GLOBAL JOURNAL OF MATHEMATICAL SCIENCES VOL. 5, NO. 1, 2006: 9-16
 ISSN 1596 - 6208

APPLICATION OF MARKOVIAN MODEL TO SCHOOL ENROLMENT PROJECTION PROCESS

V. U. EKHOSUEHI and A. A. OSAGIEDE

(Received 12 September, 2005; Revision Accepted 1 February, 2006)

ABSTRACT

This paper deals with the application of Markov chain model to student's enrolment projection in a university in Nigeria. Specifically, the paper contains the estimation of the constant rate of increase in the number of new intake into a programme of study. The estimators proposed by previous authors are used for the estimation and subsequently, the enrolment projection.

KEYWORDS: Markov Chain, school enrolment, projection, parameter estimation.

1.0 INTRODUCTION

This study focuses on enrolment profile of a new course of study in the Department of Mathematics, University of Benin, Nigeria. It employs Markov Chain Models used for analysing manpower planning systems as in Raghavendra (1991) and McClean (1991).

Uche (2000) proposed the use of Markov chain model for the estimation of future enrolment in schools in developing countries. However, there is no practical illustration of the methods of Uche (2000). Adeyemi (1998) considered enrolment structure for primary schools. There is a problem of finding an acceptable method for estimating the number of new intake into the school system in the works of these authors. In line with this problem, Osagiede and Omosigho (2004) proposed methods for estimating new intake into the first grade. The problem of the methods of Osagiede and Omosigho (2004) is the arbitrary choice of a constant rate of increase in the number of new intake into the first grade, which makes prediction to vary from one researcher to another. New and better methods were proposed by Osagiede and Ekhosuehi (2006). However, there is no practical illustration of the new methods in Osagiede and Ekhosuehi (2006). Based on the suggestion of Osagiede and Ekhosuehi (2006), this study provides a practical illustration to justify the use of the methods proposed in Osagiede and Ekhosuehi (2006).]

2.0 The Model

The Markov Chain Model for manpower planning is given by Osagiede and Omosigho (2004) as

$$E_g^t = P_{g-1} E_{g-1}^{t-1} + r_g E_g^{t-1} + N_g^t - W_g^t E_g^{t-1}, \qquad (g = 1, 2, \ldots, m) \tag{1}$$

where

E_g^t = enrolment in grade g, year t, N_g^t = new entrants to grade g, year t, W_g^t is wastage rate from grade g, year t, P_g^t, r_k^t are promotion and repetition rates, respectively.

Osagiede and Omosigho (2004) introduced the method for estimating the new intake in grade 1 as

$$N_1^t = \left(1 + \frac{\beta}{100}\right)^t N_1^0 \tag{2}$$

where β is the rate of increase in new entrant into grade 1 and

N_1^0 is the new entrant figure in the base year.

In Osagiede and Omosigho (2004), β value in equation (2) is assumed or chosen arbitrarily.

Osagiede and Ekhosuehi (2006) derived a method for estimating the rate of increase, β, in new intake as

$$\ln\left(1 + \hat{\beta}\right) = \frac{12\sum_{t=1}^{n} t \ln N_t^i - 6(n+1)\ln \prod_{t=1}^{n} N_t^i}{n\left(n^2 - 1\right)} \tag{3}$$

Where N_t^i is the number of new intake into level i, year t, i = 1, 2, and n is the number of years for which data is available.

V. U. Ekhosuehi, Department of Mathematics, University of Benin, Benin-City, Nigeria.
A. A. Osagiede, Department of Mathematics, University of Benin, Benin-City, Nigeria.

The number of new entrants in the base year is given by Osagiede and Ekhosuehi (2006) as

$$\hat{N}_o^i = (1+\beta)^{-\frac{1}{2}(n+1)}\left[\prod_{t=1}^{n} N_t^i\right]^{1/n}$$

We assume that the students' flow over time is stable and orderly so that the transition probability matrix (TPM) is stationary. That is

$P_{ij}(t) = P_{ij}$ for all t, and for given i, and for all i, j = 1, 2, …, 6.

The maximum likelihood estimates, $\hat{P}_{ij}$ is given by

$$\hat{P}_{ij}(t) = \frac{\sum_{t=1}^{n} n_{ij}(t)}{\sum_{t=1}^{n} n_i(t)} \qquad (5)$$

and the equation of projection is given by

$$(\hat{n}_i\ (t)) = Q^T\ (\bar{n}_i\ (t-1)) + \Delta^*\ N^j(t), \quad i = 1, 2, \ldots, 6$$
$$j = 1, 2. \qquad (6)$$
$$= (Q^T)^t (\bar{n}_i\ (0)) + \sum_{c=1}^{t} (Q^T)^{t-c}\ \Delta^*\ N^j(c)$$

where Δ^* is the probabilistic difference arising from fresh students admitted into level j, j = 1, 2. (See Osagiede and Ekhosuehi, 2006).

Now,

$$Q = \begin{pmatrix} (P_{10})(P_{01}) & P_{12} + (P_{10})(P_{02}) & 0 & 0 & 0 & 0 \\ (P_{20})(P_{01}) & (P_{20})(P_{02}) & P_{23} & 0 & 0 & 0 \\ (P_{30})(P_{01}) & (P_{30})(P_{02}) & 0 & P_{34} & 0 & 0 \\ (P_{40})(P_{01}) & (P_{40})(P_{02}) & 0 & 0 & P_{45} & 0 \\ (P_{50})(P_{01}) & (P_{50})(P_{02}) & 0 & 0 & 0 & P_{56} \\ (P_{60})(P_{01}) & (P_{60})(P_{02}) & 0 & 0 & 0 & P_{66} \end{pmatrix}$$

where $P_{12} + (P_{10})(P_{02})$ is the probability of promotion from level 1 to 2, and the probability that the withdrawals from level 1 would be replaced by candidates admitted through direct entry into level 2;

Q is the transpose of the projection matrix, Q^T;

$$\Delta^*\ N^j(c) = \begin{pmatrix} P_{01}\ \beta(1+\beta)^c\ N_0^1 \\ P_{02}\ \gamma^c\ (1-\gamma)\ N_0^2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

N_0^1 denotes the number of fresh students into level 1 in the base year; and

N_0^2 is the number of fresh students into level 2 in the base year.

From equations (3) and (4),

$$\ln(1+\beta) = \frac{12\sum_{t=1}^{n} t \ln N_t^1 - 6(n+1)\sum_{t=1}^{n} \ln N^1}{n(n^2-1)} \qquad (7)$$

and

$$N_0^1 = (1+\beta)^{-\frac{1}{2}(n+1)} \left(\prod_{t=1}^{n} N_t^1\right)^{\frac{1}{n}} \qquad (8)$$

Similarly,

$$\ln \gamma = \frac{12\sum_{t=1}^{n} t \ln N_t^2 - 6(n+1)\sum_{t=1}^{n} \ln N_t^2}{n(n^2-1)} \qquad (9)$$

and

$$N_0^2 = \gamma^{-\frac{1}{2}(n+1)} \left(\prod_{t=1}^{n} N_t^2\right)^{\frac{1}{n}} \qquad (10)$$

3.0 Application

To estimate the parameters of this model, data for B.Sc Statistics with Computer Science Part – Time programme in the University of Benin, Nigeria, for the period between 1998/1999 and 2003/2004 sessions were collected.

The data are presented in Tables I – V (See list of tables).

Table I: Enrolment Data for 1998/1999 Session

		i/j	1	2	3	4	5	6	w_i	$n_i(t)$
t = 1	1998/1999	r.j	52	25	0	0	0	0		77
		1	0	35	0	0	0	0	17	52
		2	0	0	25	0	0	0	0	25

Table II: Enrolment Data for 1999/2000 Session

		i/j	1	2	3	4	5	6	w_i	$n_i(t)$
t = 2	1999 / 2000	r.j	19	19	0	0	0	0		38
		1	0	14	0	0	0	0	5	19
		2	0	0	40	0	0	0	14	54
		3	0	0	0	20	0	0	5	25

Table III: Enrolment Data for 2000/2001 Session

		i/j	1	2	3	4	5	6	w_i	$n_i(t)$
t = 3	2000 / 2001	r.j	30	19	0	0	0	0		49
		1	0	28	0	0	0	0	2	30
		2	0	0	33	0	0	0	0	33
		3	0	0	0	40	0	0	0	40
		4	0	0	0	0	20	0	0	20

Table IV: Enrolment Data for 2002/2003 Session

		i/j	1	2	3	4	5	6	w_i	$n_i(t)$
t = 4	2002 / 2003	r.j	49	30	0	0	0	0		79
		1	0	49	0	0	0	0	0	49
		2	0	0	56	0	0	0	2	58
		3	0	0	0	30	0	0	3	33
		4	0	0	0	0	35	0	5	40
		5	0	0	0	0	0	18	2	20

Table V: Enrolment Data for 2003/2004 Session

		i/j	1	2	3	4	5	6	w_i	$n_i(t)$
t = 5	2003 / 2004	r.j	112	4	0	0	0	0		116
		1	0	112	0	0	0	0	0	112
		2	0	0	53	0	0	0	0	53
		3	0	0	0	56	0	0	0	56
		4	0	0	0	0	30	0	0	30
		5	0	0	0	0	0	35	0	35
		6	0	0	0	0	0	8	10	18

Source: B.Sc. Statistics with Computer Science, (Part – Time Programme) Dept. of Mathematics, University of Benin, Benin City, Nigeria

Note:

1. 2001/2002 session did not exist in the university
2. In the academic system under consideration, students who were unregistered in year (t – 1) have the opportunity to register in year t, so, the number of withdrawal cannot be ascertained in year (t – 1) for enrolment level i.
3. Withdrawal (voluntary withdrawal) can be detected when unregistered student(s) in year (t – 1) level i, fail to register in year (t + 1) for that level i, i.e. two sessions later.
4. Withdrawal from the course of study may be due to illness, expulsion, death, academic deficiency, financial insolvency and so on;
5. w_6 denotes the number of graduates.

3.1 Estimation of Parameters

To estimate the parameters of the model, we first state the assumptions of the model as given in Osagiede and Ekhosuehi (2006) as follows;

1. There are six levels in the course of study
2. Students enter the school system through level one (100 level) or through direct entry into level two (200 level).
3. The change in the number of new intake is proportional to the previous new intake.
4. Promotion from one level to the next is based on attaining a minimum of 10 credit course load; otherwise, the student is withdrawn from the university. In other words no repetition of classes or levels is allowed, except in level six. That is , $P_{ii}(t)=0$ for i=1,2,…,5.
5. No double promotion and no demotion. That is,
   $$P_{ij}(t)=0 \quad \text{for all } j>i+1 \text{ and } j\le i-1$$
6. The probability of withdrawal of student and the probability of replacement by new students are independent with probability $(P_{i\cdot})$ $(P_{\cdot j})$,
   $$j=1,2;\ i=1,2,\ldots,6.$$
7. The probability estimates are stationary;
8. It is assumed there is no withdrawal in level 6 for whatever reason.

Using equation (5), the pooled probability estimates are given in Table VI (See list of tables).

By the assumption $P_{ij} = (P_{i0})(P_{0j})$, i = 1, 2, …, 6, j = 1, 2, which is the probability that students who leave the academic system in level i will be replaced by fresh students who enter the system in level j, we obtain the matrix **Q** as:

$$Q=\begin{pmatrix} 0.0668 & 0.9332 & 0 & 0 & 0 & 0 \\ 0.0523 & 0.0194 & 0.9283 & 0 & 0 & 0 \\ 0.0379 & 0.0140 & 0 & 0.9481 & 0 & 0 \\ 0.0406 & 0.0150 & 0 & 0 & 0.9444 & 0 \\ 0.0266 & 0.0098 & 0 & 0 & 0 & 0.9636 \\ 0.4055 & 0.1501 & 0 & 0 & 0 & 0.4444 \end{pmatrix}$$

Table VI: Pooled Probability Estimates

i/j	1	2	3	4	5	6	P_i
P_{0j}	0.7298	0.2702	0	0	0	0	
1	0	0.9084	0	0	0	0	0.0916
2	0	0	0.9283	0	0	0	0.0717
3	0	0	0	0.9481	0	0	0.0519
4	0	0	0	0	0.9444	0	0.0556
5	0	0	0	0	0	0.9636	0.0364
6	0	0	0	0	0	0.4444	0.5556

From the data collected we can estimate β and γ as follows using equations (7) and (9). This is presented in Table VII.(See list of tables).

Recall equations (7) and (8) we have that

$$\ln(1+\beta) = \frac{12\sum_{t=1}^{5} t \ln N_t^1 - 6(5+1)\sum_{t=1}^{5} \ln N_t^1}{5(25-1)}$$

$$\therefore \beta = 0.2817$$

$$N_0^1 = (1.2817)^{-3}\left(\prod_{t=1}^{5} N_t^1\right)^{1/5} = 20.8401$$

Similarly, from equations (9) and (10)

$$\ln \gamma = \frac{12\sum_{t=1}^{5} t \ln N_t^2 - 6(5+1)\sum_{t=1}^{5} \ln N_t^2}{5(25-1)}$$

$$\therefore \gamma = 0.7255 \quad and \quad N_0^2 = 42.1710$$

Next,

$$\left|\Delta^* N^1(t)\right| = (1.2817)^{t-1}(5.8707)\,P_{01}$$

and

$$\left|\Delta^* N^2(t)\right| = (0.7255)^{t-1}(11.5759)\,P_{02}$$

Table VII: Logarithmic Values of N_t^1 and N_t^2

T	$\ln N_t^1$	$\ln N_t^2$
1	3.9512	3.2189
2	2.9444	2.9444
3	3.4012	2.9444
4	3.8918	3.4012
5	4.7185	1.3863

Assuming that the estimated probabilities in matrix Q are stationary, and taking the session 2003/2004 as the base year, we obtain the equation of projection as follows:

$$\begin{pmatrix} \hat{n}_1(t) \\ \hat{n}_2(t) \\ \hat{n}_3(t) \\ \hat{n}_4(t) \\ \hat{n}_5(t) \\ \hat{n}_6(t) \end{pmatrix} = \begin{pmatrix} 0.0668 & 0.0523 & 0.0379 & 0.0406 & 0.0266 & 0.4055 \\ 0.9332 & 0.0194 & 0.0140 & 0.0150 & 0.0098 & 0.1501 \\ 0 & 0.9283 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9481 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9444 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9636 & 0.4444 \end{pmatrix}^t \begin{pmatrix} 112 \\ 53 \\ 56 \\ 30 \\ 35 \\ 18 \end{pmatrix}$$

$$+\sum_{c=1}\begin{pmatrix} 0.0668 & 0.0523 & 0.0379 & 0.0406 & 0.0266 & 0.4055 \\ 0.9332 & 0.0194 & 0.0140 & 0.0150 & 0.0098 & 0.1501 \\ 0 & 0.9283 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9481 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9444 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9636 & 0.4444 \end{pmatrix}^{t-c}\begin{pmatrix} 11.5621\,(1.2817)^c \\ 0.8665\,(0.7255)^c \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The absolute change in the number of new entrants is shown in Table VIII, where t = 0 is the base year.(**See** list of Tables).

Table VIII: Difference in New Entrant Projection

T	0	1	2	3	4
$\lvert\Delta^* N^1(t)\rvert$	11.5621	14.8191	18.9937	24.3442	31.2019
$\lvert\Delta^* N^2(t)\rvert$	0.8665	0.6287	0.4561	0.3309	0.2401

Higher transition probabilities, $P_{ij}^{(n)}$, $n \leq 4$ of matrix Q are given below.

$$(Q^T)^2 = \begin{pmatrix} 0.0533 & 0.0397 & 0.0417 & 0.0248 & 0.3928 & 0.2151 \\ 0.0804 & 0.0622 & 0.0499 & 0.0474 & 0.1659 & 0.4480 \\ 0.8663 & 0.0180 & 0.0130 & 0.0139 & 0.0091 & 0.1393 \\ 0 & 0.8801 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8954 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9100 & 0.4282 & 0.1975 \end{pmatrix}$$

$$(Q^T)^3 = \begin{pmatrix} 0.0406 & 0.0423 & 0.0261 & 0.3737 & 0.2089 & 0.1232 \\ 0.0634 & 0.0517 & 0.0489 & 0.1609 & 0.4341 & 0.2410 \\ 0.0747 & 0.0577 & 0.0463 & 0.0440 & 0.1540 & 0.4159 \\ 0.8213 & 0.0171 & 0.0123 & 0.0132 & 0.0086 & 0.1321 \\ 0 & 0.8312 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8628 & 0.4044 & 0.1903 & 0.0878 \end{pmatrix}$$

$$(Q^T)^4 = \begin{pmatrix} 0.0422 & 0.0272 & 0.3564 & 0.1996 & 0.1200 & 0.0776 \\ 0.0525 & 0.0497 & 0.1557 & 0.4133 & 0.2342 & 0.1406 \\ 0.0588 & 0.0480 & 0.0454 & 0.1493 & 0.4030 & 0.2238 \\ 0.0708 & 0.0547 & 0.0439 & 0.0417 & 0.1460 & 0.3943 \\ 0.7757 & 0.0161 & 0.0116 & 0.0125 & 0.0081 & 0.1248 \\ 0 & 0.8009 & 0.3834 & 0.1797 & 0.0846 & 0.0390 \end{pmatrix}$$

These values of $P_{ij}^{n}(t)$, $2 \le n \le 4$ are subject to round off errors. Using these values in the projection equation, we obtain the enrolment profile as in Table IX.(See list of tables).

Table IX: A Four – Year Enrolment Projection

Level i	Year t 0	1	2	3	4
1	112	37	49	56	73
2	53	111	45	56	66
3	56	50	103	42	52
4	30	54	47	98	40
5	35	29	51	45	93
6	18	42	46	70	75
Graduate	**10**	**24**	**26**	**39**	**42**

4 DISCUSSION OF RESULTS

Table IX shows a general increase in enrolment. In general the transition from one level to another witnessed a general decline. This may be due to withdrawal from the programme for whatever reason. There is also a steady increase in the number of graduate throughout the projection period. The final level (level 6) experienced a steady increase through out the period. This may be due to the fact that repetition is allowed at the final level for the spill-over students. The result from this study therefore reflects reality.

A major problem in this study is that of estimating parameters for the higher levels of the academic programme. This is because, the programme is a young one and the records used are the only data available for now. Therefore, one or two year data were only available for the estimation of parameters for the higher level of the programme. This may affect the accuracy of our prediction to an extent. However, given sufficient data for all levels, the methods applied in this paper can be used satisfactorily to project future enrolment profile for any academic programme of this nature.

5. CONCLUSION

Based on the results obtained in Tables IX, the number of students admitted into level one was found to be increasing, while the number of those admitted through direct entry declined. However, the number of students in the entire academic system is expected to increase steadily in future provided other factors, such as Nigerian University Commission's (NUC) recommendation remain constant. A major feature of this result is that a unique value for the constant rate of increase in the number of new intake is obtained. This is a reasonable improvement on the work of previous authors such as Osagiede and Omosigho (2004), Adeyemi (1998) and Uche (2000).

In addition, the model shows the types of records – number of students admitted and registered, withdrawal, spill – over, graduates, etc – that should be kept in a higher institution. The Markovian model was used to forecast the expected enrolment at each level $i = 1, 2, \ldots, 6$.

Other aspects for further research include:

- The policy implication when $\beta < 0$ i.e. when the number of intake decreases at the rate of β % per year;
- The efficiency of the method used in this study compared to the ones used in previous work as that of Osagiede and Omosigho (2004);
- The application of software support such as windows – based TORA, and LINGO and AMPL to educational planning to determine the optimum manpower and technological requirements.

6. ACKNOWLEDGEMENT

The authors expressed their profound gratitude to the anonymous referee for his diligence and useful comments that resulted to the quality of work in this study.

REFERENCES

Adeyemi. J. K., 1998. Analysis of Wastage Rate in Public Primary School in Former Oredo Local Government Area of Edo State. Benin Journal of Educational Studies, 11(1&2): 92 – 100.

McClean, S. ,1991. Manpower Planning Models and Their Estimation European Journal of Operational Research 51 : 179 – 187.

Osagiede, A. A. and Ekhosuehi, V. U., 2006. Markovian Approach to School Enrolment Projection Process. Under Review for Publication in the Global Journal of Mathematical Sciences.

Osagiede, A. A. and Omosigho, S. E., 2004. School Enrolment Projection Using Spreadsheet Under Scanty Data. Nigerian Annals of Natural Sciences, 5(2): 87 – 99.

Raghavendra, B. G.,1991. A Bivariate Model for Markov Manpower Planning Systems. J. Opl. Res. Soc. 42(7): 565 – 570.

Uche, P. I., 2000. The Markovian Model of the Educational Process. In: A.O. Animalu, S.O. Iyahen and H.O. Tejumola (Editors) Contribution to the Development of Mathematics in Nigeria. National Mathematical Centre, Abuja. Pp. 235 – 245.