GLOBAL JOURNAL OF MATHEMATICAL SCIENNCES VOL. 6, NO. 2, 2007: 119 - 126
 ISSN 1596-6208

LINEAR MULTISTEP METHOD FOR SOLUTION OF SECOND ORDER INITIAL VALUE PROBLEMS OF ORDINARY DIFFERENTIAL EQUATIONS: A TRUNCATION ERROR APPROACH

M. O. UDO, G. A. OLAYI and R. A. ADEMILUYI

(Received 25 April 2007; Revision Accepted 15 November 2007)

ABSTRACT

This paper addresses the development of two linear multi-step methods for the solution of second order initial value problems of ordinary differential equations. The approach requires the construction of a truncation error term and expanding it in Taylor series, and we refer to this method as TRUNCATION ERROR APPROACH The resulting two methods are analyzed to show that they are consistent, zero-stable and hence convergent with good interval of absolute stability. The technique of derivation employed here is easier and more adaptable than those of collocation. The implementation, using the basic parameters of the derived methods, to test the practical feasibility and effectiveness proved successful.

KEY WORDS: Truncation error approach, P-stable Linear Multi-step Method, Convergence, Boundary Locus Method

1. INTRODUCTION

There are many processes in science, management and technology that involves the rate of change of one variable in relation to another modeled as differential equations. "Most differential equations in science and technology are solved by numerical methods" (Ross and Ross II; 1989,Olayi 2000) because analytical solution are not possible or useful (Lambert 1973)

There are several existing algorithms designed for the integration of

$$y'' = f(x, y, y'),\ y(a) = \eta_0,\ y'(a) = \eta_1 \qquad a, y \in R \qquad \ldots\ldots (1.1)$$

such as Runge-Kutta and Euler methods, Taylor series method as discussed in Lambert (1973), Hybrid methods by Ademiluyi (1987), Collocation methods by Awoyemi (1996,1999 and 2003) and One step method of integration by Ademiluyi and Kayode (2001), Non symmetric collocation method by Awoyemi et al (2006) and a class of Linear Multi-step Method (LMM) for special ODE's by Udo et al (2007)

The general linear multi-step method is of the form $\sum_{j=0}^{k} \alpha_j y_{n+j} = h^m \sum_{j=0}^{k} \beta_j f_{n+j},\ m = 1, 2, \ldots$ --------- (1.2)

where k is the step number and m represents the order of the differential system we are solving; α_j and β_j are constants and we assume that $\alpha_k \neq 0$ and that both α_0 and β_0 are not zero (Lambert 1973).

DEFINITION 1

According to (Fatunla 1988) a Linear multi-step method is said to be of order p, if it satisfies the condition $C_0 = C_1 = \ldots\ldots = C_p = 0; C_{p+1} \neq 0$ ......(1.3)

DEFINITION 2

A linear multi-step method is said to be consistent if and only if

(i) It has order $p \geq 1$

(ii) $\rho(r) = \rho'(r) = 0,\ \rho''(r) = 2!\,\delta(r)\ for\ r = 1$ ....(1.4)

where ρ(r) and δ(r) are called first and second characteristic polynomials of the method (see Lambert 1973, Awoyemi 1996 and Kayode 2004)

DEFINITION 3

If the roots of the characteristic polynomial ρ all have modulus less than or equal to 1 and the roots of modulus 1 are of multiplicity 1, we say that the root condition is satisfied The method is convergent if and only if it is consistent and the root condition is satisfied. Consequently, a consistent method is stable if and only if this condition is satisfied, and thus the method is convergent if and only if it is stable.
(Retrieved from http://en.wikipedia.org/wiki/Linear_multistep_method)

M. O. Udo, Department of Mathematics/ Statistics, Cross River University of Technology, Calabar, Cross River State, Nigeria
G. A. Olayi, Department of Mathematics/ Statistics, Cross River University of Technology, Calabar, Cross River State, Nigeria
R. A. Ademiluyi, Department of Mathematical Sciences, Federal University of Technology, Akure, Ondo State, Nigeria

DEFINITION 4

According to Lambert (1973),the linear multi-step method (1.2) is said to be absolutely stable for a given $\hat{h}$ (where $\hat{h} = h\lambda$) if, for that $\hat{h}$, all the roots r_s of $[\pi(r,\hat{h}) = \rho(r) - \hat{h}\sigma(r) = 0]$ satisfy $|r_s| < 1$,s=1,2,...k, and to be absolutely unstable for that $\hat{h}$ otherwise. An interval (α, β) of the real line is said to be an interval of absolute stability if the method is absolutely stable for all $\hat{h} \in (\alpha, \beta)$.

DEFINITION 5

The boundary locus method discussed in Lambert (1973) is defined as

$$h(r) = \frac{\rho(r)}{\delta(r)} \quad \text{----- (1.5)}$$

which enables us determine the interval of absolute stability of the scheme without embarking on the rigorous computation of the roots or solving simultaneous inequalities. Here we assume r = exponential ($i\theta$) where θ is the range of angles for which the stability is to be measured. Kayode (2004)

Some of these existing methods Awoyemi (1999), Udo at el (2007) and Kayode (2004) like, the collocation methods, are tedious to derive (Awoyemi 2003). Errors of different magnitudes are introduced during the process. Consequently, in this paper we present an alternative easier method. TRUNCATION ERROR APPROACH, with comparative or even better level of accuracy.

2. DERIVATION OF THE METHODS

We consider the development of two linear multi-step methods for step numbers k = 2, 3. The linear multi-step method of consideration is of the form

$$y_{n+k} = \sum_{j=0}^{k-1} \alpha_j y_{n+j} + h^2 \sum_{j=0}^{k} \beta_j f_{n+j}; \quad k = 2 \text{ or } 3 \quad \text{------ (2.1)}$$

(See Lambert 1973) where α_j's and β_j's are real constants whose values are to be determined. Note $\alpha_k = 1$ and m=2 in comparison with (1.2)

The application of (2.1) to solve (1.1) will produce a truncation error of the form.

$$T_{n+k} = y_{n+k} - \left[\sum_{j=0}^{k-1} \alpha_j y_{n+j} + h^2 \sum_{j=0}^{k} \beta_j f_{n+j} \right] \quad \text{----- (2.2)}$$

If the terms on the RHS of (2.2) are expanded in Taylor series to order p we have

$$T_{n+k} = y_n + (kh) y_n' + \frac{(kh)^2}{2!} y''_n + \ldots + \frac{(kh)^p}{p!} y^p + 0(h^{p+1}) - \sum_{j=0}^{k-1} \alpha_j \left(y_n + (jh) y_n' + \frac{(jh)^2}{2!} y_n'' + \ldots + \frac{(jh)^p}{p!} y_n^p + 0\frac{(jh)^{p+1}}{(p+1)!} \right)$$

$$- h^2 \sum_{j=0}^{k} \beta_j \left(y_n' + (jh) y_n'' + \frac{(jh)^2}{2!} y^{(4)} + \ldots + \frac{(jh)^p}{p!} y_n^{(p+2)} + 0\frac{(jh)^{p+1}}{(p+1)!} y_n^{(p+3)} \right) \ldots\ldots (2.3)$$

Combining terms of (2.3) in equal powers of h yields

$$T_{n+k} = \left(1 - \sum_{j=0}^{k-1} \alpha_j\right) y_n + \left(k - \sum_{j=0}^{k-1} j\alpha_j\right) h y_n' + \left(\frac{k^2}{2!} - \sum_{j=0}^{k-1} \frac{(j)^2}{2!}\alpha_j - \sum_{j=0}^{k} \beta_j\right) h^2 y_n''$$

$$+ \left(\frac{k^3}{3!} - \sum_{j=0}^{k-1} \frac{(j)^3}{3!}\alpha_j - \sum_{j=0}^{k} j\beta_j\right) h^3 y_n''' + \ldots + \left(\frac{k^p}{p!} - \sum_{j=0}^{k-1} \frac{j^p}{p!}\alpha_j - \sum_{j=0}^{k} \frac{j^{(p-2)}}{(p-2)!}\beta_j\right) h^p y_n^p + 0(h^{p+1}) \quad \text{------ (2.4)}$$

In the next section (2.2), (2.3) and (2.4) will be considered with k = 2 or 3.

2.1 TWO-STEP METHOD

For the derivation of a 2-step method, we set k = 2 in equation (2.4) and collect terms in equal powers of h into the form

$$T_{n+k} = C_0 + C_1 h + C_2 h^2 + \ldots + C_p h^p + 0(h^{p+1}) \quad \text{----- (2.5)}$$

$$T_{n+2} = (1 - \alpha_0 - \alpha_1) y_n + (2 - \alpha_1) h y_n' + \left(\frac{4}{2} - \frac{1}{2}\alpha_1 - \beta_0 - \beta_1 - \beta_2\right) h^2 y_n''$$

$$+ \left(\frac{8}{6} - \frac{1}{6}\alpha_1 - \beta_1 - 2\beta_2\right) h^3 y_n''' + \left(\frac{16}{24} - \frac{1}{24}\alpha_1 - \frac{1}{2}\beta_1 - \frac{4}{2}\beta_2\right) h^4 y_n^{(4)} + \ldots + 0(h^{p+1}) \quad \text{------ (2.6)}$$

Imposing order 4 (2k) accuracy on the truncation error (T_{n+2}) implies that $C_j = 0,\ j = 0(1)4$. The resulting system of equations involving the coefficients of the method using (2.5) and (2.6) are

$$\begin{aligned}
&\alpha_0 + \alpha_1 = 1 \\
&\alpha_1 = 2 \\
&\frac{1}{2}\alpha_1 + \beta_0 + \beta_1 + \beta_2 = 2 \\
&\frac{1}{6}\alpha_1 + \beta_1 + 2\beta_2 = \frac{8}{6} \\
&\frac{1}{24}\alpha_1 + \frac{1}{2}\beta_1 + 2\beta_2 = \frac{16}{24}
\end{aligned} \qquad \ldots\ldots(2.7)$$

Thus putting (2.7) in matrix form yields

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1 & 1 & 1 \\ 0 & 1/6 & 0 & 1 & 2 \\ 0 & 1/24 & 0 & 1/2 & 2 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 8/6 \\ 16/24 \end{bmatrix} \qquad \ldots(2.8)$$

Solving equation (2.8) by Gaussian elimination method, the value of the parameters are obtained as $\alpha_0 = -1, \alpha_1 = 2, \beta_0 = 1/12, \beta_1 = 10/12 \ and \ \beta_2 = 1/12$

The value of these parameters are substituted into the expansion of (2.1) above with k = 2, to obtain a two-step discrete scheme

$$y_{n+2} = 2y_{n+1} - y_n + \frac{h^2}{12}(f_{n+2} + 10f_{n+1} + f_n) \qquad \text{-------- } (2.9)$$

which is similar to the classical Numerov's method popularly called Numerov's method. (Lambert 1973, Awoyemi 1996, Olayi 1998,1999 and Kayode 2004). Both the classical Numerov's formula and (2.9) apply to 2nd order ODE initial value problems but f in (2.9) contains the first derivative of y while that in Numerov method does not contain y' (Olayi, 1998). The truncation error of the Numerov formula is O(h^6) while that of (2.9) is O (h^5)

2.2 THREE-STEP METHOD

As in section 2.1, we substitute k = 3 in (2.4) to obtain

$$T_{n+3} = [1 - \alpha_0 - \alpha_1 - \alpha_2]y_n + [3 - \alpha_1 - 2\alpha_2]hy' + \left[\frac{9}{2!} - \frac{1}{2!}\alpha_1 - \frac{4}{2!}\alpha_2 - \beta_0 - \beta_1 - \beta_2 - \beta_3\right]h^2 y''_n$$
$$+ \left[\frac{27}{3!} - \frac{1}{3!}\alpha_1 - \frac{8}{3!}\alpha_2 - \beta_1 - 2\beta_2 - 3\beta_3\right]h^3 y'''_n + \ldots + \left[\frac{3^6}{6!} - \frac{1}{6!}\alpha_1 - \frac{2^6}{6!}\alpha_2 - \frac{1}{4!}\beta_1 - \frac{2^4}{4!}\beta_2 - \frac{3^4}{4!}\beta_3\right]h^6 y^6_n$$
$$+ (0h^7) \ldots\ldots\ldots\ldots(2.10a)$$

and similarly as in (2.7) and (2.8) obtain the matrix representing the system of equation as

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 4/2 & 1 & 1 & 1 & 1 \\ 0 & 1/6 & 8/6 & 0 & 1 & 2 & 3 \\ 0 & 1/24 & 16/24 & 0 & 1/2 & 4/2 & 9/2 \\ 0 & 1/120 & 32/120 & 0 & 1/6 & 8/6 & 27/6 \\ 0 & 1/720 & 64/720 & 0 & 1/24 & 16/24 & 81/24 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 9/2 \\ 27/6 \\ 81/24 \\ 243/120 \\ 729/720 \end{bmatrix} \qquad \ldots(2.10b)$$

Using Gaussian elimination to solve (2.10b) we obtain the following coefficients

$$\alpha_0 = 1,\ \alpha_1 = -3,\ \alpha_2 = 3,\ \ \beta_0 = -\frac{1}{12},\ \beta_1 = -\frac{9}{12},\ \beta_2 = \frac{9}{12},\ \ \beta_3 = \frac{1}{12} \qquad \text{------ (2.11)}$$

Substitution of (2.11) into the expansion of (2.1) yields the implicit scheme

$$y_{n+3} = 3y_{n+2} - 3y_{n+1} + y_n + \frac{h^2}{12}(f_{n+3} + 9f_{n+2} - 9f_{n+1} - f_n) \qquad \text{.....(2.12)}$$

with truncation error O(h^7).

Equations (2.9) and (2.12) are our **two step and three step** methods respectively .Both methods agrees exactly with those derived through Collocation and interpolation by Kayode (2004).[Our method (2.12) produces a Numerov formula .In Olayi (1998) a 5-point formula was developed. It will certainly require more time to solve a 5-point formula. This is so as both (2.12) and Olayi's 5-point formular apply to 2nd order ODE initial value problems].

2.3 DEVELOPMENT OF PREDICTORS

It is noticeable of linear multi--step methods that they require two or more starting values before they can function. Hence in the methods, we need the value of y_{n+j}, j = 0, 1 2...,k-1,where k is the step number,[see (2.1)] before the value of y_{n+1} and their derivatives can be found.

2.3.1 PREDICTORS FOR THE 2-STEP METHOD

To be able to implement our 2-step method for the solution of second order initial value problems we need to develop the predictor that would be used for the evaluation

$$y_{n+2}, y'_{n+2}, y_{n+1}, y'_{n+1};\ in\ f_{n+j} = f(x_{n+j}, y_{n+j}, y'_{n+j}), j = 1,2$$

Taylor series expansion of y_{n+2} is used up to the order four (4) of the two step method to have

$$y_{n+2} = y_n + 2hy'_n + \frac{(2h)^2}{2!}y''_n + \frac{(2h)^3}{3!}y'''_n + \frac{(2h)^4}{4!}y^{(4)}_n + 0(h^5) \qquad \text{..... (2.13)}$$

Differentiating (2.13) once yields

$$y'_{n+2} = y'_n + 2hy''_n + \frac{(2h)^2}{2!}y'''_n + \frac{(2h)^3}{3!}y^{(4)}_n + 0(h^5) \qquad \text{......(2.14)}$$

Equations (2.13) and (2.14) are used for the evaluation of the y_{n+2} and y'_{n+2} respectively in the function f_{n+2} = f (x_{n+2}, y_{n+2}, y'_{n+2}), as predictors. Using Taylor series expansion, we obtain predictors for y_{n+1} and y'_{n+1} in the function $f_{n+1} = f(x_{n+1}, y_{n+1}, y'_{n+1})$ by following the procedures in (2.13) and (2.14) above respectively. The values of y_n and y'_n are as given by the initial conditions of the problem (1.1)

2.3.2 PREDICTORS FOR 3-STEP METHOD

The following values y_{n+2}, y'_{n+2}, y_{n+1}, y'_{n+1} y_n and y'_n are needed to start the evaluation of the 3-step method. They are obtained in the same way as in (2.13) and (2.14) up to the order 6 of accuracy.

3. ANALYSIS OF BASIC PROPERTIES OF THE METHODS

According to Lambert (1973), the necessary and sufficient conditions for a linear multi-step method to be convergent are that it be consistent and zero stable. Thus we now seek to establish these properties.

3.1 PROPERTIES OF THE TWO STEP METHOD

The result obtained from (2.8) is substituted back into (2.6) to have

$$\left.\begin{aligned} C_0 &= 1 + 2 - 1 = 0 \\ C_1 &= 2 - 2 = 0 \\ C_2 &= 1 + \frac{1}{12} + \frac{10}{12} + \frac{1}{12} - 2 = 0 \\ &\ \ \vdots \\ C_5 &= \frac{32}{120} - \frac{1}{60} - \frac{10}{72} - \frac{8}{72} = 0 \\ C_6 &= \tfrac{2}{720} + \tfrac{10}{288} + \tfrac{16}{288} - \tfrac{64}{720} = \tfrac{1}{240} = C_{p+1} \neq 0 \end{aligned}\right\} \qquad \text{.....(3.1)}$$

Hence by (3.1) we can deduce that our two step method is of order p = 5, with error constant $C_{p+1} = \frac{1}{240}$, see Udo et al (2007).

For our two step method (2.9)

$$\rho(r) = r^2 - 2r + 1,$$

$$\delta(r) = \frac{1}{12}\left(r^2 + 10r + 1\right) \qquad \ldots\ldots (3.2)$$

.Hence applying the conditions of definition 2 we get

$$\rho(1) = 1 - 2 + 1 = 0,\ \rho'(1) = 2 - 2 = 0,\ \rho''(1) = 2 = 2!\ \delta(1) = \frac{1}{12}(12)\ x\ 2! = 2$$

Thus from definition 2, condition (ii) holds and with p = 5, (i) also holds, implying that the two step method is consistent according to Lambert (1973)

Applying definition 3 to the two step method we see that $\rho(r) = r^2 - 2r + 1$, which implies that r = 1, 1 when $\rho(r) = 0$. Hence, the two step method satisfies the root condition, hence it is zero stable and by extension it is convergent

Using the values of ρ(r) and δ(r) as contained in (3.2), equation (1.5) becomes

$$h(r) = \frac{12(r^2 - 2r + 1)}{(r^2 + 10r + 1)} \qquad \ldots\ldots (3.3)$$

Using $r = e^{i\theta} = \cos\theta + i\sin\theta$ in (3.3) yields

$$h(\theta) = \frac{12\{(Cos2\theta - 2Cos\theta + 1) + i(Sin2\theta - 2Sin\theta)\}}{\{(Cos2\theta + 10Cos\theta + 1) + i(Sin2\theta + 10Sin\theta)\}} \qquad \ldots\ldots (3.4)$$

Rationalizing and simplifying (3.4) yields

$$h(\theta) = x(\theta) + iy(\theta). \qquad \ldots\ldots (3.5)$$

where

$$x(\theta) = \frac{12\{Cos2\theta + 8Cos\theta - 9\}}{\{Cos2\theta + 20Cos\theta + 51\}}. \qquad \ldots\ldots (3.6)$$

We only considered the real part of $h(\theta)$, hence y(θ) is ignored.

Evaluating $x(\theta)$ in (3.6) for $0 \le \theta \le 180$, we see that our two step method has an interval of absolute stability x (θ) = [-6, 0], x (θ) meaning we considered only the real part of (3.5).

3.2 ANALYSIS OF BASIC PROPERTIES OF THE THREE STEP METHOD

For us to examine the basic properties of the three step method, we are to adopt a similar approach as we have in section (3.1) above.

On substituting the result of values of our parameters obtained in (2.10) into (2.6) we have as usual that $C_0 = C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = 0$, $C_7 \neq 0$

Thus the three step method is of order p=6 and the error constant $c_{p+1} = \frac{1}{240}$

A linear multi-step method is consistent if it satisfies the conditions of consistency stated in section (3.1) above. And for the three step method we have

$$\rho(r) = r^3 - 3r^2 + 3r - 1,$$

$$\delta(r) = \frac{1}{12}\left(r^3 + 9r^2 - 9r - 1\right) \qquad \ldots\ldots (3.7)$$

It can be established that our three step method satisfies definition 2, hence it is consistent. By definition 3 the first characteristic polynomial

$$\rho(r) = r^3 - 3r^2 + 3r - 1$$

This gives the possible values of r for which ρ(r) will be zero to be $(r-1)^3 = 0$,
which implies that r = 1,1,1. Hence by definition 3, the three step method satisfies the root condition, hence it is Zero stable.

We also apply as in (1.5), (3.3), (3.4) and (3.6) above the boundary locus method of Lambert (1973) to get the following result

$$h(\theta)=\frac{12\{Cos3\theta-3Cos2\theta+3Cos\theta+1)+i(Sin3\theta-3Sin2\theta+3Sin\theta)\}}{\{(Cos3\theta+9Cos2\theta-9Cos\theta-1)+i(Sin3\theta+9Sin2\theta-9Sin\theta)\}} \quad \text{..... (3.8)}$$

Rationalizing and simplifying (3.8) yields

$h(\theta)=x(\theta)+iy(\theta)$ .....(3.9)

where

$$x(\theta)=\frac{12\{Cos3\theta+6Cos2\theta-33Cos\theta+26\}}{\{Cos3\theta+18Cos2\theta-63Cos\theta-82\}} \quad \text{..... (3.10)}$$

Hence it can be verified that the three step method (2.12) has an interval of absolute stability $x(\theta) = [-6, \infty]$, $x(\theta)$ as defined before. This interval is contained in the entire positive half part of the complex plane. Hence the three step method is P – stable, see Awoyemi (2003).

4. NUMERICAL EXPERIMENT

A numerical example on second order initial value problems is considered to test the accuracy of the derived two and three step methods using the parameters of the developed methods.

4.1 TEST PROBLEM

$y'' = x(y')^2$, ...(4.1a)

$y(0) = 1,\ y'(0)=\left(\frac{1}{2}\right)$ ......(4.1b)

Exact Solution: $y(x) = 1 + \frac{1}{2}\ln\frac{[2+x]}{[2-x]}$ .... (4.2)

4.2 RESULTS

The results of the above problem solved using a computer program can be found in Kayode (2004). Here we present a framework for solving manually; the above second order differential equation using (2.9) and (2.12) respectively. As a case study we will discuss that of the two step method (2.9) only. We see that the following starting values $y_n, y'_n, y_{n+1}, y'_{n+1}, y_{n+2}, y'_{n+2}$ are needed for the evaluation of (2.9). The first two are our given initial conditions in (4.1). The third and fourth are gotten by taking a Taylor series expansion of y_{n+1} and y'_{n+1} respectively, thus we have;

$$y_{n+1}=y_n+hy_n'+\frac{h^2}{2!}y_n''+\frac{h^3}{3!}y_n'''+\ldots+(0h^5) \quad \text{.... (4.3)}$$

$$y'_{n+1}=y'_n+(h)y''_n+\frac{h^2}{2!}y'''_n+\frac{h^3}{3!}y^{iv}_n+\ldots+(0h^5) \quad \text{..... (4.4)}$$

Our final set of starting values is as given in (2.13) and (2.14) respectively. We will need y'_{n+2} as f_{n+2} depends on it. Thus taking derivatives of (4.1a) up to the fourth derivative, values of (2.13);(2.14);(4.3)and(4.4) can be determined. Thus (2.9) at n=0 is

$$y_2=2y_1-y_0+\frac{h^2}{12}\{f_2+10f_1+f_0\} \quad \text{........ (4.5)}$$

Hence for n =1 and 2 similar schemes as (4.5) will be generated. For the solution of (4.1) we have the following,

$y(0)=1; y'_n(0)=1; y_1=1.1; y'_1=f_1=1.1; y_2=1.21; y'_2=f_2=1.2; y_3=1.33;$

$y'_3=f_3=1.3; y_4=1.46; y'_4=f_4=1.4 and y'_5=1.5$

Consequently, for n = 0, 1, 2,... we set up an iterative process with h=0.1 using (2.9), and the results are as presented in the table below.

TABLE 1 SUMARY OF PROBLEM (4.1); h=0.1 for the two step method

X	Y EXACT	Y COMPUTED	ERROR
0 1	1.0500417	1 1000000	-0.0499583
0.2	1.1003353	1.2100000	-0 1096647
0.3	1.1511404	1.3300000	-0 1788596
0.4	1.2027326	1.4600000	-0.2572674
0.5	1.2554128	1 5900000	-0 3345872

Table 2: Result of problem (4.1) using a computer programme; for two step method, h = $^1/_{40}$

VALUE OF X	Y EXACT	Y COMPUTED	ERROR
0.100000001	1.050041676	1.050002575	0.000039101
0.200000018	1.100335360	1.100005150	0.000330210
0.300000042	1.151140451	1.150007725	0.001132727
0.400000066	1.202732563	1.200010300	0.002722263
0.500000060	1.255412817	1.250012875	0.005399942

DISCUSION AND CONCLUSION

We have developed two linear multi-step methods of step two and three which agrees with those of Awoyemi (1999) and Kayode (2004) obtained through collocation method. Hence we have established that by a truncation error approach the same results gotten through collocation are possible. A study of the collocation approaches (see Awoyemi 1996, 1999 and 2003; Kayode 2004; Awoyemi et al 2006 and Udo et al 2007). for solving second order ordinary differential equations will clearly justify the advantage of the Truncation error approach over the collocation approach. For instance our two step method (2.9) was obtained by taking a Taylor series of terms in RHS of (2.2) and collecting terms in equal powers of h. Kayode (2004) in getting the same result as (2.9) sort for the solution of the constants in $y(x) = \sum_{j=0}^{k} a_j x^j$ after which the $x_{n+j}; j = 0(1)k$ terms are simplified to enable $y(x) = \sum_{j=0}^{k} a_j x^j$ depend only on x_{n+k} and not on $x_{n+j}; j = 0(1)k$. Thereafter, when the α_j's, β_j's values have been determined, they are then substituted into (2.1) before the two step method evolves.

We are sure that with this approach (TEA), time which is a very essential tool in any research will be minimized. Table 1 shows the accuracy of the two step method and by implication we know the three step method will have a better accuracy level. Researchers are encouraged to investigate the effect as the step number is increased.

REFERENCES

Ademiluyi, R. A., 1987. New Hybrid method for solving system of stiff ordinary differential equations, Ph.D Thesis, University of Benin (Unpublished).

Ademiluyi, R. A. and Kayode, S. J., 2001. Maximum order second derivative hybrid multistep methods for integration of initial value problems in ordinary differential equations'. J. Nig. Asso. Phy. . 5: 251-262.

Awoyemi, D. O., 1996. An efficient two step numerical integrator for solving general second Order ordinary differential equation. ABACUS 24(2): 31-43.

Awoyemi, D. O., 1999 A class of continuous methods for general second order initial value Problems in ordinary differential equations. Inter. J. Comp. 72, pp 29-37

Awoyemi, D. O., 2003. A P-stable linear multi-step method for solving general third order ordinary differential equations. Int. J. of Computer Maths, 80(8): 1987-1993.

Awoyemi, D. O., Udo, M. O., and Adesanya, A. O., 2006. Non symmetric collocation method for direct solution of general third order initial value problems of ordinary differential equations, J. Nat and Applied sciences, 7(1): 31-37

Fatunla, S. O., 1988. Numerical methods for IVPs' in ordinary differential Equations. Academic Press Inc, Harcourt brace Jovanovieh Publisher, New York.

Kayode, S. J., 2004. A class of maximal order linear multistep collocation methods for direct solution of second order IVPs' of ordinary differential equations. Ph.D thesis, FUTA (unpublished).

Lambert, J. D., 1973. Computational methods in ODES. John Wiley and Sons, New York.

Olayi, G. A., 1998. A survey of the Numerov methods. Global Journal of Pure and Applied Science 4(2): 175-179

Olayi, G. A., 1999. Consistency, stability and convergence of three-point Numerov type methods. Global Journal Pure and Applied Science (5)3: 409-412.

Olayi, G. A., 2000. Introductory Numerical Methods. A.B.U. Press, Zaria.

Ross, S. L. and Ross, S. L., II., 1989. Intrcduction to ordinary differential equations. John Wiley And sons, New York.

Udo, M. O., Awoyemi, D. O. and Ekpenyong, B., 2007. A class of continuous Linear Multistep Methods for solving special second order ordinary differential equations. J. Resch in Engr, 4 (1): 95-101.

www.google.com." Root condition " A necessary condition.Retrieved from ""http://en.wikipedia.org/wiki/Linear_multistep_method"