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MARKOVIAN APPROACH TO SCHOOL ENROLMENT PROJECTION PROCESS

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ABSTRACT

The use of mathematical models for educational planning has gained prominence in recent times as a means to better quantitative planning in education. This paper presents the use of Markovian model to project the future enrolment level of students in a course of study in a university in Nigeria. The study aimed at refining the method of previous authors. Further, the assumption of certain constant values in the rate of new intake by some authors is removed and a better method for calculating the constant value of this increment in new intake is introduced.

KEYWORDS: Least squares, Markov models, stationary, transition.

1.0 INTRODUCTION

The planning and implementation of higher education take route from the identified jobs and functions to be performed in the society. This is sequel to the ability of the people in a society to acquire their needs and requirements according to the desired quantities and qualities. Higher education therefore, is the main producer of higher level manpower needed to turn the economy around. The need for higher level manpower led to the introduction of many new full and part – time programmes in the country to meet this yearning need. Hence Osezuah (1998) identified manpower resources requirement and political influence as the two basic factors influencing educational planning and implementation.

This study focuses on enrolment profile of a new course programme in Department of Mathematics, University of Benin, Nigeria. This study also, includes the flow of students – internal and external – into the programme, employing Markov chain.

Markov models are being extensively used for analyzing manpower planning systems. Most of these models concentrate either on estimating the grade-wise distribution of future manpower structure given the existing structure and promotion policies or on deriving policies towards promotion given the required future structure. Limited mobility of people from one organization to another results in policies of promotion and recruitment having long – term irrevocable effects on the organization [see Raghavendra (1991), McClean (1991), Leeson (1984), and McClean and Gribbin (1987)].

In this paper Markov chain model for manpower planning is used to model the flow of students in the academic programme.

Ucho (2000) considered the use of mathematical models for estimating the future enrolment for schools in developing countries. The model considered are Markov population models that are widely used in manpower planning. Adeyemi (1998) also considered enrolment structure for primary school. These previous authors have problem in their methods of estimating new intake into the first grade. This problem is finding a suitable estimator for the rate of new intake into an educational system.

Osagiede and Omosigho (2004) attempted a solution to the method of estimating the number of new intake into the first grade. They attempted the problem by assuming an increase in the number of new intake from year to year. The snag in their method is that the value of the constant rate of increase in the number of new intake is assumed or chosen arbitrarily.

In this study an estimator for the determination of the value of the constant rate of increase is derived. In addition, we derive the projection matrix; and the transition probability of Markov chain model via the maximum likelihood method.

2.0 The Basic Markov Chain Model for Manpower Planning.

The Markov chain model for manpower planning is as given by Raghavendra (1991), as

$$N_j(t+1) = \sum_{i=1}^{k} P_{ij}(t) N_i(t) + R_j(t) \qquad (1)$$

for i, j = 1, 2, ... k

where $N_j(t)$ is the number of staff in grade i at the beginning of the period t, $P_{ij}(t)$ is the probability that a member of staff will be promoted from grade i to grade j within the time period (t, t + 1) and $R_j(t)$ is the member of new

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recruits to grade j during period (t, t + 1).

Osagiede and Omosigho (2004) modified equation (1) as

$$E_g^t = P_{g-1} E_{g-1}^{t-1} + r_g E_g^{t-1} + N_g^t - W_g^t E_g^{t-1}, \qquad (g = 1, 2, ...m) \tag{2}$$

where E_g^t = enrolment in grade g, year t, N_g^t = new entrants to grade g, year t, W_g^t is wastage rate from grade g, year t, P_k^t, r_k^t are promotion and repetition rates, respectively.

Osagiede and Omosigho (2004) introduced the method for estimating the new intake in grade 1 as

$$N_1^t = \left(1 + \frac{\beta}{100}\right) \quad N_1^0 \tag{3}$$

where β is the rate of increase in new entrant into grade 1 and N_1^0 is the new entrant figure in the base year.

The states of a Markov chain represent classes or levels in the educational system. In most cases, estimation of $P_{ij}(t)$ is usually based on the maximum likelihood.

In sub-section 2.1 and 2.2, we derive β of equation (3) using regression analysis and $P_{ij}(t)$ by the maximum likelihood method.

2.1 Derivation of the Rate of Increase (β) in the Number of New Intake

To determine this, we assumed that the change in the number of new intake is proportional to the preceding one. That is

$$\Delta N_t^i = \beta N_{t-1}^i \tag{4}$$

where N_t^i is the number of new intake into level i, year t, and β is a constant.

Thus,

$$N_t^i - N_{t-1}^i = \beta \ N_{t-1}^i$$

So,

$$N_t^i = (1+\beta) N_{t-1}^i$$

$$= (1+\beta)\{(1+\beta) N_{t-2}^i\}$$

$$N_t^i = (1+\beta)^t \ N_0^i \tag{5}$$

Equation (5) is similar to that of Osagiede and Omosigho (2004).

Let $\alpha = 1 + \beta$, then

$$\ln N_t^i = \ln N_0^i + t \ln \alpha$$

The relationship between the variables t and $\ln N_t^i$ can be determined using regression analysis. Therefore the estimated equation of (5) becomes

$$\ln N_t^i = \ln \hat{N}_0^i + t \ln \hat{\alpha} + u_t, \text{ where } u_t \sim N\left(0, \sigma^2\right) \tag{6}$$

The best values of the variables $\ln \hat{N}_0^i$ and $\ln \hat{\alpha}$ can be determined by the principle of least squares according to which of these values of unknowns S is minimum, where $S = \sum_{t=1} u_t^2$, and u_t is the error term in equation (6)

$$\sum_{t=1}^{n} u_t^2 = \sum_{t=1}^{n} \left(\ln N_t^i - \ln N_0^i - t \ln \hat{\alpha}\right)^2 \tag{7}$$

So summing over equation (6) for all t and dividing by n, we have

$$\ln \hat{N}_0' = \frac{\sum\limits_{t=1} \ln N_t'}{n} - \frac{(n+1)}{2} \ln \hat{\alpha} \tag{8}$$

Insert (8) in (7) to obtain

$$\sum_{t=1}^{n} u_t^2 = \sum_{t=1}^{n} \left\{ \ln N_t' - \frac{\sum\limits_{t=1}^{n} \ln N_t'}{n} - \ln \hat{\alpha} \left(t - \frac{(n+1)}{2} \right) \right\}^2 \tag{9}$$

For a minimum value of S,

$$\frac{dS}{d \ln \hat{\alpha}} = 0$$

$$\Rightarrow \sum_{t=1}^{n} \left(t - \frac{n+1}{2} \right) \left(\ln N_t' - \frac{\sum\limits_{t=1}^{n} \ln N_t'}{n} \right) = \ln \hat{\alpha} \sum_{t=1}^{n} \left(t - \frac{n+1}{2} \right)^2$$

This can be simplified to

$$\ln \hat{\alpha} = \frac{12 \sum\limits_{t=1}^{n} t \ln N_t' - 6(n+1) \ln \prod\limits_{t=1}^{n} N_t'}{n(n^2 - 1)} \tag{10}$$

Also from equation (8)

$$\hat{N}_0' = \alpha^{-\frac{1}{2}(n+1)} \left(\prod_{t=1}^{n} N_t' \right)^{1/n} \tag{11}$$

2.2 Transition Probability (Pij(t))

Let the flow level from level i to j during period t be denoted by $n_{ij}(t)$. Let the distribution be multinomial with probabilities $P_{ij}(t)$ for all i, j = 1(1)r (Uche and Ezepue, 1991). The distribution is

$$P(n_{i1}(t), n_{i1}(t), \ldots, n_{ir}(t)) = \frac{\left(\sum\limits_{j=1}^{r} n_{ij}(t) \right)!}{\prod\limits_{j=1}^{r} (n_{ij}(t))!} \prod P_{ij}^{n_{ij}(t)}(t) \tag{12}$$

Using the likelihood function as stated in Lindgren (1993), it is easy to show that:

$$\hat{P}_{ij}(t) = \frac{n_{ij}(t)}{n_i(t)}, \qquad i = 1(1)m, \quad j = 1(1)r, \; t = 1(1)n \tag{13}$$

Thus, given stationarity of the probabilities, the maximum likelihood estimates of P_{ij} is obtained by pooling the results in equation (13). That is,

$$\hat{P}_{ij}(t) = \frac{\sum\limits_{t=1} n_{ij}(t)}{\sum\limits^{n} n_i(t)} \tag{14}$$

3.0 The Enrolment Projection Model

In this section, we propose a model for the projection of enrolment structure of the academic department under study. To do this, some assumptions are made.

3.1 Assumptions

To develop the projection model, the following assumptions are made

1. There are six levels in the course of study
2. Students enter the school system through level one (100 level) or through direct entry into level two (200 level).
3. The change in the number of new intake is proportional to the previous new intake.
4. Promotion from one level to the next is based on attaining a minimum of 10 credit course load; otherwise, the student is withdrawn from the university. In other words no repetition of classes or levels is allowed, except in level six. That is , $P_{ii}(t)=0$ for $i=1,2,\ldots,5$.
5. No double promotion and no demotion. That is,
   $P_{ij}(t)=0 \quad \text{for all } j > i+1 \text{ and } j \leq i-1$
6. The probability of withdrawal of student and the probability of replacement by new students are independent with probability $(P_{i.})$ $(P_{.j})$,
   $j = 1, 2$; $i = 1,2,\ldots,6$.
7. The probability estimates are stationary;
8. It is assumed there is no withdrawal in level 6 for whatever reason.

From the statements of assumption above, the stocks and flows of the academic system can be represented as in figure 1.

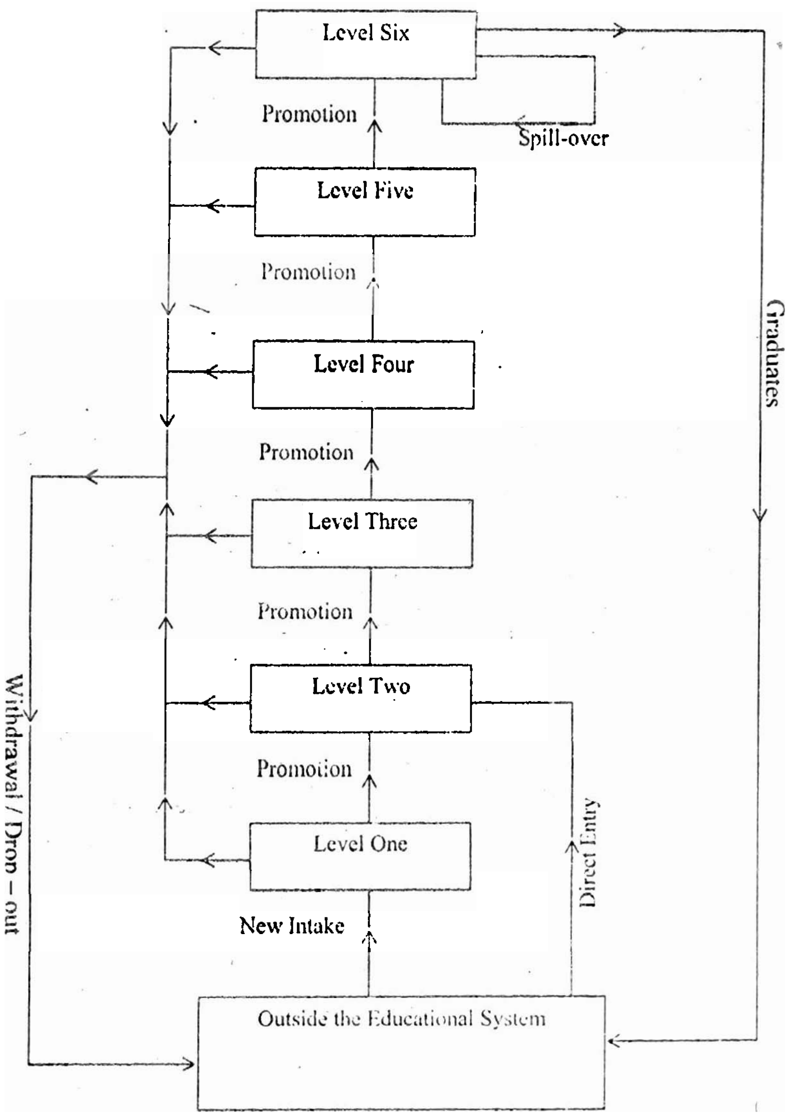

Fig. 1: Stocks and Flows in the Academic System

3.2 Notation

The following notations are used:

N_t^i	numbers of fresh students admitted into level I, year t; i = 1, 2;
t = 1, 2, ..., n	planning periods, with n being the horizon: usually each value of t represents a session;
i, j = 1, 2, ..., m	'states' of the system, representing the various levels of students in the course of study;
$P_{ij}(t)$	the transition probability of a person in level i moving to level j within the t-th session, i, j = 1,2,...,6;
$P_{io}(t)$	the probability of wastage due to a student at level i leaving the system within the t-th session, i = 1,2,...,6;
$P_{oj}(t)$	the probability of admission into level j at the beginning of the t- th session, j = 1, 2;
$n_i(t)$	the number of students in level i at the beginning of the t-th session;
N(t)	the total size of the system at the beginning of the t-th session;
$n_{ij}(t)$	the number of students (representing the promotion flow) who move from level i to j within the t-th session;
$n_{oj}(t+1)$	the admission flow to level j at the beginning of the (t + 1)th session; j = 1, 2;
β	the percentage change in the number of new intake into level one;
$(\gamma-1)$	the percentage change in the number of new intake into level two.
w_i	withdrawal from level i, i = 1(1)5
r_{oj}	the number of students admitted into level j, j = 1, 2;
Δ^*	the probabilistic difference arising from fresh students admitted into level j,j=1,2.

Based on the assumptions in sub – section 3.1 we have

$$N_t^1 = P_{01}\left\{\sum_{i=1}^{6} P_{i0}\ N_{t-1}^i + (\text{growth size})\right\}$$

Growth size = $N_t^i - N_{t-1}^i = \Delta N_t^i$ $\quad$ *where i = 1*

$$N_t^i = P_{01}\sum_{i=1}^{6} P_{i0}\ N_{t-1}^i + \Delta^* N_t^1, \tag{15}$$

where Δ^* is the probabilistic difference arising from fresh students admitted into level j, j = 1, 2.

i.e. $\Delta^* = P_{oj}\Delta$

Then

$$\Delta^* N_t^1 = P_{01}\left|\left(N_t^1 - N_{t-1}^1\right)\right| > 0$$

$$N_t^2 = P_{12}\ N_{t-1}^1 + P_{02}\sum_{i=1}^{6} P_{i0}\ N_{t-1}^i + \Delta^* N_t^2 \tag{16}$$

where

$$\Delta^* N_t^2 = P_{02}\left|\left(N_t^2 - N_{t-1}^2\right)\right|$$

$$N_t^g = P_{(g-1)g}\ \ N_{t-1}^{(g-1)} \quad ; \qquad g = 3, 4, 5 \tag{17}$$

and

$$N_t^6 = P_{56}\ N_{t-1}^5 + P_{66}\ N_{t-1}^6 \tag{18}$$

Therefore, relating equations (15), (16), (17) and (18) together give the projection matrix,

$$Q^T = \begin{pmatrix} (P_{10})(P_{01}) & (P_{20})(P_{01}) & (P_{30})(P_{01}) & (P_{40})(P_{01}) & (P_{50})(P_{01}) & (P_{60})(P_{01}) \\ (P_{10})(P_{02}) + (P_{12}) & (P_{20})(P_{02}) & (P_{30})(P_{02}) & (P_{40})(P_{02}) & (P_{50})(P_{02}) & (P_{60})(P_{02}) \\ 0 & P_{23} & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{34} & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{56} & P_{66} \end{pmatrix}$$

Recall equations (4), (5), (15) and (16) that

$$\begin{aligned} \Delta N_t^i &= \beta\ N_{t-1}^i \\ N_t^i &= (1+\beta)^t\ N_0^i \qquad , i = 1,2 \\ \Delta^{\bullet} N_t^1 &= P_{01}\left(N_t^1 - N_{t-1}^1\right) \end{aligned}$$

and

$$\Delta^{\bullet}\ N_t^2 = P_{02}\left(N_t^2 - N_{t-1}^2\right)$$

So,

$$N_t^i - N_{t-1}^i = (1+\beta)^t\ N_0^i - (1+\beta)^{t-1} N_0^i$$

$$\therefore \Delta N_t^i = \beta\,(1+\beta)^{t-1} N_0^i \tag{19}$$

Thus

$$\Delta^{\bullet}\ N_t^i = P_{.j}\ \Delta\ N_t^i \text{ for each i = j = 1, 2} \tag{20}$$

The equation of projection is given by

$$(n_i\ (t)) = Q^T(\bar{n}_i\,(t-1)) + \Delta^{\bullet}\ N^j(t)\ , \quad \begin{array}{l} i = 1,2,\dots,6 \\ j = 1,2 \end{array} \tag{21}$$

where. $(n_i\ (t))$ is a vector showing the number of students in level i at the beginning of the t th session and $(\bar{n}_i\ (t$ is a 6 x 1 vector of expected level sizes of i at time t.

$$\begin{aligned} (\hat{n}_i\ (1)) &= Q^T(\bar{n}_i\,(0)) + \Delta^{\bullet}\ N^j(1) \\ (\hat{n}_i\ (2)) &= Q^T(\bar{n}_i\,(1)) + \Delta^{\bullet}\ N^j(2) \\ (\hat{n}_i\ (3)) &= Q^T(\bar{n}_i\,(2)) + \Delta^{\bullet}\ N^j(3) \\ &= Q^T\left\{Q^T(\bar{n}_i\,(1)) + \Delta^{\bullet}\ N^j(2)\right\} + \Delta^{\bullet}\ N^j(3) \\ (\hat{n}_i\ (3)) &= \left(Q^T\right)^3(\bar{n}_i\,(0)) + \left(Q^T\right)^2\Delta^{\bullet}\ N^j(1) + Q^T\Delta^{\bullet}\ N^j(2) + \Delta^{\bullet}\ N^j(3) \end{aligned}$$

Hence, we have, in general,

$$(\hat{n}_i\ (t)) = \left(Q^T\right)^t(\bar{n}_i\ (0)) + \sum_{c=1}^{t}\left(Q^T\right)^{t-c}\Delta^{\bullet}\ N^j(c) \tag{22}$$

where the hat in the variable denotes estimates, and

$$\Delta^{*} N^{j}(c) = P_{0,j}\, \beta(1+\beta)^{c}\, N_{0}^{i} \tag{23}$$

Equation (22) is the equation of projection. This is a modification of the system in McClean (1991). The matrix Q, which is the transpose of the projection matrix Q^T in equation (22), is stochastic since,

$$\sum^{6} P_{ij} + P_{i.} = 1 \tag{24}$$

4. CONCLUSION

This work refines the work of previous authors – McClean (1991), Uche and Ezepue (1991), Osagiede and Omosigho (2004) – in the sense that given data for an educational system, we can analyse the system using a unique value for the constant percentage increase, β. Since data are affected by different uncontrollable socio – economic factors, regression method (method of least squares) is proposed for estimating the trend. The growth rate in enrolment is taken into consideration in this work. For instance, the increment in β% value that is assumed in Osagiede and Omosigho (2004) can now be calculated using the method in this paper (see equations 10 and 11). Moreover, some estimators for the parameters of the model were derived to validate their use (see equations 14, 22 and 23)

Therefore, we suggest practical illustration of the methods proposed in this paper for further research.

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