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A TEST FOR THE PARAMETERS OF MULTIPLE LINEAR REGRESSION MODELS

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ABSTRACT

A test for the parameters of multiple linear regression models is developed for conducting tests simultaneously on all the parameters of multiple linear regression models. The test is robust relative to the assumptions of homogeneity of variances and absence of serial correlation of the classical F-test. Under certain null and alternative hypotheses, the new test statistic is shown to have limiting central and noncentral chisquare distributions, respectively. A measure of efficiency due to Pitman is used to obtain the asymptotic efficiency of the new test relative to its classical counterpart. A numerical comparison of the two types of tests shows that the present test is slightly more efficient than the classical F-test.

KEY WORDS: test, multiple linear regression, parameters, robust, homogeneity of variances.

1. INTRODUCTION

Adichie (1967) developed a sign rank statistic for conducting tests simultaneously on intercepts and slopes in simple linear regression models. Jogdeo (1964) constructed a statistic for performing tests simultaneously on all the parameters of multiple linear regression models. Onuoha (1977) also generalized Adichie's technique by using a statistical model similar to that of Jogdeo in obtaining a sign-rank statistic for testing hypotheses about the complete set of parameters in multiple linear regression models.

The present paper develops an absolute–value test statistic for conducting tests similar to those of Jogdeo (1964) and Onuoha(1977)

In the above papers and in the present paper, the test statistic is a quadratic form consisting of component test statistics and the inverse of a covariance matrix, in the papers cited above, the component test statistics are defined in terms of the regression constants as well as the signs and/ or ranks of the observations, while, in this paper, they are defined in terms of the observations and the absolute values of the regression constants.

Hence, for large observations, computations of values of the component test statistics are easier to obtain in this paper than in the above papers. Moreover, the new test statistic is robust relative to the assumptions of homogeneity of variances and absence of serial correlation of the classical F- test. This is because the proposed test statistic does not depend strictly on the variance of the observations but on the variance-covariance matrix of the component test statistics defined in (2.5).

2. THE PROPOSED TEST STATISTIC

Instead of using the signs and ranks of the observations as in Onuoha (1997), we have used the observations as well as the absolute values of the regression constants to construct the present statistic. The multiple linear regression model is of the form

$$Y_{ni} = \beta_o + \beta_1\chi_{1i} + \ldots + \beta_p\chi_{pi} + Z_{ni}$$

$$Y_{ni} = \sum_{j=o}^{p} \beta_j\chi_{ji} + Z_{ni}; 1 \le i \le n < \infty, \chi_{oi} \equiv 1 \text{ for all } i \qquad (2.1)$$

Where Y_{ni} are independent random variables with distributions

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$$p(Y_{ni} \le y \mid \beta_j) = F(y - \sum_{j=o}^{p} \beta_j x_{ji}), \tag{2.2}$$

And where F is a distribution function, x_{ni} are known regression constants because they are assumed to be known without error. Z_{ni} are independently and identically distributed random variables with zero means and unit variances and the β_j's are the unknown parameters under test

$$\left(-\infty < \beta_j < \infty\right)$$

The problem is to test the following null and alternative hypotheses:

$$H_0 : \beta_j = 0 \quad \text{for all j} \tag{2.3}$$

$$H_n : \beta_j = n^{-1/2} b_j \tag{2.4}$$

Where n is the sample size of the observations.

$H_n : \beta_j = n^{-1/2} bj$ implies that the β_j's take values other than zero.

This is in line with Onuoha (1977). Thus, H_n is seen to tend to H_o at the rate of $n^{-1/2}$ as n increases. Let the component test statistics and the covariance matrix be defined respectively, as

$$T_{nj} = n^{-1/2} \sum_{i=1}^{n} \mid \chi_{ji} \mid Y_{ni}, 0 \le j \le \text{P}, \chi_{0i} \equiv 1 \quad \text{for all i} \tag{2.5}$$

and

$$\lambda_{njk} = cov\left(T_{nj}, T_{nk}\right), 0 \le j,k \le \text{P} \tag{2.6}$$

Where |. | is the absolute value symbol and $\text{cov}(T_{nj}, T_{nk})$ is the covariance of T_{nj} and T_{nk}.
The proposed test statistic is a quadratic form consisting of the component test statistics ($T_{no}, T_{n1}, \ldots, T_{np}$) and the inverse of the covariance matrix $\| \lambda_{njk} \|$ and is given by

$$\text{M}n_a = \left(T_{n0}, T_{n1}, \ldots, T_{np}\right)' \| \lambda_{njk} \|^{-1} \left(T_{n0}, T_{n1}, \ldots, T_{np}\right) = T_n^1 \| \lambda_{njk} \|^{-1} T_n \tag{2.7}$$

$$\text{Where } T'_n = \left(T_{no}, T_{n1}, \ldots, T_{np}\right) \tag{2.8}$$

and $\| \lambda_{njk} \|^{-1}$ is the inverse of the (p+1) x (p+1) matrix, $\| \lambda_{njk} \|$, with elements in (2.6) while ||.|| is a matrix notation.

3. LIMITING DISTRIBUTION OF M_n UNDER H_o

Let E_o, Var_o, Cov_o and P_o, respectively, denote the expectation that are computed under H_o. From (2.1), (2.2) and the assumptions on Z_{ni}, we get

$$E(Y_{ni}) = \sum_{j=0}^{p} \beta_j \chi_{ji} \tag{3.1}$$

Under H_o in (2.3), we have

$$E_0(Y_{ni})=0 \tag{3.2}$$

From (2.1) and the assumptions on Z_{ni}, we also obtain

$$\mathrm{Var}_o\,(Y_{ni}) = \mathrm{Var}\,(Y_{ni})=1 \tag{3.3}$$

Using (3.2) in (2.5), we get $E_0(T_{nj}) = 0, 0 \le j \le p$ (3.4)

Also using (2.5) and (3.3) in (2.6), we have

$$\lambda_{njk} = \underset{0}{\mathrm{cov}}\left(n^{-12}\sum_{i=1}^{n} |\chi_{ji}|Y_{ni}, n^{-1/2}\sum_{i=1}^{n}|\chi_{ki}|Y_{ni}\right)$$

$$= n^{-1}\sum_{i=1}^{n}|\chi_{ij}||\chi_{ik}|, \chi_{io} \equiv 1 \text{ For all i} \tag{3.5}$$

$$\mathrm{var}_0(\,T_{nj}) = \lambda_{njj} = \lambda^2{}_{nj} = n^{-1}\sum_{i=1}^{n}|\chi_{ij}|^2 = n^{-1}\sum_{i=1}^{n}\chi_{ij}^2 \tag{3.6}$$

since every T_{nj} in (2.5) is the sum of independent random variables, the central limit theorem (Meyer(1973)) applies. If we let $L(T_n|P) \to N(a, b^2)$ denote that the distribution law of $(T_n-a)/b$ tends to the standard normal distribution under P, then we obtain from (3.4) and (3.6) that

$$L\,(T_{nj}|\,P_0) \longrightarrow N(0,\ \lambda_j^2\ ), 0 \le j \le p \tag{3.7}$$

Where $\lambda_j^2 = \lim \lambda_{nj}^2$ given in (3.6).

We have shown that, under Ho, the marginal distributions of the T_{nj}'s tend to normal distributions. To prove the joint asymptotic normality of the T_{nj}'s, we use a well-known theorem of Cramer (1945). To this effect and for arbitrary constants $a_j (0 \le j \le P)$, we have

$$T_n^* = \sum_{j=0}^{p} a_j\ T_{nj} = \sum_{j=0}^{p} a_j n^{-1/2}\sum_{i=1}^{n}|\chi_{ji}|Y_{ni}$$

$$= n^{-1/2}\sum_{i=1}^{n}\left(\sum_{j=0}^{p} a_j|\chi_{ji}|\right)Y_{ni} \tag{3.8}$$

Since T_n^* is the sum of independent random variables, Y_{ni}, it follows from (3.2), (3.3) and central limit theorem that

$$L\,(T_n^*\,|P_0) \underset{n\to\infty}{\longrightarrow} N(0,\ \lambda^{*2}) \tag{3.9}$$

$$\text{Where } \lambda^{*2} = \underset{n\to\infty}{Lim}\, n^{-1}\sum_{i=1}^{n}\left(\Sigma_j a_j|\chi_{ji}|\right)^2, \chi_{0i} \equiv 1 \text{ for all i} \tag{3.10}$$

We therefore conclude that

$$\underset{n\to\infty}{L}(T_{no}, T_{n1},\ldots\ldots, T_{np})\,|P_o) \longrightarrow N_{p+1}(\underline{0}, \|\lambda_{jk}\|) \quad (3.11)$$

Applying the limit distribution of a continuous function of vector-valued random variables (Sverdrup (1952)) to (3.11),

We obtain,

$$\underset{n\to\infty}{L}(T_n \mid P_0) = \underset{n\to\infty}{L}\left((T_{n0}, T_{n1}, \ldots\ldots\ldots, T_{np}) \mid P_0\right) \to L(T \mid P_0) = N_{p+1}\,\underline{(0, \|\lambda_{jk}\|)} \quad (3.12)$$

and

$$\underset{n\to\infty}{L}(M_n \mid P_0) = \underset{n\to\infty}{L}(g(T_n) \mid P_0) \to L(g(T) \mid P_0) = L(T' \|\lambda_{jk}\|^{-1} T)$$

where $M_n = g\,(T_n) = T_n'\,\|\lambda_{njk}\| T_n$ (3.13)

From a well known theorem on quadratic forms (Adichie (1967)),

$$\underset{n\to\infty}{L}(M_n|P_o) \to \chi^2(u{=}p+1)$$

A direct consequence of this is that the critical function,

$$\varphi(M_n \mid P_0) = 1 \text{ if } M_n > \chi^2(\alpha, \mathrm{P}+1)$$

$$= 0 \text{ if } M_n \le \chi^2(\alpha; p+1)$$

Provides an asymptotic level α of H_0, where $\chi^2(\alpha, p+1)$ is the 100 (1-α)% point of the central chisquare distribution with (p+1) degrees of freedom

4. LIMITING DISTRIBUTION OF M_n UNDER H_n

Again, let E_n, Var_n, Cov_n and P_n, respectively denote that the expectation, the variance, and the probability are computed under the alternative hypothesis, H_n. From (2.4), (3.1) and (3.3), we have $E_n\,(Y_{ni}) = n^{-1/2} h_{ni}$,

and $var_n\,(Y_{ni}) = var_o\,(Y_{ni}) = 1$,

Where $h_{ni} = \sum_{j=0}^{p} b_j x_{ji}, 1 \le i \le n, \chi_{oi} = 1$ for all i (4.3)

Using (4.1) in (2.5) and (4.2) in (2.6), we get

$$E_n(T_{nj}) = n^{-1} \sum_{j=0}^{p} |\chi_{ji}|\, h_{ni} = \mu_{nj}, \quad (4.4)$$

and

$$\lambda_{njk} = Cov_n(n^{-1}\sum_{i=1}^{n} | \chi_{ji} | Y_{ni}, n^{-1/2}\sum_{i=1}^{n} | \chi_{ki} | Y_{ni}) = n^{-1}\sum_{i=1}^{n} | \chi_{ij} | . | \chi_{ik} | \qquad (4.5)$$

Equation (4.5) yie1ds

$$Var_n(T_{nj}) = \lambda_{njj} = \lambda^2_{nj} = n^{-1}\sum_{i=1}^{n} | \chi_{ij} |^2 = n^{-1}\sum_{i=1}^{n} \chi^2_{ij}. \qquad (4.6)$$

Using (4.4), (4.6) and the central limit theorem, we have

$$L(T_{nj} \mid P_n) \underset{n\to\infty}{\to} N(\mu_j, \lambda_j^2), 0 \le j \le p, \ ), \qquad (4.7)$$

Similarly,

$$L((T_{no}, T_{no}, T_{n1}, .., T_{np}) | P_n) \underset{n\to\infty}{\to} N_{p+1}(\underline{\mu}, || \lambda_{jk} ||), \qquad (4.8)$$

Where the R H S of (4.8) is the (P + 1)-variant normal distribution with mean vector $\underline{\mu}$ = lim $\underline{\mu}_n$ and covariance matrix $|| \lambda_{jk}||$ = lim $||\lambda_{njk}||$, and where $\underline{\mu}_n$ and $|| \lambda_{njk}||$ have their elements defined in (4.4) and (4.5), respectively. For the limiting distribution of M_n under H_n, we again use apply Sverdrup (1952) and another well known theorem on quadratic forms (Adichie (1967)). By these, equations (3.13) and (4.8) imply that

$$L(T_n|P_n) = L((T_{n0}, T_{n1}, \ldots, T_{np})|P_n) \underset{n\to\infty}{\to} L(T|P_n) \underset{n\to\infty}{=} N_{p+1}(\underline{\mu}, ||\lambda_{jk}||), \qquad (4.9)$$

and

$$L(M_n \mid P_n) = L((g(T_n) |P_n) \underset{n\to\infty}{\to} L((g(T) |P_n)) \underset{n\to\infty}{=} L((T'|| \lambda_{jk} ||^{-1}T)), \qquad (4.10)$$

Where T has it's usual meaning. The limiting distribution of M_n under H_n is completely specified by the well-known theorem on the distribution of quadratic forms cited earlier in this section (Adichie (1967)). Hence,

$$L(M_n|P_n) \to \chi^2(p + 1, \triangle), \qquad (4.11)$$

$$\text{Where } \triangle = \lim_{n\to\infty} \triangle_n. \qquad (4.12)$$

5 ASYMPTOTIC EFFICIENCY OF THE M_n-TEST

To obtain the asymptotic relative efficiency (ARE) of the M_n-test, ,with respect to its classical counterpart the $\hat{M}_n-test$, we employ a measure of efficiency due to Pitman (Noether (1954)) which is defined as follows : if under the same sequence of alternatives like the one stated in (2.4), two test statistics have noncentral chisquare limit distributions with the same number of degrees of freedom, it is shown in Noether (1954) that the ARE of the two tests is the ratio of their noncentrality parameters. Hence, in order to obtain the ARE of the M_n- test with respect to the $\hat{M}_n-test$, we only need to derive the noncentrality parameter, $\hat{\Delta}_n$, of the latter.

The classical test of H_0 assumes that F is the normal distribution function and uses the least squares (or the maximum likelihood) estimates, $\hat{\beta}_{nj}$, of $\beta_j (0 \le j \le p)$. The $\hat{M}_n-test$, statistic (c.f. eg. Adichie (1967)) is

$$\begin{aligned}\hat{M}_n &= n(\hat{\beta}_{no}, \hat{\beta}_{n1}, \ldots \hat{\beta}_{np})' \| \gamma_{njk} \|^{-1} (\hat{\beta}_{no}, \hat{\beta}_{n1}, \ldots \hat{\beta}_{np}) \\ &= n \hat{\beta}_n' \| \gamma_{njk} \|^{-1} \hat{\beta}_n, o \le j,k \le p,\end{aligned} \tag{5.1}$$

$$where\ \hat{\beta}_n' = (\hat{\beta}_{no}, \hat{\beta}_{n1} .. /, \hat{\beta}_{np}), \tag{5.2}$$

and $\|\gamma_{njk}\|^{-1}$ is the inverse of the $(p+1)\times(p+1)$

$$\text{matrix } \|\gamma_{njk}\| = \| \tau_{njk} \|^{-1}, o \le j,k \le p \tag{5.3}$$

$$\text{and where } \tau_{njk} = \lambda_{njk} = n^{-1} \sum_{i=1}^{n} \chi_{ijk}, o \le j,k \le p, \chi_{oi} \equiv 1 \text{ for all } i \tag{5.4}$$

The estimates $\hat{\beta}_{nj} (o \le j \le p)$, being linear functions of normal random variables, are themselves normal.

By Sverdrup (1952) and the theorem on quadratic forms used in section three of this study, $\hat{M}_n$ has a central chisquare distribution with (p + 1) degrees of freedom under H_o.
Hence,

$$\underset{n\to\infty}{L} (\hat{M}_n | P_o) \to \chi^2 (\nu = p + 1). \tag{5.5}$$

Besides, applying Adichie (1967) used in section four of this study, we have that, under any given alternatives of the form $\beta_j = \beta_{oj}$ $(0 \le j \le b)$,

$\hat{M}_n$ has a noncentral chisquare distribution with (p + 1) degrees of freedom and noncentrality parameter given by (c.f. eg Lehman (1959))

$$\hat{\Delta}_{n0} = n\underline{\beta}_0' \| \gamma_{njk} \|^{-1} \underline{\beta}_0, \tag{5.7}$$

where $\beta'_0 = (\beta_{00}, \beta_{01}, \ldots, \beta_{0p})$. (5.8)

It follows that under the sequence of near alternatives, H_n, given in (2.4), $\hat{\Delta}_{n0}$, becomes $\hat{\Delta}_n$, = (b_0, $b_1, \ldots b_p$)$'$ $\| \gamma_{njk} \|^{-1}$ (b_0, $b_1, \ldots, b_p$) $= \underline{b}' \| \gamma_{njk} \|^{-1} \underline{b}$ (5.9)

Where $\underline{b}' = (b_0\ b_1, \ldots\ b_p)$. (5.10)

The above results show that

$$L(\hat{M}_n \mid P_n) \to \chi^2 (p + 1, \hat{\Delta}) \tag{5.11}$$

Where $\hat{\Delta}$ = Lim $\hat{\Delta}_n$ is given in (5.9). Hence, the ARE of the M_n-test with respect to the $\hat{M}_n-test$, is given by ARE (M_n, $\hat{M}_n$) = $\Delta / \hat{\Delta}$. (5.12)

6. NUMERICAL COMPARISON OF THE M_n AND $\hat{M}_n$ TESTS:

Consider the data in table 6.1 below taken from Ronald (1996; P. 409) and used in Nwaigwe (2003)

Table 6.1: Younger's Advertising Data

y_{ni}	84	84	80	50	20	68
X_{1i}	13	13	8	9	9	13
X_{2i}	5	7	6	5	3	5

The model for analyzing the data is given by

$$Y_{ni} = \beta_0 + \beta_1 \chi_{1i} + \beta_2 \chi_{2i} + Z_{ni},\ 1 \le i \le 6 \tag{6.1}$$

Where β_0, β_1 and β_2 are the unknown parameters under test.

The problem is to test the following hypotheses :

$$H_0 : \beta_j = 0,\ 0 \le j \le 2. \tag{6.2}$$

$$H_n: \quad \beta_j = n^{-1/2} b_j,\ n = 6. \tag{6.3}$$

For this comparison, we take b_j ($0 \le j \le 2$) to be the least squares estimates, $\hat{\beta}_{nj}$, of β_j. These are given, respectively, by (see Nwaigwe (2003))

$$b_0 = \hat{\beta}_{n0} = -41.4654,\ b_1 = \hat{\beta}_{n1} = 2.5444,\ b_2 = \hat{\beta}_{n2} = 2.6047 \tag{6.4}$$

Using the data from Table 6.1 in (5.4),

$$||\tau_{njk}|| = 1/6 \begin{pmatrix} 6 & 65 & 31 \\ 65 & 733 & 341 \\ 31 & 341 & 169 \end{pmatrix} \quad (6.5)$$

From (5.3) and (6.5), we obtain

$$||\gamma_{njk}||^{-1} = \begin{pmatrix} 1 & 10.8333 & 5.1667 \\ 10.8333 & 122.1667 & 56.8333 \\ 5.1667 & 56.8333 & 28.1667 \end{pmatrix} \quad (6.6)$$

Equations (5.1), (6.4) and (6.6) give rise to $\hat{M}_n$=24904.8696 (6.7)

Using the data from Table 6.1 in (2.5) and (3.5), we have

T_{no} = 157.5838, T_{n1} = 1770.9811, T_{n2} = 872.8348 (6.8)

and

λ_{oo} = 1, = 10.8333, λ_{02} = 5.1667

λ_{10} = 10.8333, λ_{11} = 122.1667, λ_{12} = 56.8333 (6.9)

From (6.9) and Nwaigwe (2003), we have

$$||\lambda_{njk}||^{-1} = \begin{pmatrix} 33.3168 & -1.8154 & -2.4484 \\ -1.8154 & 0.2324 & -0.1359 \\ -2.4484 & -0.1359 & 0.7589 \end{pmatrix} \quad (6.10)$$

Using (6.8) and (6.10) in (2.7), we obtain the value of M_n under H_o as
M_n = 27452.62 (6.11)

But, under H_0, both $\hat{M}_n$ and M_n have limiting central chisquare distribution with 3 degrees of freedom. Hence, from the chisquare table, we have
χ^2 (0.05,3) = 7.815 and χ^2 (0.01,) = 11.34, (6.12)

We observe from (6.7), (6.11) and (6.12) that, if the two tests, $\hat{M}_n$ and M_n, were to be consistent while tending to the limit, both of them would reject H_0 at the 5% and 1% levels of significance. However, the M_n test would tend to reject H_o more often than its parametric counterpart, the $\hat{M}_n$ test.

Using (6.4) and (6.6) in (5.9), we have

$\hat{\Delta}_n$, = 4150.8116. (6.13)

For the noncentrality parameter, Δ_n, of the $\hat{M}_n - test$, we also use the data from Table 6.1 in (4.4) to obtain

μ_{no} = 61.1964, μ_{n1} = 687.7049, μ_{n2} = 339.7713. (6.14)

Using (6.10) and (6.14) in (4.12), we have

Δ_n = 4163.1161 (6.15)

Also using (6.13) and (6.15) in (5.12), we obtain the ARE M_n relative to $\hat{M}_n$ as ARE (M_n, $\hat{M}_n$) =

$\Delta_n / \hat{\Delta}_n$, = 100.2964% (6.16)

7. CONCLUSION

From the foregoing, we observed that if the two tests were to be consistent while tending to the limit the present M_n-test would be slightly more efficient than the classical F-test.

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