2004) result

$$y_t = 0.49423x_t \quad -0.031776x_{t-1} \quad +0.138762x_{t-2} + n_t$$

which was obtained by the direct application of the iterative multiple regression, we observed that the proposed alternative method gives a better result since it captures the contributions from many variables no matter how small.

Similarly, the noise process obtained from our proposed method (FIG.5) suggests an AR(2) given by

$$n_t = -0.655363_{n_t+1} - 0.27124_{n_t+2} + \varepsilon_t$$

which is very highly comparable to the model

$$n_t = -0.665882_{n_t+1} - 0.282361_{n_t+2} + \varepsilon_t$$

obtained by Enang (2004) with the direct method. The proposed model has proved to be highly significant for both autocorrelation and partial auto correlation residual checks.

5. CONCLUSION

The use of the method of right and left parameters, apart from giving comparable results and showing the contributions of many variables no matter their magnitude also makes use of the unique system model of both the input and output processes, a factor which was lacking by the direct application of the iterative multiple regression approach proposed by Enang (2004). We thus recommend that this method should be preferred when identifying impulse response function for a system.

6 APPENDIX
6.1 Figures

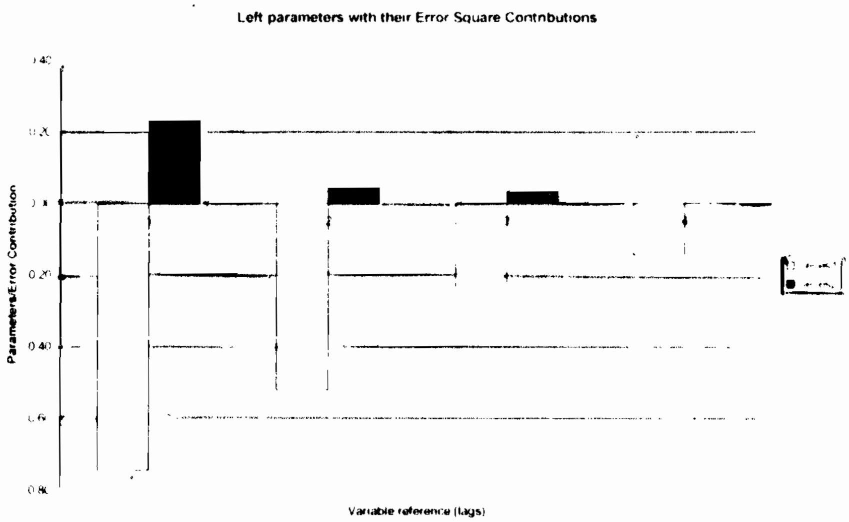

FIG. 1. Coefficient Left parameters (white) with their error square contributions (black)

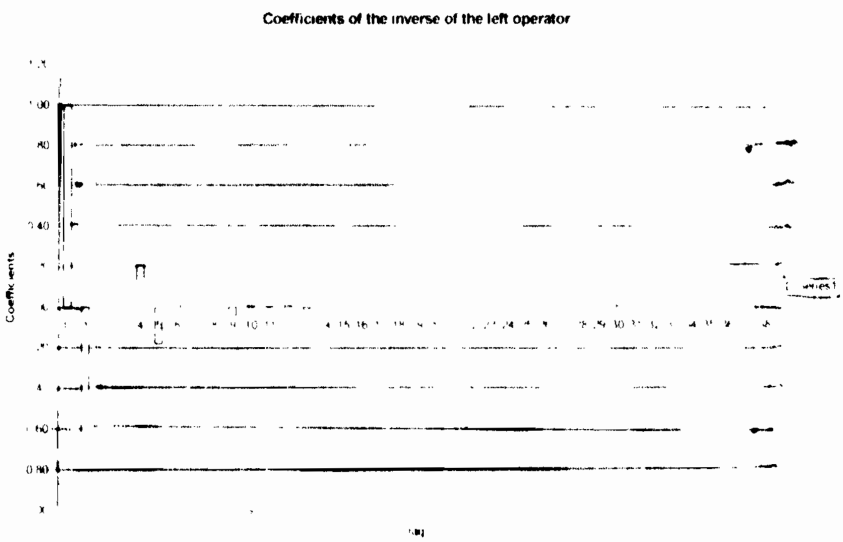

FIG. 2. Coefficients of inverse of the left operator.

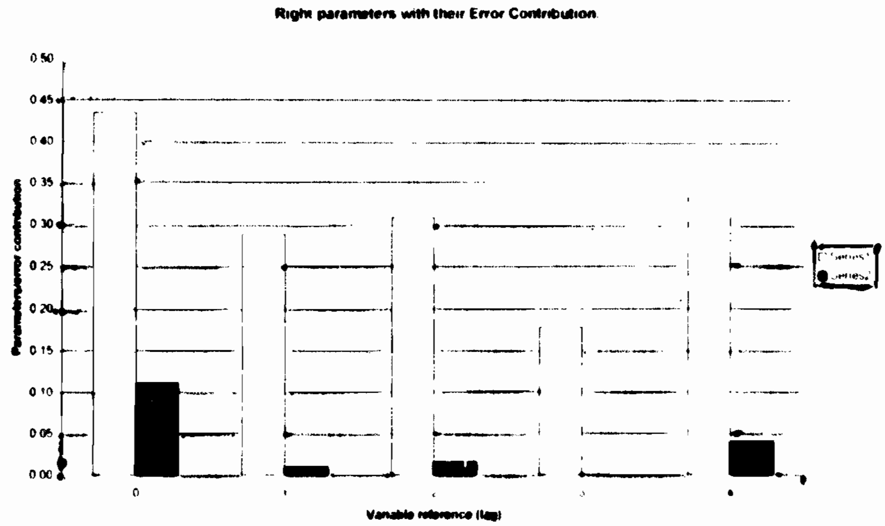

FIG. 3. Coefficients of right operator (white) with their error square contributions (black).

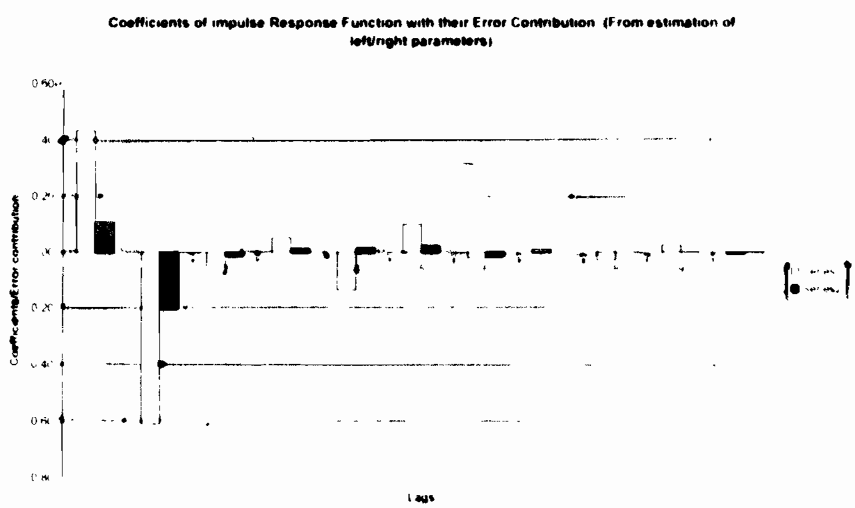

FIG. 4 Coefficients of impulse response function (white) with their error square contributions (black)