GLOBAL JOURNAL OF MATHEMATICAL SCIENNCES VOL. 6, N0. 2, 2007: 111 - 114
 ISSN 1596-6208

A MATHEMATICAL MODEL OF HIV/AIDS TRANSMISSION WITH PREVENTIVE PARAMETER

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(Received 17 July 2007; Revision Accepted 9 October 2007)

ABSTRACT

The formulation of a mathematical model for the transmission of HIV/AIDS by making good assumption on a free non-infected HIV/AIDS population is considered. The existence and uniqueness of solution of the model is established. And the effect of introducing a single infected person into a free HIV/AIDS population is analyzed for a given period of time. The result shows that transmission occurred during contact between infected person and non-infected person, and increases in a ratio to the number of contact. The introduction of a preventive measure decreases infection.

KEYWORD: HIV/AIDS infection, mathematical model, existence and uniqueness, finite approximation.

1.0 INTRODUCTION

The epidemic of the human immune-deficiency virus (HIV) infection in our society is at alarming rate, and the resultant effect is the acquire immune-deficiency syndrome (AIDS) by the body. This rendered the body system incapable of efficient clearing of all kind of infections, (Hansen et al 1985). The consequence of this dreaded disease is of great concerned to all stakeholders in the society and this has encouraged many research works on; courses of the infection, dynamic of the transmission, preventive measures, pathogenesis and treatment of the infection. Infection only occurred if there is contact between infected cell and a non-infected cell (Duffin and Tullis, 2002). The infection is exclusively on the CD4 + T cells (immune cells) such that a drop in the T cells results in an increased in viral load (Duffin and Tullis, 2002).

Santos and Continho (2001), formulated the lymph nodes lattice model that predicts all the three phases of the diseases transmission. The model shows that HIV infected is characterized by, a peak in viral load, following initial infection, a quasi steady state where viral load rises slowly, and finally a dramatic rise in virus and loss of CD4 + T (immune cells) which results in AIDS infection. Kimbir (2005), presented a two-sex version of a mathematical model for the prevention of HIV transmission dynamics in a varying population. The results shows that diseases infection is prevented if sexually active males and females uses preventive measures such as condom, sex abstinence etc.

The understanding of the dynamic between the virus infection and the immune system (pathogenesis) enhanced the procedure for treatment of the disease (Wodarz and Nowak, 2002). A mathematical model was used to understand correlates of long-term immunological control of HIV, and to design therapy requires that convert a progressing patient into a state of long-term non-progression.

Mathematical model have been of great important in the analysis of epidemiological phenomenon. But most of the model suffered from making good assumptions of the operating factors in relation to the subject, which result in inaccurate predictions. (Anderson and May, 1991). The efficiency of a good model lies in its precise prediction capability.

This research is aimed at making good assumptions on a free HIV/AIDS infected population, to formulate HIV/AIDS transmission model. A case of a single infected person introduce into the population, and considering the fact that all members of population are susceptible to the infection. While some susceptible person uses preventive measures such as condoms, sterilization of needles etc.

The establishment of the existence and uniqueness of the solutions of the model is an aided tool in the formulation of the solutions of the model, which enhanced its predictive capability.

2.0 BASIC ASSUMPTION

The following assumptions were made:

1. An HIV/AIDS free population (N) is considered.
2. A single infected person is introduced into the population at time t_0
3. Infection is by contact between infected person (z) and a susceptible person (x).
4. Rate of infection is constrained by a preventive measure (αx) on susceptible persons.

2.1 MATHEMATICAL FORMULATION

The basic model of the virus transmission has three major variables; the population of the infected person (z), the population of the non infected (susceptible) persons (x), the population of non-infected (susceptible) persons who uses preventive measure (y).

2.2 MODEL EQUATION

$$\dot{x} = N - x(d - a) - B\,x\,z$$

$$\dot{y} = ax$$

$$\dot{z} = uz - cz$$

$$x(0) = \frac{N}{d},\ y(0) = y_0,\ z(0) = 1$$

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The non-infected person (x), in the absent of any transmission increases at a constant rate N and die naturally at a rate d. This implies that at a point where there is no contact between infected person and non-infected person, x = N/d. αx measure the rate of prevention of any form of non-infected persons. The infected persons transmit the virus to the non-infected person at a rate proportional to the product of their abundance Bxz. The rate constant B measures the efficacy of the process, (rate of which infected person finding the non-infected person, rate of actual contact and rate of successful transmission). The infected persons increased at a rate uz and are removed or die because of the infection at a rate cz.

We can represent system (1) in its matrix form as,

$$\dot{V}(t) = AV(t) + h(t) \qquad (2)$$
$$V(t_0) = V_0,$$

where

$$\dot{V}(t) = \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{pmatrix},\ A = \begin{bmatrix} -(d-a) & 0 & -\varphi \\ a & & 0 \\ 0 & 0 & u-c \end{bmatrix},\ h = \begin{pmatrix} N \\ 0 \\ 0 \end{pmatrix},\ V(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} and\ V(t)_0 = \begin{bmatrix} x(t_0) \\ y(t_0) \\ z(t_0) \end{bmatrix},$$

and $\varphi = Bx$.

2.3 EXISTENCE AND UNIQUENESS OF SOLUTION OF (2)

Suppose $D = \{(x, y, z, t); x \geq N/d,\ y \geq 0,\ z \geq 1,\ 0 \leq t \leq t_n,\}$
and $H = \{0 < \alpha - d \leq k_1,\ 0 < \varphi \leq k_2,\ 0 < \alpha \leq k_3,\ 0 < a \leq k_4,\ 0 < u - c \leq k_5\}$.

Theorem 1.0

The solution of system (2) with domain defined in D and H, for k_i i = 1, 2, 3, 4, 5 being real constants, exists in the interval $0 < t < t_n$.

Proof;

Let us considered system (2) in the form,

$$\dot{V}(t) = A\,V(x, y, z, t) + h(t),$$
$$V(t_0) = (x_0, y_0, z_0). \qquad (3)$$

Then $V(x, y, z, t)$ is continuous and also integrable on $0 \leq t \leq t_n$.

Assume $V = \eta \exp(\lambda t)$ is a solution of (3), then

$$A\eta = \lambda\eta. \qquad (4)$$

Where η and λ are n–linearly independent eigen vectors and eigen-values of A, such that $A\eta_i = \lambda\eta_i$, i = 1,...n. Introducing η_i into (3) by setting $V = \eta q$ and $\dot{V}(t) = \eta\dot{q}$,, we obtained

$$\eta\dot{q} = A\eta q + h \qquad (5)$$
$$\dot{q} = \eta^{-1} A\eta q + \eta^{-1} h \qquad (6)$$

Let $\eta^{-1} A\eta = D$, (diagonal matrix with λ as the diagonal element), we obtained

$\dot{q} = \lambda_i q + r_i(t)$, where $r_i(t)$ is a linear combination with constant coefficients $h_n(t)$ ...$h_n(t)$. Then

$$q_i(t) = e^{\lambda_i t}\left(\int e^{\lambda_i t} r_i(t)dt + c_i\right) \qquad (7)$$

Equation (7) is the solution of the system (2) on $0 \leq t \leq t_n$.

Theorem 1.1

Let $0 \leq k \leq M$, k is any real constant. Also let matrix A have n-linearly independent eigen vectors, then the solution satisfying the initial

conditions $\begin{pmatrix} x(t_0) \\ y(t_0) \\ z(t_0) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ is unique.

Proof 1.1

Considering system (1) such that,

$$V_1(x_1, y_1, z_1,) = N - x(d - \alpha) - \varphi z$$
$$V_2(x_2, y_2, z_2) = \alpha x$$
$$V_3(x_3, y_3, z_3,) = uz - cz$$
$$x \geq x_0,\ y \geq y_0,\ z \geq z_0. \qquad (8)$$

Then for any real constant k such that $0 \leq k \leq M$, and M is an integer,

$$\frac{\partial vi}{\partial xi} \leq xk,\ \frac{\partial vi}{\partial yi} \leq yk,\ \frac{\partial vi}{\partial zi} \leq zk;\ i = 1,2,3 \qquad (9)$$

holds.

From (9), system (1) satisfies Lipschitz conditions of boundedness, and therefore has a unique solution. Also since system (1) satisfied equation (4), then the evaluation of equation (7) at $\begin{pmatrix} x(t_0) \\ y(t_0) \\ z(t_0) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$, yield the desired unique solution.

3.0 METHOD OF SOLUTION

(Finite Differences Approximation Method)
Considering system (1) as

$$\dot{x} = N - x(d - \alpha) - Bxz$$
$$\dot{y} = \alpha x$$
$$\dot{z} = uz - cz$$

with $x(0) = \dfrac{N}{d}$, $y(0) = y_0$, $z(0) = 1$ (10)

Solving for z in (10)

$$\frac{dz(t)}{dt} = uz - cz. \qquad (11)$$

Let

$$\frac{dz}{dt} = \frac{z_{i+1} - z_i}{h},$$ for h being equal interval between z_{i+1} and z_i.

Substituting the above in (11), we obtained,

$$\frac{z_{i+1} - z_i}{h} = z_i(u - c)$$
$$z_{i+1} - z_i = z_i h(u - c)$$
$$z_{i+1} = z_i + z_i h(u - c)$$
$$z_{i+1} = z_i(1 + hu - hc). \qquad (13)$$
$$\dot{x}(t) = N - x(d - \alpha) - Bxz.$$

While $\dfrac{x_{i+1} - x_i}{h} = N - x_i(d - \alpha) - Bx_i z_i$

$$x_{i+1} - x_i = hN + x_i(d - \alpha) - Bx_i z_i$$

implies $x_{i+1} = hN + x_i(1 - h(d - \alpha) - hBz_i)$

$$\dot{y}(t) = \alpha x,$$
$$\frac{y_{i+1} - y_i}{h} = \alpha x_i$$

implies $y_{i+1} = h\alpha x_i + y_i$ (14)

3.1 ANALYSIS

Recall the following constants:

N	=	abundant of non-infected
d	=	rate of death of non-infected
B	=	contact rate between infected and non-infected
α	=	the rate of using preventive measures on non-infected
u	=	rate of increased of infection
c	=	rate of death or removal of infected

We assume and vary parameters for four difference cases:

Case 1: (no interaction between infected and non infected persons).
B = 0, d = 0.1, α = 0, u = 0, c = 0, N = 3, h = 0.1

Case 2: (there is contact between infected and non-infected persons, there is no death or removal of infected persons, and no used of preventive measured).
B = 0, d = 0.1, α = 0, u = 0.4, c = 0, N = 3, h = 0.1

Case 3: (there is contact between infected persons and non-infected persons, death or removal of infected persons occurred, but there is no used of preventive measured by non-infected persons).
B = 0.2, d = 0.1, α = 0, u = 0.4, c = 0.2, N = 3, h = 0.1

Case 4: (there is contact between infected persons and non-infected persons, death or removal of infected persons occurred, there exists the used of preventive measured)
B = 0.2, d = 0.1, α = 0.3, u = 0.4, c = 0.2, N = 3, h = 0.1

Using equation (12), (13) and (14), we generate the following table for each of the case above.

Table

T	z_1	z_2	z_3	z_4	y_1	y_2	y_3	y_4	x_1	x_2	x_3	x_4
0	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	30.0000	30.0000	30.0000	30.0000
0.1	1.0000	1.0400	1.0200	1.0200	0.0000	0.0000	0.0000	0.9000	30.0000	29.4000	29.4000	30.3000
0.2	1.0000	1.0820	1.0404	1.0404	0.0000	0.0000	0.0000	1.8090	30.0000	28.7900	28.8062	30.5879
0.3	1.0000	1.1250	1.0617	1.0612	0.0000	0.0000	0.0000	2.7266	30.0000	28.1790	28.2187	30.8631
0.4	1.0000	1.1700	1.0854	1.0824	0.0000	0.0000	0.0000	3.6524	30.0000	27.5630	27.6377	31.1254
0.5	1.0000	1.2170	1.1040	1.1040	0.0000	0.0000	0.0000	4.5861	30.0000	26.9420	27.0630	31.3740
0.6	1.0000	1.2660	1.1261	1.1261	0.0000	0.0000	0.0000	5.5273	30.0000	26.3180	26.4948	31.6087
0.7	1.0000	1.3170	1.1486	1.1486	0.0000	0.0000	0.0000	6.4755	30.0000	25.6890	25.9331	31.8290
0.8	1.0000	1.3700	1.1716	1.1716	0.0000	0.0000	0.0000	7.4304	30.0000	25.0560	25.3780	32.0345
0.9	1.0000	1.4250	1.1950	1.1950	0.0000	0.0000	0.0000	8.3914	30.0000	24.4190	24.8296	32.2246
1.0	1.0000	1.4820	1.2189	1.2189	0.0000	0.0000	0.0000	9.3561	30.0000	23.7790	24.2879	32.55 70

3.2 DISCUSSION OF RESULT

In the table, it is observed that when the contact rate is zero (even when an infected person has been introduced into the population), the infected person (z_1) and the non-infected person (x_1) remained constant as at initial stage all the time. When there is contact (B = 0.2), the infected person (z_2) increases with time and the non-infected persons (x_2) decreases. In cases 3 and 4, because some of the infected persons (z) are dying or removed at rate c = 0.2, the infected persons (z_3 and z_4) increases at the same rate but less than case 2, while the non-infected person (x_3) decreases but not as in x_2, still because of the dying or removal rate of infected person. The case of x_4 differs from x_1, x_2 and x_3 because of the preventive measure, which increases the number of non-infected persons (x) as well as the population. This difference is seen in y_4.

3.3 CONCLUSION

In this paper, a mathematical model of HIV/AIDS transmission was formulated. The establishment of existence and uniqueness theory for the solution of the model is an aided tool in the analysis. The effect of introducing an individual with the virus into an HIV/AIDS free population was observed for various cases. Result shows that when there is no contact between infected person and non-infected person, transmission of the virus remained zero. Also, when there is contact between infected person and non-infected person, transmission increases and the number of person infected increases in a ratio to the number of contacts made, which is the opposite of the non-infected. The introduction of preventive measured increases the number of non-infected persons. The results obtained conformed to the reality of diseases transmission model as reported in (Anderson and May 1991).

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