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ANALYSING WAREHOUSE PROBLEM HAVING PERIODIC NONNEGATIVE EXCESS DEMAND AND NO INITIAL STOCKS.

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ABSTRACT

In this paper we have examined the warehouse problem in which the warehouse has a fixed capacity with no initial stock and supply and demand are periodically carried out for a finite time horizon. In addition to the usual constraints in the linear programming (LP) formulation of the warehouse problem, we included the periodic excess demand constraints in the LP formulation, which by a theorem reduced the LP problem to a more sparse LP problem. The resulting sparse LP problem has the observed advantage of less computational time when solving either manually or by computer. The sparse LP problem is numerically illustrated and solved using a computer program. The results show that in the LP problem augmented with excess demand constraints, no items are supplied and sold in periods when the unit selling price is lower than the unit cost price. We also observed that the level of effectiveness which is measured by the objective function is much lower than that of LP problem which is not augmented with excess demand constraints. This agrees with what generally happens when additional constraints are incorporated into a LP problem.

KEYWORDS: Warehouse problem, inventory, linear programming and computer programming.

1. INTRODUCTION

The management of warehouse is directly connected to both the transportation model and inventory control. Ignizio (1982) stated that the traditional transportation problem is concerned with the movement of a homogeneous product from several supply nodes to several destination nodes. Takakuwa (1998) and Takakuwa and Fujii (1999) referred to both the supply and destination nodes as warehouses. Warehouses are very important in the control of inventory and their treatment in relation to inventory theory is the focus of this paper. Therefore, a good understanding of inventory is essential for the consideration of the warehouse problem. Kothari (1982) defined inventory as the physical stock of goods in the warehouse which when it is too little (Taha, 2002) results in expensive opportunity cost in the form of emergency production for accelerated service or loss of goodwill and profit from potential sales. The simplest way of satisfying customers is to hold large stocks. But holding large stocks according to Shroeder (1981) and Ekoko (1999) will mean high inventory-carrying charges (such as storage, deterioration, pilferage and insurance) and possible losses caused by price declines. Kootz and Weigrich (1988) and Agbadudu (1996) identified the following as some of the benefits of inventory control: Procurement of goods in economic quantities to attact discount and reduced transportation cost; elimination of delays in production caused by the non availability of required materials; and checking the over accumulation of inventories so that investment is consistent with production requirements.

The stocks in the warehouse could be raw materials, semi-finished and finished goods, Riggs (1970) and Williams (1988). It should be understood that the finished goods of one organization could be raw materials of another firm. For example, planks which are the finished goods of a sawmiller are the raw material inventory of a furniture maker, Unugbro (1994).

The commonest inventory question most researchers have always addressed over the years in the literature is: What is the optimal quantity to be produced or ordered that will minimize total inventory cost, Hillier and Lieberman (2001). But as pointed out in Kothari (1982), "New and new techniques are helping to answer inventory questions." Hence the unique inventory question in this paper is what quantities of goods are to be ordered and sold from the warehouse in each period in order to maximize total profit. The solution is obtained using a computer program. The types of inventory costs are discussed in Lucey (1996) and Evertte and Ronald (1978) as well as other literature. But Gandner and Deannenbrig (1979) described these costs as "serious gap that exists between theory and practice in inventory management", because the holding and shortage costs typically assumed in the theory are difficult, if not impossible to measure in practice. The warehouse problem in this paper has no such difficulties as we seek for the optimal ordering and selling quantities of the periods that will maximize total profit. Furthermore, the warehouse problem under consideration is assumed to have no initial quantity with nonnegative periodic excess demand. The inclusion of these assumptions resulted in a more sparse linear programming problem, Ekoko (2005).

2. THE WAREHOUSE PROBLEM AND ITS FORMULATION AS A LP PROBLEM

A warehouse is a secured house or place where goods that are produced or ordered are possibly kept and preserved until they are sold. A warehouse also serves a secondary purpose as depot from where retailers can buy their wares for sale to their customers. Thus the warehouse problem is the determination of the periodic quantities of items ordered and sold that will maximize the total profit.

We now proceed to formulate the warehouse problem as a LP problem. Let E *and* e be the capacity

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of the warehouse and initial quantity of items in the warehouse respectively. Furthermore, let x_j denote the quantity of items ordered in period j $(j = 1(1)n)$ at a unit ordering cost of c_j while x'_j is the quantity of items sold in period j at a unit selling price of c'_j. Given that E *and* e are known and demand is periodic in excess, we seek to determine the optimal quantity of x_j *and* x'_j that will maximize the total profit of operating the warehouse. Before continuing we state the following assumptions of the model.

Some assumptions

(a) We can order for items and sell items at any time period subject to space and commodity availability respectively in the warehouse.

(b) During each small time period the ordering cost and selling price are constant and may be different from those of another period.

(c) The warehouse problem is to be examined for a given length of time made up of n finite time periods.

(d) For items of social services that are essential, there is a legislature that in each period prices of the items should not be increased even when the items are in short supply.

The warehouse problem has two linear constraints, which are based on periodic excess demand. By excess demand, we refer to the expression $(x'_j - x_j) \geq 0$.

The first constraint based on excess demand stipulates that the sum of the first i periodic excess demand should not exceed the initial quantity in the warehouse. Mathematically, this can be stated as:

$$\sum_{j=1}^{i}(x'_j - x_j) \leq e, \quad i = 1(1)\,n \qquad (1)$$

The second constraint states that the quantity ordered during the ith period cannot exceed the leftover space in the warehouse after the sum of excess demand from the first $(i-1)$ periods has been satisfied. Mathematically this can be stated thus:

$$x_i \leq E - \left[e - \sum_{j=1}^{i-1}(x'_j - x_j)\right]$$

This simplifies to

$$\sum_{j=1}^{i} x_j - \sum_{j=1}^{i-1} x'_j \leq E - e, \qquad i = 1(1)n \qquad (2)$$

Where it is understood that the second summation in equation (2) does not exist when $i = 1$, so that $x_1 \leq E - e$.

Since we cannot order for or sell negative quantities, it is clear that

$$x_j \geq 0, \; x'_j \geq 0 \qquad (3)$$

The total profit from all the n periods is

$$\sum_{j=1}^{n}(c'_j x'_j - c_j x_j) \qquad (4)$$

(1) - (4) constitute a LP problem which is stated thus:

$$\left.\begin{array}{l} Maximize\ z = \sum_{j=1}^{n}(c'_j x'_j - c_j x_j) \\ subject\ to \\ \sum_{j=1}^{i}(x'_j - x_j) \leq e \quad or \sum_{j=1}^{i} x'_j - \sum_{j=1}^{i} x_j \leq e, \ (i = 1(1)n) \\ -\sum_{j=1}^{i-1} x'_j + \sum_{j=1}^{i} x_j \leq E - e, \ (i = 1(1)n) \\ \quad x'_j \geq 0, \ x_j \geq 0 \end{array}\right\} \qquad (5)$$

The system (5) is the LP model of the warehouse problem. The LP model in (5) has $2n$ linear constraints, $2n$ nonnegativity constraints in $2n$ variables. Further simplification of (5) yields the system in (6).

$$\left.\begin{array}{llllllllll}
Max\ z = & c'_1x'_1 + c'_2x'_2 + c'_3x'_3 + \cdots + c'_nx'_n - c_1x_1 - c_2x_2 - c_3x_3 - \cdots - c_nx_n & & & & & & & & \\
s.t. & & & & & & & & & \\
& x'_1 & & & & -x_1 & & & & \leq e \\
& x'_1 & +x'_2 & & & -x_1 & -x_2 & & & \leq e \\
& x'_1 & +x'_2 & +x'_3 & & -x_1 & -x_2 & -x_3 & & \leq e \\
& \cdots & & & & & & & & \\
& x'_1 & +x'_2 & +x'_3 + \cdots + & x'_n & -x_1 & -x_2 & -x_3 - \cdots & x_n & \leq e \\
& & & & & x_1 & & & & \leq E - e \\
& -x'_1 & & & & +x_1 & +x_2 & & & \leq E - e \\
& -x'_1 & -x'_2 & & & +x_1 & +x_2 & +x_3 & & \leq E - e \\
& \cdots & & & & & & & & \\
& -x'_1 & -x'_2 - \cdots - & x_{n-1} & & +x_1 & +x_2 + \cdots & & +x_n & \leq E - e \\
& x'_1, x'_2, \cdots, x'_n, x_1, x_2, \cdots, x_n \geq 0 & & & & & & & &
\end{array}\right\} \qquad (6)$$

The optimal solution of the LP model is also the optimal solution of the warehouse problem.

3. Imposition of Additional Constraints on the Warehouse Problem.

The two requirements to be added to the LP problem in (6) are:

(c)

(a) that the excess demand should be nonnegative periodically i.e. $(x_i' - x_i) \geq 0$

(b) the warehouse initial quantity should be zero i.e. $e = 0$

So we can augment the LP problem in (6) with these added constraints as follows:

$$Max\ z = c_1'x_1' + c_2'x_2' + c_3'x_3' + \cdots + c_n'x_n' - c_1x_1 - c_2x_2 - c_3x_3 - \cdots - c_nx_n \qquad 7(a)$$

s.t.

$$x_1' - x_1 \leq 0 \qquad 7(b_1)$$

$$x_1' + x_2' - x_1 - x_2 \leq 0 \qquad 7(b_2)$$

$$x_1' + x_2' + x_3' - x_1 - x_2 - x_3 \leq 0 \qquad 7(b_3)$$

....

$$x_1' + x_2' + x_3' + \cdots + x_n' - x_1 - x_2 - x_3 - \cdots\cdots - x_n \leq 0 \qquad 7(b_n)$$

$$x_1 \leq E \qquad 7(d_1)$$

$$-x_1' + x_1 + x_2 \leq E \qquad 7(d_2)$$

$$-x_1' - x_2' + x_1 + x_2 + x_3 \leq E \qquad 7(d_3)$$

....

$$-x_1' - x_2' - \cdots\cdots\cdots - x_{n-1} + x_1 + x_2 + \cdots\cdots\cdots + x_n \leq E \qquad 7(d_n)$$

$$x_1' - x_1 \geq 0 \qquad 7(f_1)$$

$$x_2' - x_2 \geq 0 \qquad 7(f_2)$$

$$+x_3' - x_3 \qquad 7(f_3)$$

....

$$x_n' - x_n \geq 0 \qquad 7(f_n)$$

$$x_1', x_2', \cdots, x_n', x_1, x_2, \cdots, x_n \geq 0 \qquad 7(g)$$

In correspondence with the specifications in Ekoko (1999), equation 7(a) is called the objective function, which is the measure of effectiveness. Equations $7(b_1)$ - $7(f_n)$ constitute the set of linear constraints while 7(g) is the set of nonnegativity constraints. Specifically, equations $7(f_1)$ - $7(f_n)$ constitute the n periodic nonnegative excess demand requirements. The warehouse problem in LP form which is expressed in 7(a) - 7(g) is compared to the former LP in (6) as follows: The objective functions and nonnegativity constraints are the same in both LP problems. The initial warehouse quantity, e is specified to be zero in the latter LP problem. By the addition of n extra nonnegativity excess demand constraints to the latter LP problem the latter LP problem has a total of $3n$ linear constraints (as against $2n$ in the former) in $2n$ variables.

Before continuing, let us state and prove a theorem which is based on latter LP problem in 7(a) - 7(g).

Theorem

Given that the warehouse has no initial quantity $(\text{i.e. } e = 0)$ and that the periodic excess demand is nonnegative $i.e.$ $(x_i' - x_i) \geq 0$ then,

(i) $x_i' = x_i$

(ii) $0 \leq x_i \leq E \quad and \quad 0 \leq x_i' \leq E$

and (iii) The LP problem in 7(a) - 7(g) can be reduced to have only n variables, x_i which are the periodic quantities sold.

Proof

The proof of the theorem is given as follows:

From equation $7(b_1)$ and $7(f_1)$, we have

$$x_1' - x_1 = 0 \qquad 8(a_1)$$

$$i.e.\ x_1' = x_1$$

Substituting equation $8(a_1)$ into equation $7(b_2)$ and considering equation $7(f_2)$, we have

$$x_2' - x_2 = 0 \qquad 8(a_2)$$

$$i.e.\ x_2' = x_2$$

Continuing this way, we have

$$x_n' - x_n = 0 \qquad 8(a_n)$$

$$i.e.\ x_n' = x_n$$

From equation $7(d_1)$

$$x_1 \leq E \qquad 9(a_1)$$

By substituting equation 8(a_1) into equation 7(d_2), we have

$$x_2 \le E \qquad 9(a_2)$$

Similarly, substituting 8(a_1) and 8(a_2) into 7(d_1), we have

$$x_3 \le E \qquad 9(a_3)$$

And continuing this way, by substituting equation 8(a_1) - 8(a_{n-1}) in equation 7(d_n), we have

$$x_n \le E \qquad 9(a_n)$$

Since $x'_j = x_j \ (\forall \ j = 1(1)n)$, the objective function can be expressed only in terms of the x_j variables and the LP problem in 7(a) - 7(g) is now reduced to:

$$\left.\begin{array}{l} Max \quad z = (c'_1 - c_1)x_1 + (c'_2 - c_2)x_2 + \cdots + (c'_n - c_n)x_n \\ s.t. \\ \qquad x_1 \le E \\ \qquad x_2 \le E \\ \qquad x_3 \le E \\ \qquad \vdots \quad \vdots \quad \vdots \\ \qquad x_n \le E \\ x_1, x_2, \cdots, x_n \ge 0 \end{array}\right\} \qquad (10)$$

This completes the required proof and we proceed to numerically illustrate the system (10) in section 4

4. Numerical Illustration

The unit cost prices and unit selling prices of a commodity for five time periods are tabulated as follow:

Period	1	2	3	4	5
c_j (₦)	25	25	25	35	45
c'_j (₦)	20	35	30	25	50

If initially there are no items in the warehouse which has capacity for 200 units, determine the quantities to be ordered and sold that will maximize the total profit from all the periods.

Solution

The LP form of the warehouse problem is as follows:

$$Max \ z = -5x_1 + 10x_2 + 5x_3 - 10x_4 + 5x_5$$

$s.t.$

$$x_1 \le 200$$
$$x_2 \le 200$$
$$x_3 \le 200$$
$$x_4 \le 200$$
$$x_5 \le 200$$
$$x_1, x_2, \cdots, x_5 \ge 0$$

The computer program in PASCAL, which is used to solve the numerical problem is in fig. 1 while the initial and optimal tableaux of the solution are in fig. 2.

Program

```
PROGRAM Simplex(input,output);
CONST
  n=20; m=10;          {No. of variables and constraints }               {**}
  ncols=21;            {Maximum no. of columns in tableau }              {**}
  fwt=8; dpt=2;        {Output format constants for tableau values}      {**}
  fwi=1;               {Output format constant for indices }             {**}
  largevalue = 1.0E20;    smallvalue=1.0E-10;
TYPE  mrange = 1..m; ncolsrange = 1..n;
   matrix = ARRAY [mrange,ncolsrange] OF real;
   column = ARRAY [mrange] OF real;
   baseindex = ARRAY [mrange] OF integer;
   row = ARRAY [ncolsrange] OF real;
   rowboolean = ARRAY [ncolsrange] OF boolean;
VAR
   a : matrix;              { Matrix A in Standard form of problem }
   b : column;              { Vector b in standard form of problem }
   c : row;                 { Coefficients of objective function }
```

```
    basic : baseindex;          { Basic variables at each stage }
    nonbasic : rowboolean;      { Status indicators for variables }
    z0 : real;                  { Value of objective function }
    it : integer;               { Iteration counter }
    solution,
    unbounded : boolean;        { Iteration process terminators }
    r, s : integer;             { Row and column of pivot element }
    f3,f4:text;
    PROCEDURE inputdata;
    VAR i,j :integer;
    BEGIN
      FOR i := 1 TO m DO
      BEGIN FOR j := 1 TO n DO read(f3,a[i,j]); read(f3,b[i]) END;
      FOR j := 1 TO n DO read(f3,c[j]);  read(f3,z0);
      FOR i := 1 TO m DO read(f3,basic[i]);
    END; { Inputdata }
PROCEDURE initialise;
   VAR i,j :integer;
   BEGIN it := 0; solution := false; unbounded := false;
     FOR j := 1 TO n DO nonbasic[j] := true;
     FOR i := 1 TO m DO nonbasic[basic[i]] := false;
     END; { Initialise }
   PROCEDURE outputtableau;
   VAR i, j :integer;
   BEGIN   writeln(f4); writeln(f4,'     ITERATION', it: 2);
     write(f4,'     BASE VAR.   ', ' ' :fwt-5, 'VALUE');
     FOR j := 1 TO n DO write(f4,' ':fwt-fwi-1, 'X', j:fwi); writeln(f4);
     FOR i := 1 TO m DO
     BEGIN
       write(f4,' ':8-fwi,'X',basic[i]:fwi,' ':8,b[i]:fwt:dpt);
       FOR j := 1 TO n DO write(f4,a[i,j]:fwt:dpt); writeln(f4)
       END;
       write(f4,' ':7,' Z',' ' :8,z0:fwt:dpt);
       FOR j := 1 TO n DO write(f4,c[j]:fwt:dpt); writeln(f4)
     END; {outputtableau }
    PROCEDURE nextbasicvariable (VAR r,s: integer);
  VAR i, j :integer; min : real; unbounded : boolean;
  BEGIN  min := largevalue; { Find the variable, s, }
    FOR j := 1 TO n DO      { to enter the basis.   }
      IF nonbasic[j] THEN IF c[j] < min THEN BEGIN min := c[j]; s := j END;
      solution := c[s] > - smallvalue;
      IF NOT solution THEN
      BEGIN unbounded := true; i:= 1;      { Check that at least one value }
        WHILE unbounded AND (i <= m ) DO  { in column s is positive.      }
        BEGIN unbounded := a[i,s] < smallvalue; i:= i + 1  END;
        IF NOT unbounded THEN
        BEGIN min := largevalue;   { Find the variable, basic[r],  }
        FOR i := 1 TO m DO         { to leave the basis.           }
          IF a[i,s] > smallvalue THEN
            IF b[i]/a[i,s] < min THEN BEGIN min := b[i]/a[i,s]; r := i END;
        nonbasic[basic[r]] := true; nonbasic[s] := false; basic[r] := s; writeln(f4);
        writeln(f4,'      PIVOT IS AT ROW ', r:fwi, ' COL ', s:fwi)
      END
    END
 END; { nextbasicvariable }
     PROCEDURE transformtableau ( r,s: integer);
 { Construct the new canonical form, implementing }
 VAR i,j :integer; pivot, savec : real; savecol : column;
 BEGIN
   FOR i := 1 TO m DO savecol[i] := a[i,s]; savec := c[s]; pivot := a[r,s];
   b[r] := b[r]/pivot;
   FOR j := 1 TO n DO  a[r,j] := a[r,j]/pivot;
   FOR i :=1 TO m DO
     IF i <> r THEN
     BEGIN b[i] := b[i] - savecol[i] * b[r];
       FOR j := 1 TO n DO a[i,j] := a[i,j] - savecol[i] * a[r,j]
```

```
        END;
        FOR j: = 1 TO n DO c[j]:= c[j] - savec * a[r,j];
        z0 := z0 - savec * b[r]; it := it + 1;
    END; {transformtableau }

    BEGIN {Main Program }
    assign(f3,'ekoko.in'); reset(f3);
    assign(f4,'outdata4.out'); rewrite(f4);
      writeln(f4); writeln(f4,'     SIMPLEX METHOD'); writeln(f4);
      inputdata; initialise;
      REPEAT
        outputtableau;
        nextbasicvariable(r,s);
        IF NOT (solution OR unbounded) THEN transformtableau(r,s)
         UNTIL solution OR unbounded;
         { Output results } writeln(f4);
         IF unbounded THEN writeln(f4,'    VARIABLE',s:fwi, ' IS UNBOUNDED')
         else writeln(f4,'    MAXIMUM AT Z = ', z0:fwt:dpt);
         close(f3); close(f4);
         END. {Simplex }
```

Fig. 1: Program Simplex

SIMPLEX METHOD

Initial Tableau

BASE VAR.	VALUE	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
X6	200.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
X7	200.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
X8	200.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
X9	200.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00
X10	200.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00
Z	0.00	5.00	-10.00	-5.00	10.00	-5.00	0.00	0.00	0.00	0.00	0.00

Optimal Tableau

BASE VAR.	VALUE	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
X6	200.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
X2	200.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
X3	200.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
X9	200.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00
X5	200.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00
Z	4000.00	5.00	0.00	0.00	10.00	0.00	0.00	10.00	5.00	0.00	5.00

Fig. 2: Initial and Optimal Tableaux

Where $X_1, X_2, \cdots, X_5$ represent $x_1, x_2, \cdots, x_5$ respectively and $X_6, X_7, \cdots, X_{10}$ represent $x_6, x_7, \cdots, x_{10}$ respectively. The slack variables of the LP problem are $X_6, X_7, \cdots, X_{10}$ for 1st, 2nd,..., 10th linear constraints respectively.
Hence the optimal solution is given as:

$x_1 = x_1' = 0$, $x_2 = x_2' = 200$, $x_3 = x_3' = 200$ $x_4 = x_4' = 0$.

and the maximum total profit $=$ N4,000.00

5. DISCUSSION

By the theorem and as expressed in equations 8(a_1) - 8(a_n) the quantity ordered and the quantity sold in each period should be equal. This is as a result of the requirement that there should be no initial stock and that the periodic excess demand should be nonenegative.

The solution to the numerical example show that both the ordering and selling quantities of the 1st and 4th periods are zero. This can be explained from the data in the table of the numerical illustration where in the 1st and 4th periods there are always losses since $c'_i < c_i$ with $x'_i = x_i$. And during such periods it is better not to operate the warehouse. The ordering quantity and selling quantity are both equal to the capacity of the warehouse in the 2nd, 3rd and 5th periods. This means that during periods 2, 3 and 5 the warehouse should be operated to full capacity because $c'_i > c_i$. In each of these three periods, profits are made and the total profit is

$\sum(c'_i x'_i - c_i x_i) =$ ₦4,000.00 for $j = 2, 3$ *and* 5.

This profit agrees with the maximum total profit obtained from the computer implementation of the solution.

6. CONCLUSION

The LP model of the general warehouse problem has $2n$ variables, $2n$ linear constraints and $2n$ nonnegativity constraints. By the introduction of no initial quantity and nonnegativity excess demand as requirements the LP model is now reduced to n variables, n linear constraints and n nonnegativity constraints. This reduced LP model of the modified warehouse problem is more sparse than the LP model of the general warehouse problem. By the advantage of sparsity property, the computer stores only non-zero coefficients and arithmetic calculations involving many zeros are often more quickly done by first testing for the presence or absence of a zero before doing the arithmetic operations of addition, subtraction or multiplication. And based on (i) in the theorem $(i.e.\ x'_i = x_i)$, we observed from the solution of the numerical illustration that the warehouse should not be operated during periods whose selling price (c'_i) is less than ordering cost (c_i) and the warehouse should be operated to full capacity during periods whose selling price (c'_i) is more than the ordering cost (c_i).

REFERENCES

Agbadudu, A. B., 1996. Elementary Operations Research, Vol. 1, A. B. Mudiaga Ltd. Benin City.

Ekoko, P. O., 1999. Basic Operations Research for Science and Social Sciences, United City Press. Benin City.

Ekoko, P. O., 2005. Expanding the Scope of Usage of Computer in Solving Practical LP Problem, A.M.S.E. Vol. 42, (2): 59-67.

Evertte, E. A. and Ronald. J. E., 1978. Production and Operations Management, Prentice-Hall.New Jersey

Gandner, E. S. and Deannenbrig, D. G., 1979. Analysing Aggregate Inventory Trade Off, Management Science. Vol. 25. (2): 189- 197.

Hillier, F. S. and Lieberman, G. J., 2001. An Introduction to Operations Research. Holden Day. San Francisco.

Ignizio, J. P., 1982. Linear Programming in Single and Multiple-Objective Systems Prentice-Hall, Inc. New Jersey.

Kootz, H. and Weihrich, H., 1988, Management, 9th Edition,: McGraw-Hill Book Co. San Francisco

Kothari, C. R., 1982. An Introduction to Operations Research,: Vikas Publishing House PVT Ltd. New Delhi

Lucey, T., 1996. Quantitative Techniques. London: ELST.

Riggs, J. L., 1970. Production Systems: Planning, Analysis, and Control,: John Wiley and Sons, Inc. London

Shroeder, P. G., 1981. Operations Management, McGraw-Hill, Inc. Toronto

Taha, H. A., 2002. Operations Research an Introduction. Prentice-Hall, Inc. New Jersey.

Takakuwa, S., 1998. A Practical Module-Based Simulation Model for Transportation-Inventory System. In Proceedings of the 1998 Winter Simulation Conference, ed. D.J. Medeiros, E.F. Watson, J.S. Carson, and M.S. Manivannan, 1239-1246. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers.

Takakuwa, S. and Fujii, T., 1999. A Practical Module-Based Simulation Model for Transshipment-Inventory Systems. In Proceedings of the 1999 Winter Simulation Conference, ed. P.A. Farrington, H.B. Nembhard, D.T. Sturrock, and G.W. Evans, 1324-1332, Institute of Electrical and Electronics Engineers. Piscataway, New Jersey

Unugbro, A. O., 1994. Principles of Economics and Business," Aniko Printers, Benin City.

Williams, A., 1986. Quantitative Methods for Business, West Publishing Co. California.