AN APPLICATION OF OPTIMAL TRANSPORTATION ALGORITHM (OTA) TO THE DISTRIBUTION OF PETROLEUM PRODUCTS IN NIGERIA

P. E. CHIGBU, and N. S UDOH

(Received 23 February, 2006; Revision Accepted 28 August, 2006)

ABSTRACT

An alternative therapy to "deregulation" as the only panacea to petroleum products availability and sufficiency in the petroleum industry of a nation with incessant scarcity of one petroleum product or the other is sought in this work. To achieve this, an optimal Transportation Algorithm (OTA), is applied to the distribution of petrol with the depots as sources and five cities as destinations. The results obtained show among others an optimum allocation of the available products to the cities according to their needs at minimum cost. The insufficiency of storage facilities (Depots) at strategic locations is determined by making allocation from dummy depot to serve the ever-growing need of the population.

KEYWORDS: Basic feasible solution, minimum cost, optimal solution, optimal transportation algorithm, petroleum product.

1 INTRODUCTION

Petroleum products' supply and distribution have direct impact on all aspects of any nation's economy. This is also true for the Nigerian economy right from the peasant farming of the rural areas to the workplaces of urban centres and up to the most sophisticated industrial production environment. The distribution of petroleum products is a typical transportation problem in Operations Research or Management Sciences, which, in this work, we seek to solve optimally using the distribution of petrol in Akwa Ibom State of Nigeria as a case study.

The transportation (or distribution) problem was an early example of linear network optimization and is now a standard application for industrial firms having several manufacturing plants, warehouses, sales territories, depots and distribution outlets: see Wagner (1969), chapter six. The transportation problem is defined as that in which it is required to find a method of apportioning a homogeneous product which is available in unit quantities at certain sources to a number of destinations which have known requirements such that the total cost of transportation is a minimum: see also Wagener (1972). Here, the strategic decision involves selecting transportation routes so as to allocate the production / stock of various refineries / depots to various terminals (filling stations). Such a distribution plan and its review are necessary if efficiency in products distribution is to be guaranteed.

The supply of petroleum products, namely, Premier Motor Spirit (PMS) – Petrol, Double Purpose Kerosene (DPK)-Kerosene, Automotive Gas Oil (AGO) – Diesel, etc. by the four local refineries in the country – Nigeria at Port Harcourt I & II, Warri and Kaduna, as at the year 2000 stood at 21% while importation stood at 79%. However, in the first quarter of year 2001, the percentage of local supply stood at 74% of PMS, 43% of AGO and 45% of DPK while 26% of PMS, 57% of AGO and 55% of DPK were imported. There are twenty- three (23) Depots / facilities owned by the Nigerian National Petroleum Corporation (NNPC) and located in various parts of the country. The combined capacity of the storage depots represents a nation-wide sufficiency of 71 days, 99 days and 108 days on the average at the consumption level of 18 million litres, 8 million litres and 10 million litres per day for PMS, DPK and AGO, respectively: see The Report of the Federal Government of Nigeria's special committee on petroleum product; supply and distribution (October, 2000).

The Pipeline and Products Marketing Company (PPMC) uses pipeline to transport petroleum products from refineries to depots. It also uses chartered coastal vessels to convey products to some storage facilities by sea. In order to ensure continuous availability of petroleum products to areas of need, PPMC has now resorted to trucking of products by road beyond the transport differential zone (TDZ) i.e from the southern depots to filling stations in other parts of the country. This is called "Bridging".

1.1 THE DATA

Akwa Ibom State was selected for this study and cities with filling stations in the state were considered. All the Local Government Areas in the State were also considered. Attention was given to those Local Government Areas with high-rated urban standard using a combination of factors as selection index. Factors that were considered are those that related to the demand/consumption of petrol in the cities such as economic/industrial standards, business/commercial standards, seats of administration and population. Consequent upon these, five Local Government Areas were selected out of a total of thirty-one Local Government Areas in the state.

Questionnaires were administered to 40% of the total number of filling stations owned by independent marketers selected at random. Data on the actual monthly allocation of petrol from depot to filling stations in each of the five selected cities were obtained from the Monitoring Unit of the Department of Petroleum Resources, Akwa Ibom State. Also, written reports in periodicals were used to complement the overall data need for this work.

2 APPLICATION OF THE STANDARD TRANSPORTATION PROBLEM MODEL TO THE DISTRIBUTION OF PMS-PETROL

Of the three common white products of petroleum (PMS, AGO, DPK), PMS is chosen for study because of its relative importance in the industrial life of the economy, its relative high demand to other products and its recurrence scarcity over the past decade. Again, during the first quarter of year 2001 under consideration, all the refineries were operational and operated at improved capacity utilization compared to the year 2000. This enabled NNPC to meet and even exceed the national demand for AGO and DPK to a large extent. It is observed however, that the availability of PMS during the period under consideration was still below the national demand, and hence, the interest to take a more detailed study of its distribution.

Petrol is distributed at twenty-two depots including refineries where tankage is also dedicated to finished products. In this work, the depots at Calabar and Port Harcourt (P/H) refineries were used as sources of supply while the destinations are the filling stations in each of the selected five cities in Akwa Ibom State.

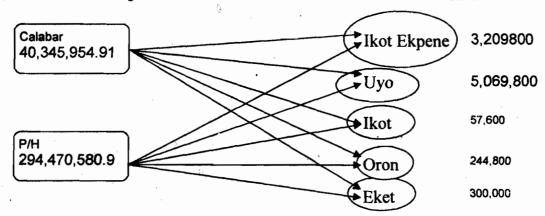


Figure 1.0: Network Representation of the distribution of petrol (in litres) in Akwa Ibom State with stored allocation and supply to destination

In order to complement the supply/distribution of petrol to curb its frequent scarcity within a TDZ, we introduce a dummy source to satisfy the excess demand of 4,409,785 litres of petrol which is the actual difference between demand and supply of petrol. Therefore, the supply and demand data in figure 1.0 is modified to accommodate a dummy source in figure 2.0 with estimated demand values; see section 2.3 of this paper.

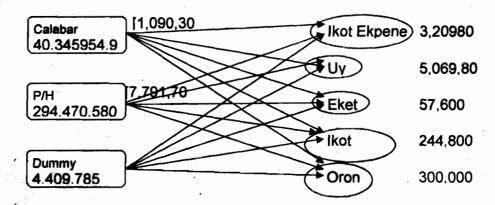


Figure 2.0: A network representation of the distribution of petrol in litres including dummy source (depot) in Akwa Ibom State with stored allocation and supply to destination

The problem depicted in figure 2.0 is actually a linear programming problem of the transportation type. To formulate the model, let Z denote total shipping cost, and let X_{ij} (i = 1, 2, 3 ; j = 1, 2, 3, 4, 5) be the quantity of petrol to be shipped from depot i to city j. Thus, the objective is to choose the values of these fifteen decision variables (X_{ij}) so as to

Minimize $Z = 1.10 X_{11} + 1.10 X_{12} + 1.20 X_{13} + 1.20 X_{14} + 1.40 X_{15} + 1.00 X_{21} + 1.20 X_{22} + 1.20 X_{23} + 1.00 X_{24} + 1.50 X_{25}$

This problem can be expressed in terms of constraints coefficients for the distribution of petrol in the five cities derived from the above linear programming problem: see table 1.0.

Table 1.0: Constraint coefficient for the distribution of Petrol

It is this special structure in the pattern of these coefficients in table 1.0 that distinguishes this problem as a transportation problem. Any linear programming problem that fits this special formulation is of the transportation problem type regardless of its physical context: see Hillier & Lieberman (1995), chapter eight. The enormous advantages of transportation problem method: see Udoh (2002) and also Chigbu and Udoh (2002) justify the utilization of transportation problem model in this work.

Hence, the standard transportation problem model applies but with the following definition of parameters;

C_a = cost of transportation of petrol from source i, i=1,2 (depot) to destination j, j=1,2,3,4,5

 S_i = supply of petrol from source i (i = 1, 2),

 $D_i = destination j (j = 1, 2, 3, 4, 5).$

ASSUMPTION

This work assumes that the total supply from the depots is delivered to specific destinations (filling stations) and as such do not take into account the effect of hoarding and smuggling of products to neighbouring countries or diversion of products for black marketeering as claimed by NNPC since it is assumed that existing laws; taskforce(s) and uniformed law-enforcement personnel guarantee the absence of such practices / operations or at worst their reduction to its barest minimum.

2.2 TRANSPORTATION PROBLEM MODELS

The mathematical description of the classical transportation problem is;

Minimize
$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} C_{ij}$$
 (1.0)

subject to
$$\sum_{i=1}^{n} X_{ij} \le S_i \quad \nabla_i = 1,2,...,m$$
 (supply) - - - - (2.0)

n

$$\Sigma \quad X_{ij} \ge d_{j} \quad \forall_{j} = 1, 2, ..., n \text{ (demand)}$$
 - - - (3.0)
 $j=1$

Xii ≥ 0 Vij

The programming formulation of this problem becomes;

Minimize
$$Z = \sum_{j=i}^{m} \sum_{j=j}^{n} Z X_{ij} C_{ij}$$

subject to
$$\sum_{i=1}^{n} X_{ij} = S_{i} \forall i = 1, 2, ..., m$$

$$\sum_{j=1}^{n} X_{ij} = d_{j} \forall i = 1, 2, ..., m$$

$$X_{ij} \ge 0 \forall i, j$$
(4.0)

Equations (5.0) and (6.0) imply that the total supply must at least be equal to total demand. (Σ S_i = Σ D_j). The resulting formulation in equations (4.0), (5.0) and (6.0) is called a balanced transportation problem model. It differs from the classical transportation model only in the fact that all constraints are equations. The model assumes that the transportation cost on a given route is directly proportional to the number of units transported on that route.

In case of unbalanced system (model) where S_i or d_j represents a bound rather than exact requirement, a dummy source or destination can be introduced to take up the slack in order to convert the inequalities to equalities and satisfy the feasibility condition; see, for example, Hillier & Lieberman (1995), chapter eight.

SUPPLY CONSTRAINT

This constraint stipulates that the total shipments from a source cannot exceed its supply. The supply constraint in this work (equation 2.0) represents depot capacity constraint. Its use depends on the particular application of the transportation problem model. In this application problem of petrol distribution, the capacity constraint is satisfied since storage facility can support about 71 days nation-wide sufficiency of petrol: see also The Report of the Federal Government of Nigeria's Special Committee on Petroleum Products Supply and Distribution (October, 2000).

DEMAND CONSTRAINT

The demand constraint stipulates that the total shipments to a destination must satisfy its demand. The demand constraint in this work (equation 3.0) requires that the sum of the total shipments to a destination must satisfy its demand. The transportation problem model requires that total supply ΣS_i must at least be equal to the total demand Σd_i . This is the condition for feasible solution.

In case of unbalanced model like ours where demand exceeds supply and demand digrepresents a bound rather than exact requirement, a dummy source is introduced to take the slack, that is, to augment the supply in order to convert the inequality to equality and satisfy the feasibility condition. The frequent scarcity of petrol within a transport differential zone (TDZ) actually justifies the need for the introduction of a dummy source in order to satisfy the demand. Stock at depots for the period under consideration are: Calabar =13,448,652 litres and Port Harcourt = 98,098,381 litres: see Agbese (2001). Based on data collected from the department of Petroleum Resources of Akwa (bom State Ministry of Petroleum and National Resources, out of the total stock in each depot, a total supply of 3209800, 5069800, 57,600, 300,000 and 244800 litres were respectively allocated to filling stations in Ikot Ekpene, Uyo, Ikot Abasi, Eket and Oron of Akwa Ibom State; see Table 2.0.

The total supply of 8,882,000 litres does not equal the total estimated demand of 13,291,785 litres. This unbalanced situation implies that it will not be possible to fill all the demand at the distribution centres (filling stations in each city). Our objective is to formulate the transportation model in a manner that will optimally distribute the shortage quantity of petrol, if it were to be available, among the distribution centres.

Table 2.0: Incomplete Cost and Requirement Table (N stands for Naira which is Nigerian Currency)

	Cost in N per litre								
, t	Ikot Ekpene (1)	Uyo (2)	Ikot Abasi (3)	Eket (4)	Oron (5)	Supply			
Source: Calabar	1.10	1.10	1.20	1.20	1.40	1,090,300			
Port Harcourt	1.00	1.20	1.20	.0	1.50	7,791,700			
Minimum needed	3,209,800	50,69,800	57,600	300,000	7,791,700				

2.3 FORMULATIONS

2.3.1 Estimate of demand of petrol during the Planning Horizon (January - March, 2001):

According to Hillier & Lieberman (1995), chapter eight, demand is a decision variable with both lower bound (minimum requested) and upper bound. This upper bound is the quantity requested unless the request exceeds the total supply remaining after the minimum needs are met. Again, according to Wagner (1972), chapter six, the numerical values of the X_{ij} are inherently approximate, since in most real applications the values of demand (d_j) are only forecasts of requirements during the planning horizon. In this application problem, let D represent the estimated demand by a filling station per period. The mathematical relation built for the estimation of the demand of petrol in this work is based on the number of sales day for a given quantity of petrol and is given by

$$D = \left(\begin{array}{c} \underline{Q} \\ \underline{s}_d \end{array}\right) T_d$$

where T_d is the total possible number of sales day per month or quarter (excluding Sundays);

s_d is the average number of sales day per truck (i.e. duration of sales in days) for a quantity of petrol supplied;

Q is the quantity or allocation of petrol supplied in a given period.

The average sales day of petrol per truck of 30,000 litres in each city from the field survey are as follows:

Table 3.0: Average sales day of petrol per truck

City	Average sales day per truck
Ikot Ekpene	5
Uyo	4
Ikot Abasi	5
Eket	4
Oron	5

Therefore, the estimated demand, D, for the first quarter of 2001 are given below as:

 $D = 3209800 \times 78 = 42172,740$ litres for lkot Ekpene;

60

D = 8,238,425 litres for Uyo;

D = 74,800 litres for Ikot Abasi;

D = 487,500 litres for Eket and;

D = 318,240 litres for Oron.

Hence, the cost and requirement table for the distribution of petrol with estimated demand is shown in table 4.0.

Table 4.0: Cost and requirement Table with estimated demand

	Cost in N per litre							
	1	2	3	4	5	Supply		
Calabar 1	1.10	1.10	1.20	1.20	1.40	1,090,300		
Port Harcourt 2	1.00	1.20	1.10	1.20	1.50	7,791,700		
Dummy 3 (D)	0	0	0	0	0	4,409,785		
Minimum needed	3,209,800	5,069,800	57,600	300,000	244,800			
Requested	4,172,740	8,238,425	74,880	487,500	318,240			

The imaginary supply quantity for the dummy source (depot) in table 4.0 is the amount by which the total of all demand exceeds the total of all real supply. The cost entries in the dummy row are zero because there is no cost incurred by the fictitious allocations from this dummy source. Again, the dummy source has an adequate (fictitious) supply to provide at least some of the minimum need in addition to its extra requested amount in lkot Ekpene, lkot Abasi, Eket and Oron destinations. Therefore, since for instance, lkot Ekpene's minimum need is 3,209,800 litres, adjustment must be made to prevent the dummy source from contributing more than 962,740 litres representing the difference between the minimum need and the requested amount. This adjustment is accompanied by grouping lkot Ekpene into two destinations; one having a demand of 3,209,800 litres — minimum needed quantity with a huge unit cost "M" for any allocation from the dummy source to ensure that this allocation will be zero in the optimal solution, and the other group having a demand of 962,940 litres — requested quantity with a unit cost of zero for the dummy source allocation. The same adjustment applies to lkot Abasi, Eket and Oron destinations. Uyo does not require any adjustment because its demand (8,238,425 litres) exceeds the dummy source's supply (4,409,785 litres) by 3,828,540 litres. So the amount supplied to Uyo from real source will be at least 3, 828,540 litres in any feasible solution. Consequently, its minimum need of 5,069,800 litres is guaranteed. This formulation gives the final cost and requirement shown in table 5.0.

Table 5.0: Complete Cost and Requirement Table for the distribution of Petrol.

	Cost (N) per litre of petrol distributed to destinations									
	lk. Ek (A)	lk. Ek (B)	Uyc	lk. Ab.	ik. Ab.	Eket (A)	Eket (B)	Oron (A)	Oron. (B)	4.74
	1	2	3	4	5	6	7	8	9	Supply
Source: Calabar 1	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.4	1.4	1,090,300
Port Harcourt 2	1.0	1.0	1.2	1.0	1.0	1.2	1.2	1.5	1.5	7,791,700
Dummy 3 (D)	M	0	0	М .	0	М	0	М	0	4,410,785
Demand .	3,209,800	96,240	8,238,425	57,600	17,280	300,000	187,500	244,800	74,400	

3.0 AN OPTIMAL TRANSPORTATION ALGORITHM (OTA)

STEP ONE: INITIALIZATION

Construct an initial basic feasible (BF) solution using

- (i) Vogel's Approximation Method (VAM)
- (ii) Russell's Approximation Method (RAM)

Go to step two

STEP TWO: DECISION

(i) Compare the optimal BF solution of VAM and RAM

(ii) Choose the result of the method with the smaller value of the objective function (Z) i.e. one with a smaller minimum cost value.

STEP THREE: OPTIMALITY TEST

Derive U_i and V_i by selecting the row having the largest number of allocation.

(ii) Set its U_i = 0, where U_i is the row having the largest number of allocation and V_j here refers to a unit cost of the jth demand (Column) constraint.

(iii) Solve the set of equations $C_{ij} = U_i + V_j$ for each (i,j) such that X_{ij} is basic.

(iv) If $C_{ij} - U_i - V_j \ge 0$ $V_{ij} \in X_{ij}$ is non-basic, then the current solution is optimal. So stop, otherwise go to step four.

STEP FOUR: ITERATION

- (i) Determine the entering basic variable: select the non-basic variable X_{ij} having the largest (in absolute terms) negative value of $C_{ij} U_i V_j \in C_{ij} U_i V_j < 0$.
- (ii) Determine the leaving basic variable: identify the chain reaction required to retain feasibility when the entering basic variable is increased. The donor cell (a cell with a basic variable that has another basic variable in the same row to serve as a recipient cell in such a way that the chain reaction tends to completion) in the chain reaction having the smallest allocation automatically provides the leaving basic variable (ties are broken arbitrarily).
- (iii) Determine the new BF solution: Add the value of the leaving basic variable to the allocation for each recipient cell (a cell with a basic variable that has another basic variable in the same column to serve as a donor cell in such a way that the chain reaction tends to completion). Subtract this value from the allocation for each donor cell. Chain reaction here is a sequence of compensating changes in other basic variables (allocations), in order to continue satisfying the supply and demand constraints.
- (iv) Repeat the optimality test (in step two) at the end of every iteration until there is no basic entering variable. That is, $C_{ii} U_i V_i > 0$ V_{ii} hence, optimality is attained.
- 4 APPLICATION OF OTA TO THE DISTRIBUTION OF PETROLEUM PRODUCTS IN NIGERIA USING THE DISTRIBUTION OF PETROL IN AKWA IBOM STATE AS A CASE STUDY

STEP ONE: INITIALIZATION

Construct an initial basic feasible (BF) solution using VAM and RAM for the problem in table 5.0:

(i) Vogel's Approximation Method (VAM): the solution of the problem using this method yields the following initial basic feasible solution; $X_{39} = 74,440$, $X_{37} = 187,500$, $X_{33} = 4,148,845$, $X_{24} = 57,600$, $X_{25} = 17,280$, $X_{21} = 209,800$, $X_{22} = 962,940$,

 $X_{13} = 1,090,300, X_{23} = 2,999,280, X_{26} = 300,000, X_{28} = 244,800$ with Z = 46,533,286.

(ii) Russel's Approximation Method (RAM): This method yields the following initial BF solution for the problem; X₃₈ = 74,400,

 $X_{37} = 187,500, X_{33} = 4,148,845, X_{24} = 57,600, X_{25} = 17,280,$

 $X_{21} = 209,800$, $X_{22} = 962,940$, $X_{13} = 1,090,300$, $X_{23} = 3,016,560$, $X_{26} = 300,000$ with Z = 1,090,776,742. Go to step two of the OTA.

STEP TWO: DECISION

Select VAM and go to step three since $Z_{VAM} = 6,533,268 < Z_{RAM} = 9,776,742$.

STEP THREE: OPTIMALITY TEST

Select the row with the highest number of allocations. Therefore, select row 2 (U₂). Select U₂ = 0; then solve $C_{ij} = U_i + V_j$ for U_i and $V_j \vee X_{ij} \ni C_{ij} = U_i - V_j \ge 0$. The basic variables are: X_{21} , X_{22} , X_{23} , X_{24} , X_{25} , X_{26} , X_{28} , X_{13} , X_{33} , X_{37} , X_{36} ; with the following results: $U_2 = 0$, $V_1 = 1.0$, $V_2 = 1.0$, $V_3 = 1.2$, $V_4 = 1.0$, $V_5 = 1.0$, $V_6 = 1.2$, $V_8 = 1.5$, $U_i = 0.1$, $U_3 = -1.2$, $V_7 = 1.2$ and $V_9 = 1.2$

Table 6.0: Optimal Solution of the distribution of Petrol

		Destination									
	1	2	3	4	5	6	7	8	9	Supply	Ui
Source ¹ !	1.1	1.1	1.2	1.2	1.2	1.2	1.2	1.4	1.4	1090300	- 0.1
	0.2	0.2	1090300	0.3	0.3	0.3	0.1		0.3		
2.	1.0	1.0	1.2	10	1.0	1.2).2	1.5	1.5	7791700	. 0
	320980	962940	299280	575600	17280	300.000	_ 0	244800	0.3		
3 (D)	М	_0	.0		0	М	لما	۵	۵	.4410785	- 1.2
·	M-2.2	0.2	4148845	M-2.2	0.2	M-2.4	187500	M-2.7	74440		
Demand	320980	962940	823842	575600	17280	300.000	187500	244800	74440	Z=6533286	
V_{i}	1.0	1.0	1.2	1.0	1.0	1.2	1.2	1.5	1.2		

Fill the values of U_i and V_j for rows and columns, respectively. Also, calculate and fill the values of $C_{ij} - U_i - V_j$ for each non-basic variable (X_{ij}) written in the corner of each cell as shown in Table 6.0.

Apply the optimality test by checking the value of $C_{ij} - U_i - V_j$ in table 6.0 An optimal solution is reached since all the $C_{ij} - U_i - V_j$ values are non-negative in table 6.0. Therefore, the optimality test identifies the set of allocations in table 6.0 as being optimal, which concludes the OTA.

4.1 SOME INTERPRETATION OF RESULTS

The interpretation is that Ikot Ekpene city can have optimal allocation of 3,209,800 litres and 962,940 litres all from Port Harcourt refinery/depot to meet its total demand at the transportation cost of N1.00 per litre. Uyo could have optimal allocation of 1,090,300, 299,280 and 4,148,845 litres of petrol from Calabar depot, Port Harcourt refinery / depot and the dummy depot, respectively and at the cost of N1.00, N1.20 and N1.00 per litre, respectively to meet its minimum need of 5,069,800 litre and additional 468,625 litres. This situation calls for the establishment of an additional depot to supply Uyo a lion share of her need (4,148,845 litres), which is hitherto being supplied by the dummy depot.

Ikot Abasi city could have an optimal allocation of 74,800 litres of petrol from Port Harcourt refinery/depot to meet its total demand of 74,800 litres at the transport cost of N 1.00 per litre. Also, Eket city could have an optimal allocation of 300,000 litres at the cost of N1.20 from Port Harcourt refinery/depot and 187,500 litres at N0.00 per litre from the dummy source to meet its total demand of 487,500 litres. Again, this calls for the establishment of a depot to meet the demand of the city of Eket. Oron city could also have an optimal allocation of 244,800 litres and 74,400 litres from Port Harcourt refinery/depot and the dummy depot, respectively, to meet its total demand at the cost of N1.50 and N0.00, respectively.

However, should the dummy depot be replaced by a real depot, the imaginary cost of transportation put at N0.00 to prevent preference supply from the dummy depot, will be removed to allow for barest competitive transport cost as is the case in any other real source.

6. CONCLUSION

This is an attempt to proffer solution to the ever crisis-ridden petroleum sector in Nigeria and its attendant products scarcity and consequent overpricing of products by marketers using the OTA. The OTA reveals the

contribution/impact of distribution network to the petroleum sector. Again, the OTA applied in this work produced optimum allocation of petrol to the five cities in Akwa Ibom State considered in this work subject to the demand / supply constraints.

However, the wide gap between the demand and supply of petrol shown in this survey is narrowed in practice by the complementary supply/distribution of petrol by major marketers. This is because the data used in the analysis only covers the supply/distribution of petrol by independent marketers as highlighted earlier. More so, the demand and supply requirements of Uyo, Eket and Oron cities in Akwa Ibom State calls for an urgent need to establish additional depot within the state to cater for the growing need of petrol.

On the whole, the way and manner of application of OTA in this work could easily be adapted for any other programming or transportation problem with similar characteristics.

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