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WAGE MODEL FOR A FIRM IN A MINIMUM WAGE SCENERIO

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ABSTRACT

In most capitalist economies, trade unions exist and do exert a great influence on the functioning of the labour market. High cost of living and humanitarian consideration make a prudent government to institute minimum wage legislations. This paper presents a wage model for a firm experiencing a constant rate of change of number employed with time.

KEYWORDS: Minimum wage, inflation rate, free market economy and trade union.

1. INTRODUCTION

The problem of collective bargaining and the attendant strike actions have continued to pose questions in the minds of employers, public administrators and the public as a whole. Now wages include fees, commissions and salaries (Jhingan, 2003a). Factors such as the productivity of labour, the efforts by trade union, the capacity of the industry to pay, and regional variations (as the cost of living differs from one region to another) may result to unequal pay for the same job (see Jhingan, 2003a, Starr, 1981, and Ekanem and Iyoha, 1999).

Marshall (1920) as cited by Ekanem and Iyoha (1999) maintains that if the demand for labour is inelastic, the union can win large wage gains without standing the risk of reduction in employment. In most cases, a minimum wage is set by the government.

Lewis (1954) as cited by Iyoha (2004) and Jhingan (2003b) posited that the subsistence sector set a floor to wage in the capitalist sector, and presented a model of the form:

$$Cw = Sw + \delta, \ \delta > 0$$

where Cw = capitalist wage, Sw = subsistence earnings and δ is the urban – rural wage differential.

Harris and Tadoro (1970) as cited by Jhingan (2003b) asserted that the urban expected wage differential leads to migration of workers from the rural to the urban sector.

Also, Fei and Ranis (1961) as cited by Jhingan (2003b) analyses the transition process through which an underdeveloped economy hopes to move from a condition of stagnation to one of self – sustained growth. One of the snags in their model is based on the assumption of a constant institutional wage which is above the marginal physical productivity (MPP) of labour during phases I and II of the development process without empirical validation.

In Nurkse's Theory of Disguised Unemployment as a Saving Potential, as cited by Jhingan (2003b), the entire process of capital formation is assumed to be 'self – financing'. This is unrealistic, for, unless, wages are paid, workers cannot be attracted to new capital projects (Jhingan, 2003b).

Therefore, this study focuses on the derivation of wage model for a firm having a constant rate of change of number employed with time, incorporating a fixed minimum wage, and to remedy the defects in the assumptions of previous authors.

2.0 Model Development

Some of the assumptions of the model are given in this section.

2.1 Assumptions

The model is based on the following assumptions:

1. The industry supply curve for labour is upward slopping, while the demand curve is downward slopping.
2. The minimum wage is set at the point of intersection between the industry's demand and supply curve for labour.
3. The wage paid by the employer, w_m and the additional wage bill due to the established minimum wage and efforts by the trade union, w, are functions of number employed as well as time.
4. The minimum wage instituted is fixed
5. When no labour is employed, no wage is paid
6. Where the labour force corresponds to the minimum wage, the additional wage is zero.
7. Wage is paid after labourer has worked for a specific period t and the labourer expects to receive at least the minimum wage.
8. The rate of change of wage with time prior to and up to the institution of minimum wage is a function of the number employed up to that period.

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The rate of change of number employed, N, with time, t, is constant. i.e. $\frac{dN}{dt} = \rho$. Assumption (9) rhymes with the 'stylized facts' of growth stated in Jhingan (2003b) and Iyoha (2004).

10. The model operates in the short – run

2.2 Model Construction

Given the above assumptions, the model is explained as follows:

Output is a function of labour, land and capital, so that the production function is

$$Q = f(N, L, K), \quad (1)$$

where Q is the output or commodity produced, N is labour units employed to produce this output, L is fixed land and K is the fixed quantity of available capital. (Jhingan, 2003b).

In most free market economies, trade unions exist and do exert a great influence on the functioning of the labour market. Through a system of collective bargaining, unionized workers have always succeeded in obtaining higher wages and better working conditions for their workers. The ultimate weapon of the union is the threat of a strike (Ekanem and Iyoha, 1999). It is within the powers of the government to institute minimum wage laws in the economy.

Let the minimum wage at the equilibrium labour force, ℓ^*, be w* (see figure 1). Let $w_m(N, t)$ be the firm's wage. From the Harris – Todaro Model of Migration and Unemployment, as cited by Jhingan (2003b), and the fact that producers are wage – takers and they aim at profit – maximization, then the firm's wage is

$$W_m(N, t) = MP_L \quad (2)$$

where MP_L is the marginal product of labour (Jhingan, 2003b).

Equation (2) is called the equation of the wage – rigidity axiom. (Jhingan, 2003b)

From equation (1)

$$MP_L = \frac{\partial Q}{\partial N} = \frac{\partial f}{\partial N}, \frac{\partial f}{\partial N} > 0; \frac{\partial^2 f}{\partial N^2} < 0 \quad (3)$$

equation (3) is similar to that stated in Jhingan, (2003b)

However, in an economy, the minimum wage (w*) is at a lower level due to institutional or political factors (Jhingan, 2003b), so that

$$w_m(N, t) \geq w^* \quad (4)$$

$$\text{Thus, } w_m(N, t) = w(N, t) + w^*, \; w(N, t) \geq 0 \quad (5)$$

where w(N, t) is the additional wage bill due to the minimum wage and efforts by the trade union

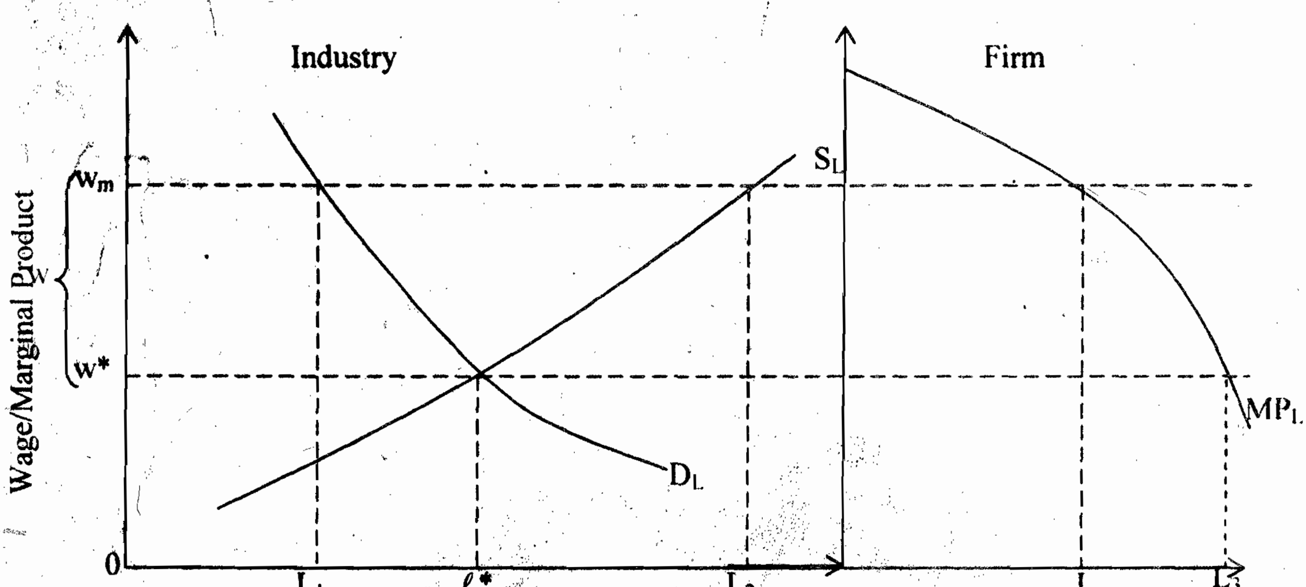

Figure 1: Quantity of Labour

(See Jhingan, 2003a)

S_L is the industry's supply curve of labour, and
D_L is the industry's marginal productivity curve
Equation (5) is similar to the Lewis model as cited by Iyoha (2004). Combining (2), (3) and (5), we have

$$w(N, t) + w^* = \frac{\partial f}{\partial N} \tag{6}$$

Differentiating (6) with respect to t,

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial N}\left(\frac{\partial f}{\partial N}\right)\frac{dN}{dt} = \rho\frac{\partial}{\partial N}\left(\frac{\partial f}{\partial N}\right). \tag{7}$$

where $\rho = \frac{\partial N}{\partial t}$ is a constant

$\frac{\partial w^*}{\partial t} = 0$ is possible, since by assumption (4), the minimum wage, w*, is fixed (or constant) in the short – run.

Again, $\frac{\partial^2 w}{\partial t^2} = \rho\frac{\partial^2}{\partial N^2}\left(\frac{\partial f}{\partial N}\right)\frac{dN}{dt}$

$$\therefore \frac{\partial^2 w}{\partial t^2} = \rho^2\frac{\partial^2 w}{\partial N^2}, \tag{8}$$

as $\frac{\partial^2}{\partial N^2}\left(\frac{\partial f}{\partial N}\right) = \frac{\partial^2}{\partial N^2}(w_m(N, t))$

$$= \frac{\partial^2}{\partial N^2}(w(N, t) + w^*) = \frac{\partial^2 w(N, t)}{\partial N^2}$$

where L and K are assumed fixed, in the short – run.

Now, if no labour is employed, then no wage is paid at any time t. This means that $w_m(0, t) = 0$ and

$$w(0, t) = 0 \tag{9}$$

At the equilibrium labour force ℓ^*, $w_m(\ell^*, t) = w^*$,

which implies that $w(\ell^*, t) = 0$ (see figure 1) (10)

Usually, wage is paid after a labourer has worked for a specific period t. Thus, w(N, 0) = 0 (11)

At the initial period, t = 0, the inflation rate bears a non – linear relationship with the rate of unemployment as illustrated by the Phillip's curve (see Kelejian, and Oates, 1981). Therefore,

$$\frac{\partial w}{\partial t}\Big|_0 = g(N), \quad \frac{dg(N)}{dN} > 0 \tag{12}$$

where g(N) is a function of the number employed, which bears a systematic relationship with the rate of excess demand at t = 0. As $N \to \ell^*$, $g(N) \to g(\ell^*)$ implies that excess demand for labour tends to zero (see figure 1). The economic implication of this is that there exist some frictional unemployment. So, some people will be in the process of moving from one job to another where the number of vacancies is equal to the number of people seeking jobs. (Kelejian and Oates, 1981).

2.3 Model Solution

To solve equation (8), which is a partial differential equation, the method of separation of variables is applied by setting

$$w(N, t) = X(N)\,T(t), \tag{13}$$

where X is a function of N alone and T is a function of t alone. (Gupta, 1993, and Stephenson, 1973).

Taking the partial derivatives of equation (13), we obtain

$$\left.\begin{aligned} \frac{\partial w}{\partial N} &= T\frac{dX}{dN}, \quad \frac{\partial^2 w}{\partial N^2} = T\frac{d^2X}{dN^2} \\ \frac{\partial w}{\partial t} &= X\frac{dT}{dt}, \quad \frac{\partial^2 w}{\partial t^2} = X\frac{d^2T}{dt^2} \end{aligned}\right\} \tag{14}$$

Substituting equation (14) into equation (8), we have

$$X\frac{d^2T}{dt^2} = \rho^2 T\frac{d^2X}{dN^2}$$

Or

$$\frac{1}{\rho^2 T}\frac{d^2T}{dt^2} = \frac{1}{X}\frac{d^2X}{dN^2} \tag{15}$$

Taking λ as constant of separation, we get

$$\frac{1}{\rho^2 T}\frac{d^2T}{dt^2} = \frac{1}{X}\frac{d^2X}{dN^2} = \lambda \tag{16}$$

There arises three cases for the real value λ: (Gupta, 1993).

Case (i) $\lambda = 0$, so that by equation (16), $\frac{d^2X}{dN^2} = 0$, $\frac{d^2T}{dt^2} = 0$, giving

X = AN + B, T = Ct + D, where A, B, C, D, are constants.

Case (ii) $\lambda = \mu^2$, *so that by* (16), $\frac{d^2X}{dN^2} - \mu^2 X = 0$, $\frac{d^2T}{dt^2} - \mu^2\rho^2 T = 0$, giving

$$X = Ae^{\mu N} + Be^{-\mu N}, \quad T = Ce^{\mu\rho t} + De^{-\mu\rho t}$$

Case (iii) $\lambda = -\mu^2$; $\frac{d^2X}{dN^2} + \mu^2 X = 0$, $\frac{d^2T}{dt^2} + \mu^2\rho^2 T = 0$, giving

$$X = A\cos\mu N + B\sin\mu N, \quad T = C\cos\mu\rho t + D\sin\mu\rho t$$

If we impose the conditions of equations (9), (10), (11) and (12), we have for each case, as follows:

A = B = 0 for case (i) and case (ii) when w(0, t) = 0 and $w(\ell^*, t) = 0$. Therefore Case(i) and Case (ii) fail to give that solution of (8). But the solution of case (iii) which is periodic in time is capable of giving a solution of (8). We have therefore that

$$w(N,t) = (A\cos\mu N + B\sin\mu N)(C\cos\mu\rho t + D\sin\mu\rho t) \tag{17}$$

Applying conditions of equations (9), (10), (11) and (12), we can determine the constants A B. C, D, and μ.

Moreover, we obtain A = 0 for w(0, t) = 0;

$$w(\ell^*,t) = 0 = B\sin\mu\ell^*(C\cos\mu\rho t + D\sin\mu\rho t),\ B \neq 0$$

$$\therefore \sin\mu\ell^* = 0 \Rightarrow \mu = \frac{n\pi}{\ell^*},\ n = 1,2,\ldots$$

$$w(N,0) = 0 = \left(B\sin\frac{n\pi}{\ell^*}N\right)C \Rightarrow C = 0;$$

So, $w(N,t) = (B\sin\mu N)(D\sin\mu\rho t)$

$$\frac{\partial w}{\partial t} = (\sin\mu N)(BD\mu\rho\cos\mu\rho t)$$

As $\mu = \dfrac{n\pi}{\ell^*}$ and at t = 0,

$$\frac{\partial w}{\partial t}\Big/_{t=0} = \left(\sin\frac{n\pi}{\ell^*}N\right)\left(\frac{n\pi}{\ell^*}\rho\alpha Cos\frac{n\pi}{\ell^*}\rho t\right)\Big/_{t=0} = g(N),$$

where

$$\alpha = BD$$

$$\therefore \frac{n\pi}{\ell^*}\rho\alpha\sin\frac{n\pi}{\ell^*}N = g(N) \qquad (18)$$

Suppose equation (13) is linear and homogeneous. Then the sum of any number of distinct solutions of equation (8) is also a solution of equation (8). As such the solution of (8) in place of (14) may be taken as

$$w(N,t) = \sum_{n=1}^{\infty}\alpha_n \sin\frac{n\pi}{\ell^*}N\sin\frac{n\pi}{\ell^*}t \qquad (19)$$

and

$$g(N) = \sum_{n=1}^{\infty}\alpha_n\frac{n\pi}{\ell^*}\rho\sin\frac{n\pi}{\ell^*}N \qquad (20)$$

(see Gupta, 1993 and Stephenson, 1973)

Now, to find an expression for α_n, multiply equation (20) by $\sin\dfrac{m\pi}{\ell^*}N$ and integrate from N = 0 to ℓ^* wrt N, we obtain

$$\int_0^{\ell^*} g(N)\sin\frac{m\pi}{\ell^*}N\,dN = \sum_{n=1}^{\infty}\alpha_n\frac{n\pi}{\ell^*}\rho\int_0^{\ell^*}\sin\frac{m\pi}{\ell^*}N\sin\frac{n\pi}{\ell^*}N\,dN,$$

where

$$\int_0^{\ell^*} \sin\frac{m\pi}{\ell^*}N \sin\frac{n\pi}{\ell^*}N\,dN = \begin{cases} 0\ , & m \neq n \\ 0\ , & m = n = 0 \\ \dfrac{\ell^*}{2}\ , & m = n > 0 \end{cases}$$

$$\therefore\ \alpha_n \frac{n\pi}{\ell^*}\rho = \frac{2}{\ell^*}\int_0^{\ell^*} g(N)\sin\frac{n\pi}{\ell^*}N\,dN,$$

Or

$$\alpha_n = \frac{2}{n\pi\rho}\int_0^{\ell^*} g(N)\sin\frac{n\pi}{\ell^*}N\,dN \qquad (21)$$

Hence, the wage model for the firm is given by

$$w_m(N,t) = \begin{cases} \displaystyle\sum_{n=1}^{\infty}\frac{2}{n\pi\rho}\sin\frac{n\pi}{\ell^*}N\sin\frac{n\pi}{\ell^*}\rho t\int_0^{\ell^*} g(N)\sin\frac{n\pi}{\ell^*}N\,dN + w^*, & N > 0 \\ 0\ , & \text{otherwise} \end{cases} \qquad (22)$$

The first term on the right side of equation (22) is the additional wage bill due to the instituted minimum wage and efforts by trade union. This term depends largely on N, the labour units. The additional wage bill w(N,t), due to instituted minimum wage increases with increase in N to a certain level. Thereafter, w(N,t) starts declining. This can be interpreted to be the period when the firm financial standing can not cope effectively with the wage bill for additional labour. In this case, a further increase in the unit of labour within the limit of the available resources, may lead to a fall in output and the firm experiences difficulty to pay the wage bill (Jhingan, 2003a). This may mean a slash in the workers' wages with its associate problem of trade dispute by labour union. This situation is rampant among firms especially in developing nations. The presence of periodic functions in equation (22) makes this possible. Our model therefore reflects reality.

3.0 Analysis of the Model

In practice, the higher wage bill caused by the institution of minimum wage laws imposes tremendous financial pressures on the marginal firms. As responses by firms, new technologies are introduced in the production process and management practices. In addition quality of supervision is upgraded, the work force is pruned by the termination of inefficient workers and elimination of "dead-woods" as well as the upgrading of employee selection standards and qualifications. Thus, the new wage will be paid to employees left in the firm at time t. So, the firm's wage is hypothesized to depend on number employed and time i.e. $w_m = w_m(N,t)$.

The model also shows that the entire process of capital formation in an organization involves the payment of wages. i.e. $w_m>0\ \ \forall N > 0$. This overcomes one of the limitations of the Nurkse's Theory of Disguised Unemployment where the entire process of capital formation is assumed to be 'self – financing' (Jhingan, 2003b).

Assuming that working capital is available, employment of surplus labour is likely to lead to inflation in the economy, for when the newly employed workers are paid wages, their demand for consumer goods increases without a corresponding increase in output of consumer goods (Jhingan, 2003b). Therefore, $0 \le N \le \ell^*$. $N = 0$ captures the exit of firms as a result of the institutional wage.

The concept of minimum wage used in the model is realistic (see Jhingan, 2003a, Jhingan, 2003b and Starr, 1981). The rise in wage and employment with development explains why $w_m > w^*$ for some employment level L. w^* is established at the point of intersection of D_L and S_L has empirical validation and that $w_m = MP_L$. (see Ekanem and Iyoha, 1999, Jhingan, 2003a and Jhingan, 2003b). This gives a better view than Fei – Ranis Theory (1961) as cited by Jhingan (2003b), that is based on the assumption of a constant institutional wage which is above the marginal physical productivity of labour (MPP) during phases I and II of the development process.

The inflation rate prior to and up to the period of institution of minimum wage is encapsulated in the model as

$$\left.\frac{\partial w}{\partial t}\right/_{t=0} = g(N).$$

where g(N) is a function of the number employed, which bears a systematic relationship with the rate of excess demand at t = 0. This is sequel to the fact that inflation is not self destructive as stated in Jhingan (2003b). It also captures the most comprehensive conception of the proper role of minimum wage fixing as an instrument of macro – economic policy for economic stability (Starr, 1981).

4.0 CONCLUSION

This paper presents a wage model, which is a function of number employed, N, and time, t. It uses a fixed minimum wage legislation set by the government. It is periodic in time, thus reflecting reality.

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