

ON THE MEASURE OF ENTROPY VALUE OF EXTERNAL DEBT STRUCTURE FOR AN ECONOMY

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ABSTRACT

This paper considers the use of external debt structure to measure the stability of an economy. This is achieved by modifying the existing entropy model to capture the debt service ratio. The modified entropy model is then used to estimate the entropy value of the Nigerian economy.

KEYWORDS: debt service ratio; development planning; entropy; likelihood function.

1.0 INTRODUCTION

To accelerate the rate of economic development, less developed countries (LDCs) borrow to import capital goods, components, raw materials, technical – know – how, etc (Jhingan, 2003). Repayment of this loan and interest constitutes external debt. Anyanwu (1997) stipulated that, to manage external debt effectively, authorities must project the time profile of debt service obligation, they must accurately forecast export earnings, domestic revenues and future access to finance. Iyoha (2004) maintains that external debts has a deleterious effect on economic growth and development (also see Obadan, 2002, Iyoha, 2005, and Iyoha, 2002).

McClellan and Abodunde (1978) define entropy as the thermodynamic variable of the system under consideration. They also view entropy as a measure of "the number of ways in which the elementary particles of the system may be arranged in the given circumstances". Entropy has been used in the study of manpower system, combat degradation and queuing systems (see Omosigho and Osagiede, 1999).

In this paper, we modify the model of McClellan and Abodunde (1978) to capture the debt service ratio to estimate entropy of the Nigerian economy based on the external debt structure. We do this by using data on external debt outstanding from 1983 to 1996, and the debt service ratio for the same period.

2.0 BASIC ENTROPY FOR MANPOWER SYSTEM

McClellan and Abodunde (1978) used entropy to measure the stability of the length of service in a system by applying the formula:

$$H = \frac{-\sum_{i=1}^k P_i \log P_i}{\log k} \quad (1)$$

where k is the maximum number of tenure classes;

P_i is the probability that a member of staff is in tenure class i .

The tenure profile, for a firm, is defined as the proportion of staff in each length of service class.

Entropy, H , lies in the closed interval $[0, 1]$. $H = 0$ when $P_1 = 1$ and $P_2 = P_3 = \dots = P_k = 0$ and $H = 1$ only when

$P_1 = P_2 = \dots = P_k = \frac{1}{k}$. Low entropy (i.e $0 \leq H < 0.5$) implies instability, while high entropy, for $0.5 \leq H \leq 1$, indicates

stability in the organization. Omosigho and Osagiede (1999) stated that, when $0 < H < 1$, the entropy is a good measure of the distribution of experience in the organization.

3.0 MODEL DEVELOPMENT

Rostow (1960) as cited by Jhingan (2003) distinguished five stages of economic growth, viz: the traditional society, the pre – conditions for take – off, the take – off, the drive to maturity, and the age of high mass consumption. Okojie (2002) categorises development planning in Nigeria since the post – independence period into the following:

- Fixed medium –term planning in Nigeria
 - First National Development Plan, 1962 – 1968;
 - Second National Development Plan, 1970 – 1974;
 - Third National Development Plan, 1975 – 1980;
 - Fourth National Development Plan, 1981 – 1985;
- The Structural Adjustment Programme Period, 1986 – 1988
- Perspective Planning in Nigeria
 - The First National Rolling Plan, 1990 – 1992, and
 - Vision 2010, 1997 – 2010.

In the light of the stages of development, an entropy formula can be developed for an economy. Let each stage of development represent the tenure class.

Let the debt level in stage i owed to creditor j during period t be denoted by $n_{ij}(t)$. Let $P_{ij}(t)$ be the probability of debt in stage i owed to creditor j during period t , $i = 1, 2, \dots, m$; $j = 1, 2, \dots, r$ and $t = 1, 2, \dots, n$.

Where m is the maximum developmental stage, r is the r th creditor and n is the maximum period for which data are available.

Omosigbo and Osagiede (1999) posited that the estimation of entropy is normally based on the use of relative frequencies as probabilities without proof. With a view to validating this statement, we assume that the distribution of the system is multinomial, so that we can write

$$P(n_{11}(t), n_{12}(t), \dots, n_{ir}(t)) = \frac{\left(\sum_{j=1}^r n_{ij}(t)\right)!}{\prod_{j=1}^r (n_{ij}(t))!} \prod_{j=1}^r P_{ij}^{n_{ij}(t)}(t) \quad (2)$$

(see Uche and Ezepeue, 1991)

Using the likelihood function as stated in Lindgren (1993), it is easy to see that,

$$\hat{P}_{ij}(t) = \frac{n_{ij}(t)}{n_i(t)}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, r; \quad t = 1, 2, \dots, n \quad (3)$$

where $n_i(t)$ is the debt level in stage i in period t .

The maximum likelihood estimates of P_{ij} is obtained by pooling the results in equation (3). That is

$$\hat{P}_{ij} = \frac{\sum_{t=1}^n n_{ij}(t)}{\sum_{t=1}^n n_i(t)} \quad (4)$$

Besides, the effect of Nigeria's escalating external debt has been aggravated by domestic macroeconomic policy deficiencies which have resulted in a stagnating income per capita, uneven growth of real gross domestic product (GDP) and rising inflation (Iyoha, 2005). Accompanying the escalating external debt has been a crushing debt – service burden (Iyoha, 2005). High debt – service ratios make the country extremely vulnerable to pressures on its balance of payments, thus hampering the development effort (Iyoha, 2002, 2004, 2005, and Obadan, 2002).

We therefore modify the formula in equation (1) to capture the debt service ratio as

$$H^* = \frac{1}{2m} \sum_{i=1}^m \sum_{j=1}^r \left(\frac{\alpha_{ij}}{r} - \frac{P_{ij} \log P_{ij}}{\log r} \right) \quad (5)$$

where α_{ij} is the debt service ratio in the i th developmental stage to the j th creditor. Debt service ratio is the ratio of debt – service payment to exports of goods and services.

$H^* = 0$ if $P_{i1} = 1, P_{i2} = P_{i3} = \dots = P_{ir} = 0$ and $\alpha_{i1} = 0$ for all i . This means that all debt were owed to creditor 1 (say) through out the various developmental stages, but the economy failed to service her debt. This indicates a severe economic instability, since the debt stock increases as a result of interest, and the economy possibly experiences debt overhang problem.

$H^* = 1$ if $P_{ij} = \frac{1}{r}$ for every i, j , which means that there exists a uniform limit on total external borrowing from

each creditor j , and the receipts from export of goods and services are used for debt service payment i.e. $\alpha_{ij} = 1$. This implies stability. Thus, for economic stability, the rate at which external debt is secured should be related to the foreign exchange earning capacity of the nation (see Anyanwu, 1997). The notation α_i is given by

$$\alpha_i = \sum_{j=1}^r \alpha_{ij} \quad (6)$$

4.0 APPLICATION

The entropy value of the Nigerian economy is calculated using available data for the years between 1983 and 1996.

Equation (5) is used to estimate the entropy value. The source of data is from Obadan (2002), but the value for 1986 was not used because it does not add up to the total given (see Table I in List of Tables). Therefore, we obtained the results in Table II (see List of Tables).

The debt service ratio for the i th developmental stage, $\hat{\alpha}_i$ is computed from Iyoha (2005) by the formula

$$\hat{\alpha}_i = \frac{\sum_{t=1}^{n_i} \alpha_i(t)}{n_i} \tag{7}$$

where n_i depends on i since the periods of the development stage are unequal.

From Table III (see List of Tables), we obtain $\hat{\alpha}_1$ as follows:

$$\hat{\alpha}_1 = \frac{\sum_{t=1}^3 \alpha_1(t)}{3},$$

where the coding for t is from 1983 to 1985.

$$\hat{\alpha}_1 = \left(\frac{17.8 + 29.1 + 31.7}{3} \right) \% = 0.262$$

Similarly, $\hat{\alpha}_2 = 0.205$, $\hat{\alpha}_3 = 0.226$ and $\hat{\alpha}_4 = 0.159$ are obtained.

By considering each of the developmental stages in Nigeria, we have the following:

$$H_i^* = \frac{1}{2} \left(\frac{\hat{\alpha}_i}{r} - \frac{\sum_{j=1}^r P_{ij} \log P_{ij}}{\log r} \right) \tag{8}$$

For the period 1983 to 1985, a period of fixed medium – term planning in Nigeria, we have

$$H_1^* = \frac{1}{2} \left(\frac{0.262}{5} + \frac{1.4427}{\text{Log}_e 5} \right) = 0.4744$$

(see Table II in List of Tables)

Similarly, for the SAP period, $H_2^* = 0.4524$,

the rolling plan period, $H_3^* = 0.4217$,

and the perspective planning period, $H_4^* = 0.3565$

Therefore, the entropy of the Nigerian economy from 1983 to 1996 can be estimated from the formula in equation (5) as:

$$\begin{aligned} H^* &= \frac{1}{2m} \sum_{i=1}^m \sum_{j=1}^r \left(\frac{\hat{\alpha}_{ij}}{r} - \frac{P_{ij} \log P_{ij}}{\log r} \right) \\ &= \frac{1}{m} \sum_{i=1}^m \frac{1}{2} \left(\frac{\hat{\alpha}_i}{r} - \frac{\sum_{j=1}^r P_{ij} \log P_{ij}}{\log r} \right) \end{aligned}$$

Thus, for $m = 4$ and $r = 5$,

$$\begin{aligned} H^* &= \frac{1}{4} (0.4744 + 0.4524 + 0.4217 + 0.3565) \\ &= 0.4263. \end{aligned}$$

Table I: Structure of External Public Debt Outstanding, 1983 – 96 (US \$ million)

	1983	1984	1985	1986	1987	1988	1989
Multilateral	884 (5.0)	1097 (6.3)	1317 (7.0)	1887 (7.4)	2985 (10.50)	2838 (9.2)	3171 (10.0)
Paris Club	5390 (30.3)	5811 (33.5)	7833 (41.4)	12589 (44.5)	12589 (44.5)	14400 (46.9)	15871 (50.2)
London Club	6263 (35.3)	4996 (28.8)	3560 (18.8)	5860 (20.7)	5860 (20.7)	5960 (19.4)	5680 (18.0)
Promissory Notes	3702 (20.8)	4125 (23.8)	4255 (22.5)	4850 (17.1)	4850 (17.1)	4810 (15.7)	4553 (14.4)
Others	1526 (8.6)	1318 (7.6)	1939 (10.2)	2032 (7.2)	2032 (7.2)	2685 (8.7)	2311 (7.3)
TOTAL	17765	17347	18904	28316	28316	30693	31586

	1990	1991	1992	1993	1994	1995	1996
Multilateral	3842 (11.6)	3650 (10.9)	4518.0 (16.4)	3694.7 (12.9)	4402.3 (14.9)	4411.0 (13.5)	4665.0 (16.6)
Paris Club	17171 (51.9)	17793 (53.3)	16433.9 (60.0)	18160.5 (63.2)	18334.3 (62.3)	21669.6 (66.5)	19019.0 (68.0)
London Club	5861 (17.7)	5988 (17.9)	2120.0 (7.7)	2055.8 (13.97)	2057.8 (6.9)	2045.0 (6.3)	2043.0 (7.3)
Promissory Notes	4550 (13.7)	4479 (13.4)	3246.0 (11.8)	3159.9 (11.0)	3178.2 (10.8)	3148.0 (9.7)	2140.0 (7.6)
Others	1675 (5.1)	1454 (4.4)	1246.9 (4.4)	1647.3 (5.7)	1456.3 (4.9)	1311.2 (4.0)	121.0 (0.04)
TOTAL	33099	33364	27564.8	28718.2	29428.9	32584.8	28060.0

Note: Figures in parentheses are percentage shares while those above them are in million US dollars
Source: Obadan, 2002, citing from CBN Annual Report.

Table II: Probability Estimates for Various Stages

Creditor j	Fixed Medium – Term Plan			SAP Period		
	P_{ij}	$\text{Log } P_{ij}$	$P_{ij} \log P_{ij}$	P_{2i}	$\text{Log } P_{2i}$	$P_{2i} \log P_{2i}$
1. Multilateral	0.0611	-2.7960	-0.1708	0.0993	-2.3098	-0.2294
2. Paris Club	0.3524	-1.0431	-0.3676	0.4731	-0.7485	-0.3541
3. London Club	0.2743	-1.2934	-0.3548	0.1932	-1.6442	-0.3177
4. Promissory Note	0.2237	-1.4976	-0.3350	0.1569	-1.8522	-0.2906
5. Others	0.0885	-2.4242	-0.2145	0.0776	-2.5565	-0.1984
Total			-1.4427			-1.3902

j	Rolling Plan			Perspective Plan		
	P_{3i}	$\text{Log } P_{3i}$	$P_{3i} \log P_{3i}$	P_{4i}	$\text{Log } P_{4i}$	$P_{4i} \log P_{4i}$
1	0.1277	-2.0579	-0.2628	0.1446	-1.9340	-0.2797
2	0.5466	-0.6040	-0.3301	0.6497	-0.4312	-0.2801
3	0.1486	-1.9067	-0.2833	0.0690	-2.6730	-0.1844
4	0.1305	-2.0360	-0.2657	0.0979	-2.3241	-0.2275
5	0.0465	-3.0675	-0.1426	0.0382	-3.2654	-0.1247
Total			-1.2845			-1.0964

Tables III: Trends in External Debt Burden Indicators, 1971 – 2003

Year	External Debt (US \$ Million)	Debt Per Capital (US \$)	Debt / GDP Ratio (Percentage)	Debt Service Ratio (Percentage)
1971	309	6.4	2.7	2.3
1972	401	8.0	3.7	1.8
1973	421	8.2	2.5	1.8
1974	523	9.8	1.8	1.3
1975	559	10.2	1.6	0.5
1976	594	10.5	1.4	0.7
1977	763	13.1	1.1	0.3
1978	2164	36.1	3.5	2.7
1979	2825	45.6	3.7	1.7
1980	3444	53.2	3.7	0.8
1981	3668	54.9	4.6	5.0
1982	13124	191.9	17.1	8.9
1983	17765	251.3	18.5	17.8
1984	17347	237.6	23.3	29.1
1985	18904	250.4	23.9	31.7
1986	25574	328.3	56.7	28.0
1987	28316	351.8	92.6	11.9
1988	30693	368.9	92.2	24.2
1989	31589	372.1	106.9	25.3
1990	33099	381.8	114.6	23.9
1991	33730	381.1	101.4	25.7
1992	27565	302.9	99.0	18.2
1993	28718	305.5	90.8	16.2
1994	29429	304.0	71.1	18.8
1995	32585	327.5	36.7	15.2
1996	28060	274.6	22.5	13.4
1997	27087	258.0	21.0	10.9
1998	28773	265.4	22.9	16.1
1999	28039	179.3	77.2	9.0
	28274	245.4	86.4	9.0
2000				
2001	28347	236.6	57.9	11.9
2002	30990	253.7	62.3	7.5
2003*	32917	261.3	64.4	9.8

* Estimated

Source: Iyoha (2005) citing from Debt Management Office Nigeria, 2004. Annual Report and State of Account for the year ended December 31st, 2003.

5.0 DISCUSSION

From the analysis in section 4.0, $H^* = 0.4263$. This result shows an economic instability for the Nigerian economy. This may be as a result of accumulated interest arising from the debt. Additionally, there is a strong empirical evidence of the negative effect of high external debt for the Nigerian economy, as H^* is low, and low debt service ratio, α_i , below 30% in the period under study. This is in line with Anyanwu (1997); Iyoha (2002; 2004; 2005) and Obadan (2002). This result also illustrates a declining trend in the level of stability in the Nigerian economy as indicated by the values of H_i^* . It is worthy of note that the adoption of the Structural Adjustment Programme did not remedy the situation as indicated by the low value of H_2^* ($H_2^* = 0.4524$).

6.0 CONCLUSION

We have applied entropy to measure the external debt structure of an economy with Nigeria as a case study. The basic entropy model is modified to accommodate some variables as the debt service ratio as well as developmental stages in the economy. The low entropy value obtained in this work signifies low debt servicing and hence economic instability. Therefore, we suggest that the government should formulate macroeconomic policies to facilitate payment of debt.

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