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FORECASTING PERFORMANCE OF A SEASONAL BILINEAR ARIMA TIME SERIES MODEL

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ABSTRACT

In this paper we concentrated on the seasonal bilinear model:

$$X_t = \alpha X_{t-s} + \beta e_{t-s} + \gamma X_{t-s} e_{t-s} + e_t \qquad (1.5)$$

where X_t, $t \in Z$ and e_t, $t \in Z$ are two stochastic processes defined on some probability space (Ω, $\mathcal{F}$, P), where Z = (..., -1, 0, 1, ...), α, β, γ are the parameters of the model whereas αX_{t-s}, βe_{t-s} and $\gamma X_{t-s} e_{t-s}$ are the autoregressive (AR) part, moving average (MA) part and the bilinear part respectively, which had been proved to be both stationary and invertible by Iwueze (1986) and Iwueze and Chikezie (2005). Furthermore, we adopted the Box-Jenkins procedure and generated forecasts for one period; one-step, two-step, three-step and four-step ahead. Results obtained gave strong indications of good and fine forecasts as they are not far away from the original values.

KEY WORDS: seasonal bilinear models, Box-Jenkins procedure, forecasts.

1. INTRODUCTION

Time series models are important in that they provide forecasts of future behaviour of a given phenomenon. Sometimes, the performance of a time series model is judged on the basis of its forecasting performance. Suppose that X_t, $t \in Z$ is a discrete parameter time series and when at time $t = t_0$, a forecast is required of the future value X_{t_0+k}. Such a forecast has to be based on the past and present of the series, ie X_s, $s \leq t_0$. Denote the forecast made at time $t = t_0$ for k-steps ahead by $X_{t_0}(k)$. The forecast error is defined by

$$e_{t_0}(k) = X_{t_0}(k) + k - X'_{t_0}(k) \qquad (1.1)$$

while the k-step forecast error variance or expected mean square error is defined by

$$\sigma^2_e(k) = E[X_{t_0}(k) + k - X_{t_0}(k)]^2 / n \qquad (1.2)$$

Then it is well known that $\sigma^2_e(k)$ is minimum if and only if

$$X_{t_0}(k) = E[X_{t_0}(k) + k / X_s(k)], \quad s \leq t_0 \qquad (1.3)$$

2. THEORETICAL FRAMEWORK AND METHODOLOGY

Linear time series models are widely used in many fields because these models can be analysed with considerable ease and they provide fairly good approximations for the true underlying generating random process. However, the underlying structure of the series may not be linear and what is more, the series may not be Gaussian. In these situations, second-order properties, such as covariances and spectra, can no longer adequately characterize the properties of the series and one is led then to consider non-linear models which can provide a better fit. A particular class of non-linear models which has been found to be useful in many fields is the bilinear models. Bilinear models have been extensively discussed in the control theory literature, one could check Ruberti, Isidori and d'Alessandro (1972) and Brunni, Duppilo and Koch (1974) for further details. Until recently, the theory of bilinear models dealt with the structural theory of deterministic bilinear differential equations, that is more mathematical (exactness), without the uncertainty of the error term e_t. The study of bilinear models as stochastic models was initiated by Granger and Andersen (1978) and Subba Rao (1981).

Let e_t, $t \in Z$ be a sequence of independent and identically distributed random variables with $E(e_t) = 0$ and $E(e_t^2) = \sigma^2 < \infty$. Let $a_1, a_2, \ldots, a_p$; $c_1, c_2, \ldots, c_q$ and b_{ij}, $1 \leq i \leq m$, $1 \leq j \leq k$ be real constants. The general form of the bilinear model, as defined in Granger and Andersen (1978) is:

$$X_t = \sum_{j=1}^{p} a_j X_{t-j} + \sum_{j=1}^{q} c_j e_{t-j} + \sum_{i=1}^{m} \sum_{j=1}^{k} b_{ij} X_{t-i} e_{t-j} + e_t \qquad (1.4)$$

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for every $t \in Z$. If X_t, $t \in Z$ satisfies (1.4), Subba Rao (1981) uses the notation that X_t, $t \in Z$, is BL (p, q, m, k) where BL is the abbreviation for bilinear model.

Various simple forms of (1.4) are discussed in the literature by the following authors: Granger and Andersen (1978); Subba Rao (1981); Pham and Tran (1981); Subba Rao and Gabr (1981); Tong (1981); Quinn (1982); Bhaskara Rao, Subba Rao and Walker (1983); Akamanam (1983), Akamanam, Bhaskara Rao and Subramanyam (1986).

For the purposes of this paper we are going to concentrate on the seasonal bilinear model:

$X_t = \alpha X_{t-s} + \beta e_{t-s} + \gamma X_{t-s} e_{t-s} + e_t$ (1.5)

where X_t, $t \in Z$ and e_t, $t \in Z$ are two stochastic processes defined on some probability space $(\Omega, \mathcal{F}, P)$, where $Z = (\ldots, -1, 0, 1, \ldots)$, α, β, γ are the parameters of the model whereas αX_{t-s}, βe_{t-s} and $\gamma X_{t-s} e_{t-s}$ are the autoregressive (AR) part, moving average (MA) part and the bilinear part respectively, which has been proved to be both stationary and invertible by Iwueze (1986) and Iwueze and Chikezie (2005).

On forecasting procedures, it must be stated according to Chatfield (1980) that there are several forecasting procedures which include Intuitive or judgmental forecasting - this is quite subjective. We also have methods of Extrapolation of trend curves. Exponential smoothing Box and Jenkins forecasting procedure and others.

For the purposes of this paper, we are going to adopt the Box-Jenkins procedure and generate forecasts for one period; one-step, two-step, three-step and four-step ahead forecasts. When the adequacy of the residuals obtained is not in doubt, then applying Box and Jenkins method, meaningful forecasts could be obtained. It should be noted that this was the case for this work, hence, the application of Box and Jenkins method.

For a bilinear model of the form (1.5), formula (1.3) can be used to form forecasts, provided the model is invertible. Our rule for forming forecasts is as follows: "write down the equation for X_{t_0+k}, everything on the right-hand side that has already occurred at time t_0 is given its observed value, any thing that is yet to occur is replaced by its conditioned expectation" {Box and Jenkins (1976)}.

For model (1.5), we obtain

$$X_{t_0+k} = \alpha X_{t_0+k-s} + \beta e_{t_0+k-s} + \gamma X_{t_0+k-s} + e_{t_0+k-s} + e_{t_0+k} \quad (1.6)$$

Applying rule (1.6), the forecasting function is :

$$\hat{X}_{t0}(k) = \begin{cases} \alpha X_{t_0+k-s} + \beta \hat{e}_{t_0+k-s} + \gamma X_{t_0+k-s} + \hat{e}_{t_0+k-s} \\ \mu, \quad k > s \end{cases} \quad (1.7)$$

3. NUMERICAL ILLUSTRATION

The program for the computation of the parameters of the bilinear time series model (1.5) is written in Fortran Language. With good initial estimates obtained from the six regions described by Nwogu and Iwueze (2003) and Chikezie and Iwueze (2005), [Describes the values which the parameters of Model (1.5) can take within the unit circle divided into six regions thus.

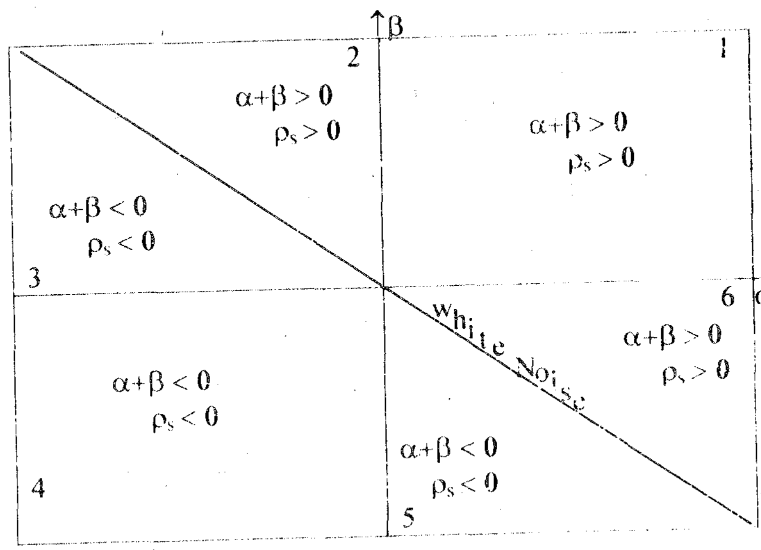

Figure 1.1: Autocorrelations(ρ_s) for various values of α, β, γ.

It was discovered that the sign of γ do not affect the value and sign of ρ_s] reliable estimates of the parameters of model (1.5) could be obtained. Using these estimated parameters of the model, we attempted to forecast the last four values of the series in three regions – 1, 4 and 7 and compared them with the original series.

Table 1.1: Forecasting in the Regions: Regions 1, 4 and 7 for the last four (4) theoretical values

	Region 1		Region Region 4		Region 7	
	THE.	EST.	THE.	EST.	THE.	EST.
	(X)	X	(X)	X	(X)	X
97	1.31	2.80	-5.86	-4.07	1.34	2.88
98	0.51	0.87	-1.32	-0.88	-0.31	0.16
99	0.44	1.65	0.52	1.75	-0.64	0.52
100	1.82	2.30	-2.39	-1.91	1.69	2.29

4. CONCLUDING REMARKS

We have examined the forecasting performance of a bilinear ARIMA time series model, applying the Box-Jenkins procedure discussed in this paper and obtained some useful results. Table 1.1 revealed that forecasts made in the regions generally and in particular the reported regions; 1, 4 and 7 are very close to the original values which give strong indications of good and fine forecasts.

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