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ESTIMATION OF THE PARAMETERS OF THE BILINEAR SEASONAL ARIMA TIME SERIES MODEL

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ABSTRACT

The parameters of the seasonal bilinear time series model were estimated analytically. This was followed by some numerical illustrations. The closeness of the estimated values to the true values attested to the adequacy of the least square method of minimizing error and the Newton-Raphson iterative procedure.

KEY WORDS: Seasonal Bilinear Time Series model, Least Square Method, Newton-Raphson iterative procedure.

1. INTRODUCTION

The general form of the bilinear model, as defined in Granger and Andersen (1978) is:

$$X_t = \sum_{j=1}^{p} a_j X_{t-j} + \sum_{j=1}^{q} c_j e_{t-j} + \sum_{i=1}^{m} \sum_{j=1}^{k} b_{ij} X_{t-i} e_{t-j} + e_t \quad (1.1)$$

for every $t \in Z$. Several sub-classes of the general bilinear model (1.1) have been studied and their parameters estimated. For instance, Subba Rao (1981) considered the estimation of the parameters of the bilinear model

$$X_t = a_0 + \sum_{j-1}^{p} a_j X_{t-j} + \sum_{i=1}^{p} \sum_{j=1}^{q} b_{ij} X_{t-i} e_{t-j} + e_t \quad (1.2)$$

Gabr and Subba Rao (1981) considered the estimation of the parameters of the bilinear model

$$X_t = a0 + \sum_{i=1}^{l} a_{k_i} X_{t-k_i} + \sum_{1}^{m} b_{r_i s_j} X_{t-r_j} e_{t-s_j} + e_t \quad (1.3)$$

where $k_1, k_2, \ldots, k_l$ are subsets of the integers (1,2, ..., p) and Akamanam, (1983) considered the estimation of the parameters of the vector – valued bilinear process

$$\underline{X}_t = A\underline{X}_t + \underline{b}e_{t-1} + B\underline{X}_{t-1} e_{t-1} + Ce_t \quad (1.4)$$

It is worthy of note that all the authors of equations (1.2), (1.3) and (1.4) adopted the Newton-Raphson iterative procedure in their estimations.

The object of this paper is to consider the estimation of the parameters of the seasonal bilinear time series model

$$X_t = \alpha X_{t-s} + \beta e_{t-s} + \gamma X_{t-s} e_{t-s} + e_t, \quad s > 1 \quad (1.5)$$

which is a subset of (1.1) and first studied by Iwueze and Chikezie (2005) for every $t \in Z$, for some constants α, β, and γ which are the parameters of the model (1.5) since its invertibility condition has been stated by Iwueze (1996), p118.

2. FIRST AND SECOND-ORDER PARTIAL DERIVATIVES

From (1.5),

$$e_t = X_t - \theta_1 X_{t-s} - \theta_2 e_{t-s} - \theta_3 X_{t-s} e_{t-s} \quad (1.6)$$

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where $\theta_1 = \alpha$, $\theta_2 = \beta$ and $\theta_3 = \gamma$. Obtaining the first and second–order partial derivatives of (1.6) with respect to θ_1, θ_2, θ_3 shows that

$$\frac{\partial e_t}{\partial \theta_1} = -X_{t-s} - (\theta_2 + \theta_3 X_{t-s}) \frac{\partial e_{t-s}}{\partial \theta_1} \quad (1.7)$$

$$\frac{\partial e_t}{\partial \theta_2} = -e_{t-s} - (\theta_2 + \theta_3 X_{t-s}) \frac{\partial e_{t-s}}{\partial \theta_2} \quad (1.8)$$

$$\frac{\partial e_t}{\partial \theta_3} = -X_{t-s} e_{t-s} - (\theta_2 + \theta_3 X_{t-s}) \frac{\partial e_{t-s}}{\partial \theta_3} \quad (1.9)$$

$$\frac{\partial^2 e_t}{\partial \theta_1^2} = -(\theta_2 + \theta_3 X_{t-s}) \frac{\partial^2 e_{t-s}}{\partial \theta_1^2} \quad (1.10)$$

$$\frac{\partial^2 e_t}{\partial \theta_1 \partial \theta_2} = -\frac{\partial e_{t-s}}{\partial \theta_1} - (\theta_2 + \theta_3 X_{t-s}) \frac{\partial^2 e_{t-s}}{\partial \theta_1 \partial \theta_2} \quad (1.11)$$

$$\frac{\partial^2 e_t}{\partial \theta_1 \partial \theta_3} = -X_{t-s} \frac{\partial e_{t-s}}{\partial \theta_1} - (\theta_2 + \theta_3 X_{t-s}) \frac{\partial^2 e_{t-s}}{\partial \theta_1 \partial \theta_2} \quad (1.12)$$

$$\frac{\partial^2 e_t}{\partial \theta_2^2} = -2 \frac{\partial e_{t-s}}{\partial \theta_2} - (\theta_2 + \theta_3 X_{t-s}) \frac{\partial^2 e_{t-s}}{\partial \theta_2^2} \quad (1.13)$$

$$\frac{\partial^2 e_t}{\partial \theta_2 \partial \theta_3} = -\frac{\partial e_{t-s}}{\partial \theta_2} - X_{t-s} \frac{\partial e_{t-s}}{\partial \theta_2} - (\theta_2 + \theta_3 X_{t-s}) \frac{\partial^2 e_{t-s}}{\partial \theta_2 \, d\theta_3} \quad (1.14)$$

$$\frac{\partial^2 e_t}{\partial \theta_3^2} = -2 X_{t-s} \frac{\partial e_{t-s}}{\partial \theta_3} - (\theta_2 + \theta_3 X_{t-s}) \frac{\partial^2 e_{t-s}}{\partial \theta_3^2} \quad (1.15)$$

Proceeding as in Subba Rao (1981), we can note that maximizing the likelihood function of $(X_{\eta}, X_{\eta+1}, \ldots, X_n)$ is the same as minimizing the function

$$S = S(\theta) = \sum_{t\ 1}^{n} e_t^2 \quad (1.16)$$

with respect to the parameters

$$\underline{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \quad (1.17)$$

Then the first and second-order partial derivatives of $S(\underline{\theta})$ are solved to obtain the components of (1.19) and (1.20). When minimizing $S(\theta)$ with respect to $\underline{\theta}$, the normal equations are non-linear in θ. The solution of these equations require the use of non-linear algorithm such as Newton-Raphson algorithm which is described below. The Newton-Raphson iterative equation given as

$$\theta_{k+1} = \theta_k - H^{-1}(\theta_k)G(\theta_k) \tag{1.18}$$

can be adopted to obtain the (k+1)th iteration ($\hat{\theta}_{k+1}$) of the estimates from the kth estimate ($\hat{\theta}_k$) where

$$G(\theta) = \begin{pmatrix} \frac{\partial s}{\partial \theta_1} \\ \frac{\partial s}{\partial \theta_2} \\ \frac{\partial s}{\partial \theta_3} \end{pmatrix} \tag{1.19}$$

and
$$H(\theta) = \begin{pmatrix} \frac{\partial^2 s}{\partial \theta_1^2} & \frac{\partial^2 s}{\partial \theta_1 \partial \theta_2} & \frac{\partial^2 s}{\partial \theta_1 \partial \theta_3} \\ & \frac{\partial^2 s}{\partial \theta_2^2} & \frac{\partial^2 s}{\partial \theta_2 \partial \theta_3} \\ & & \frac{\partial^2 s}{\partial \theta_3^2} \end{pmatrix} \tag{1.20}$$

The estimates obtained by the iterative equations (1 18) usually converge, but to obtain a good set of estimates it is necessary that we have good sets of initial values of the parameters [(Iwueze and Chikezie (2005)]

3. NUMERICAL ILLUSTRATION

The program for the computation of the parameters of the bilinear time series model (1 5) is written in Fortran Language. With good initial estimates obtained from the six regions described by Nwogu and Iwueze (2003) and Iwueze and Chikezie (2005), reliable estimates of the parameters of model (1.5) could be obtained However for want of space only Region 1 is illustrated for n =100 and 500 Offcourse the central limit theorem and the law of large numbers played out as the estimates for n = 500 are quite closer to true values when compared with that of n = 100

Table 1 – Initial and Final estimates for Region 1, s = 1, 2, 3, 4, 6, 12 and n = 100.

	INITIAL ESTIMATES				FINAL ESTIMATES			
S	α	β	γ	σ^2	α	β	γ	σ^2
1	0 70	0 50	0 20	1.30	0 78	0 27	0 25	0 97
2	0 76	0 36	0 16	1 17	0 76	0 36	0 16	0 96
3	0 82	0 26	0 12	1 40	0.80	0 31	0 07	1 31
4	0 74	0.50	0 21	1 00	0.80	0.33	0 23	0 97
6	0 77	0 41	0 18	1 24	0.83	0 35	0 05	1 40
12	0 66	0 36	0 13	1 72	0 79	0 43	0 19	0 99

Table 2 – Initial and Final estimates for Region 1, s = 1, 2, 3, 4, 6, 12 and n = 500

	INITIAL ESTIMATES				FINAL ESTIMATES			
S	α	β	γ	σ^2	α	β	γ	σ^2
1	0 82	0.40	0 20	1 01	0 81	0 40	0 20	1 00
2	0 74	0 41	0 23	1 04	0 80	0 40	0 20	1 00
3	0.77	0.38	0 13	1 36	0 79	0 37	0 05	1 34
4	0.74	0 39	0.21	1.14	0.76	0 42	0 11	1 25
6	0.76	0.18	0 14	1 41	0.78	0.35	0 02	1 37
12	0.80	0.16	0 10	1 64	0.81	0 36	0 04	1.37

4 CONCLUDING REMARKS

In this paper we have examined the estimation of the parameters of the bilinear ARIMA Time Series model The closeness to the true values of the estimates obtained in the numerical illustration showed that the entire procedure of minimizing error (Least Square Method) and Newton-Raphson iteration are quite adequate since the Final Estimates are not far from the Initial Estimates comparably.

REFERENCES

Akamanam, S. I., 1983. Some contributions to the study of bilinear time series models. Ph.D Thesis, University of Sheffield.

Box, G. E. P and Jenkins, G. M., 1976. Time Series Analysis, Forecasting and Control 2nd ed Holden-Day, San Francisco.

Gabr and Subba Rao., 1981 The estimation and prediction of subset bilinear time series models with applications. Journal of Time Series Analysis. 2,3: 115- 171

Granger, C. W. J. and Andersen, A. P., 1978 An introduction to Bilinear Time Series Models Vanderhoeck and Reprecht, Gottingen

Iwueze, I.S., 1996. On the invertibility of stationary and ergodic bilinear time series models ABACUS. Vol. 24, No. 2 113-119.

Iwueze, I.S. and Chikezie, D.C., 2005. A bilinear representation of the multiplicative seasonal ARIMA (0,d,0)x(1,D,1)s time series model, Global Journal of Mathematical Sciences, 4 (1 & 2):-------

Nwogu, E. C. and Iwueze, I. S., 2003 On the purely seasonal ARIMA process. Journal of the Nigerian Association of Mathematical Physics, Vol 7: 115- 126.

Subba Rao T and Gabr, M. M., 1981. The estimation and prediction of subset bilinear time series models with applications. Journal of Time Series Analysis, 3. pp 155-171.