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CHARACTERISTICS OF MICROWAVE PROPAGATION IN VAPOURISED TROPOSPHERE

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ABSTRACT

Microwave communication equipment design in temperate region may not be very suitable in the tropics because the characteristics of the troposphere as the medium of propagation differ appreciably. This paper present an investigation on effect of vertical gradient of water vapour on refractivity gradient and microwave propagation in a typical troposphere of the tropics. The meteorological vertical data taken from radiosonde confirms evidence of super-refraction and ducting of the microwave signal.

KEY WORDS: Ducting, humidity, inversion, refraction, variation.

1.0 INTRODUCTION

The effects of the properties of the lower troposphere on very high frequency (VHF) Burrows, (1968) radio wave are manifested in a number of different forms. Some are used to provide anomalous long-distance transmissions beyond the line-of-sight by "scatter" or 'ducting' phenomena and other forms of degenerate conditions on line-of-sight transmissions. It is argued that the radio refractive index of the various parts of the troposphere in its various states is perhaps the single all-embracing parameter to which any influence of the V.H.F wave may be attributed.

The meteorological phenomena involved in the formation of tropospheric ducts or layering are abnormal vertical structures of temperature inversion and humidity, caused by such processes as subsidence, evaporation, advection and radiation cooling. Each of these by itself is effective and of sufficient magnitude.

The effects of the properties of the troposphere on VHF and microwave transmission result in time variations of the received signal strength. Variations in the level of the received signal are assumed to result from various forms of interference due to multiple path propagation. It is the intent of this paper to present the effects of humidity on microwave propagation in typical troposphere of the tropics.

2. Wave Propagation in the Idealized Medium

For the purpose of investigating electromagnetic propagation in the troposphere in its simplest form, an idealized atmosphere must be assumed for this region. Such medium would be considered completely stable, with constant gradients of dry-air density, temperature and specific humidity from the ground to the tropopause. These conditions will allow the variations of the dielectric constant, and more particularly, the refractivity with altitude to be estimated.

For a gaseous medium of dry-air, Debye, (1957) suggested that the dielectric constant may be expressed in the form:

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{4\pi\rho A}{3M}\left[\frac{\alpha_o + \mu^2}{3KT}\right] \qquad ....(1)$$

where ε = dielectric constant of the medium ,
ρ = density of the medium in kilograms/cm^3,
M = molecular weight in kilograms,
A = Avogadros constant (6.061×10^{26}), per kg
α_0 = Polarization factor,
μ = electric moment of a molecule,
K = Boltzman's constant (1.372×10^{-23}) joules/°K),

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T = absolute temperature in °K,

For most gases, $\varepsilon + 2 = 3$ to a high degree of accuracy so that equation (1) may be given in the form:

$$\varepsilon - 1 = \frac{4\pi\rho A}{M} \left(\frac{\alpha_o + \mu^2}{3KT}\right) \qquad \ldots(2)$$

Also for the perfect gas,

$$PV = \frac{RT}{M} \qquad \ldots(3)$$

where

P = gas pressure in millibars (mb)
V = specific volume in cc/kg.

R = gas constant,

Since

$\rho = \frac{1}{V} = \frac{PM}{RT}$ equation(2) may be written as

$$\varepsilon - 1 = \frac{4\pi A\rho}{RT} \left(\frac{\alpha_o + \mu^2}{3KT}\right) \qquad \ldots(4)$$

Since the constants α_o and μ are not readily separately measurable, equation (4) may be written in the simplified form:-

$$\varepsilon - 1 = \frac{KP}{T} \qquad \ldots(5)$$

The constant K can be calculated for a particular gas from measurable parameters P and T.

For a composite gas, the atmosphere is considered to contain water vapour, the linear addition theorem (partial pressures) may be applied to give,

$$\varepsilon - 1 = \frac{K_1 P_{g1}}{T} + \frac{K_2 P_{g2}}{T} + \frac{K_3 P_{g3}}{T} + \ldots\ldots \frac{K_n P_{gn}}{T} \qquad \ldots(6)$$

where P_{gn} are partial gas pressures.

For a sample of moist air of total pressure P (mb) and containing water vapour of partial pressure e (mb), equation (6) may be given as

$$\varepsilon - 1 = \frac{K_1}{T}(p - e) + \frac{K_2 e}{T} \qquad \ldots(7)$$

The velocity of propagation of a wave in any medium (V_m) other than free space is,

$$v_m = 1/\sqrt{\mu_r \ \varepsilon_r \ \varepsilon_x} \qquad \ldots(8)$$

where

μ_r = relative permeability of the medium.

ε_r = relative permitivity of the medium.

The velocity of propagation of the same wave in free space is given **as**:

$$v_s = 1/\sqrt{\mu_a \ \varepsilon_a} \qquad \ldots(9)$$

The refractive index n of the medium is thus

$$n = \frac{v_s}{v_m} = \sqrt{\frac{\mu_r \ \varepsilon_r}{\mu_a \ \varepsilon_a}} \qquad \ldots(10)$$

$$= \sqrt{\varepsilon_r} \text{ for air } = \mu_r = \mu_o$$

Since $\varepsilon = n^2$, then substituting n^2 for ε in equation (7)

$$n^2 - 1 = \frac{k_1}{T}(p-e) + k_2\frac{e}{T}$$

Then~ n^2 - 1~ =~ ε-1

n^2~ =~[1~+~(ε-1)]^{½}

n~=~1 + ½ (ε-1),

∴ 2(n-1) ≈ε-1

Substituting for ε-1 in (7),

$$\varepsilon - 1 = 2(n-1) = \{\frac{k_1}{T}(p-e) + k_2\frac{e}{T}\} \therefore n-1 = ½\{\frac{k_1}{T}(p-e) + k_2\frac{e}{T}\}$$

Then equation (7) may thus be written,

$$n - 1 = ½\{\frac{K_1}{T}(p-e) + \frac{K_2 e}{T}\} \qquad \ldots(11)$$

From the experimental results of many investigators such as Schelleng, (1933), Englund (1935),

Barrell (1951), Birnbaum (1951) and Smith (1953), the values of the coefficients K_1 and K_2 have been evaluated as
$K_1 = 158.10^{-9}$ °k/mb and

$$K_2 = 136.10^{-9} \left[\frac{T + 5582}{T}\right] \text{°k/mb.}$$

Substituting these values in equation (11) gives

$$(n - 1).\ 10^6 = \frac{77.6}{T}(P + 4810\frac{e}{T}) \qquad ...(12)$$

$$\text{Thus } N = (n - 1).\ 10^6 = \frac{P}{T} + 3.73 \ x \ 10^5 \ \frac{e}{T^2})$$

$$\text{or } N = (n - 1).\ 10^6 = 77.6 \ \frac{P}{T} + 3.73 \ x \ 10^5 \ \frac{e}{T^2})$$

2.1 Ideal Atmosphere

Equation (12) suggests that a decrease of temperature with altitude will give rise to an increase in the refractive index, and that the converse will apply to both partial air and water vapour pressures. However, the impossibility of formulating a universal law relating actual air pressure, water vapour pressure, temperature and refractive index with altitude, the International Radio Consultative Committee (CCIR) (1959) has adopted a basic empirical relationship of refractive index with altitude in the form,

$$n = 1 + 289 \times 10^{-6} \times e^{-0.136h} \qquad ...(13)$$

where h = altitude above mean sea level in km.

Alternatively, equation (13) may be given in the form:

$$N = (n - 1)\ 10^6 = 289e^{-0.136h} \qquad ...(14)$$

The refractive index gradient of this atmosphere is given as

$$dN/dh = d(n - 1)\ 10^6 = -\ 0.136 \times 289e^{-0.136h}$$

$$= -39.3e^{-0.136h} \text{ units/km.}$$

For low altitudes (less than 2000m), the vertical refractive index gradient is universally taken as the refractivity gradient for an ideal atmosphere is:

$$\frac{dn}{dh} = -39.3 \ units/km$$

$$\approx -4.0 \ x \ 10^{-6} \ units/km$$

$$= -40N \text{ units/km.} \qquad ...(15)$$

For an idealized atmosphere considered as the propagation medium, the actual ray path will not be straight but curved, since a constant refractive index gradient with altitude is assumed.. With a known value of this gradient, a modified radius of the earth may be determined in order that straight -line ray paths may agin be assumed.

Assuming a^1 to be the modified radius of the earth, it can be proved that

$$1/a^1 = 1/a + dn/dh \qquad ...(16)$$

where a is the true radius of the earth (=6370 km).

nd,

$$\frac{dn}{dh} = -40 \times 10^{-6} \text{ units/km} \qquad ...(17)$$

Thus, for an earth of radius 6370km which is surrounded by an atmosphere of constant refractive ndex gradient, the modified radius (a^1) allowing for the use of straight ray paths is found from,

$$1/a^1 = \frac{1}{6370} - 40 \ x \ 10^{-6}$$

$a^1 = 4a/3 = 8500$km ...(18)

ie $a^1 = k\,a$

where $k = \frac{4}{3}$ = the effective earth radius factor.

3.0 Wave Propagation in the Practical Atmosphere

The refractive index of the practical atmosphere is given by,

$$N = 77.6 \ \frac{P}{T} + 3.73 \ x \ 10^5 \ \frac{e}{T^2}) \text{ from equation 12.}$$

The first term is referred to as the "dry" component, and the second as the "wet" component.

Equation (12) suggests that changes in temperature will give rise to changes on the refractive index and its gradient, with altitude. In an atmosphere where the refractive index gradient is changing with altitude, a stratified form may be assumed and propagation over a spherical earth surface may be considered as propagation over a flat earth surface by using a modified refractive index, Dougherty, Dutton (1981)

Changes in the temperature gradient or specific humidity of the atmosphere can give rise to marked changes in the refractive index gradient of the troposphere which in turn may cause either sub-standard or super-standard refraction of a ray.

3.1 The Vertical Refractivity Gradient

The conditions that are conducive to super-refraction can be deduced from the differential of the refra index expression of equation (12) with height,

$$\frac{dN}{dh} = 77.6 \left[\frac{1dp}{Tdh} - \left(\frac{P}{T^2} + \frac{9620}{T^3}\right)\frac{dT}{dh} + \frac{4810}{T^2}\frac{de}{dh}\right]$$

Near the ground, for typical atmospheric conditions P = 1000 mb, T = 288^0 1C, e = 12 mb and H = 70%. The vertical refractivity gradient becomes

$$\frac{dN}{dh} = 0.27\frac{dp}{dh} - 1.3\frac{dT}{dh} + 4.5\frac{de}{dh} \qquad ...(13)$$

It follows that the vertical gradient of refractive index, dN/dh, at a height (h) and the lapse rate of this gradient are mainly determined by the gradients of pressure (P), temperature (T), and humidity (e H). Although P, T, and e vary with height, their numerical values have a much smaller effect on the gradient of refractive index.

The pressure of the air decreases with height always, and its gradient is dependent on weather conditions slightly. Therefore the first term in equation (13) is nearly constant and always negative. On the other hand, the vertical gradients of temperature and humidity are subject to strong variations, which are markedly dependent on weather conditions and may even change sign owing to temperature inversions, inside clouds and in moisture pockets. In the standard troposphere the temperature and humidity always decrease with height, which makes the reason for the two derivatives dT/dh and de/dh negative. Therefore, the absolute value of dN/dh takes a minus sign, and is obtained by subtracting the absolute value of the first and third terms.

Under abnormal weather conditions, temperature inversions may occur; instead of decreasing with height as is usual, the temperature increases within a particular height interval. With temperature inversion, dT/dh > o, and the second term in equation (13) takes a minus sign. The absolute value of dN/dh is obtained by adding together three negative terms.

Thus, among the conditions conducive to super-refraction, that is abnormally high negative values of $\frac{dN}{dh}$, are above all temperature inversions and an extremely high lapse rate of humidity with height.

Of these two factors, temperature inversion is the decisive one. However, the vertical vapour gradient could be so low as to cause super-refraction alone. Thus, the objective of this paper is to highlight the effects of vapour pressure gradient on the vertical refractivity gradient particularly, within the temperature inversion region.

3.2 Radio Ducts

The stratification of the troposphere due to the changing of the refractive index gradient with height gives rise to tropospheric ducts in which the radio wave is trapped resulting in a region of abnormally high field strength extending along the surface of the earth without limitation by the horizon, Stephansen, (1980). This is the phenomenon of super-refraction or trapping or ducting. Ducting occurs when the vertical refractive index gradient is less than - 157N/km. The radio ray is bent downward so as to have a radius of curvature less than that of the earth, Ajayi, (1989), Alfred et al, (2000) and the refractivity gradient $\frac{dN}{dh}$ > -157 N/km.

The first necessary condition for a duct to occur is that the refractive index gradient ($\frac{dN}{dh}$) shall be equal to or less than -157N/km. The second necessary condition is that the gradient ($\frac{dN}{dh}$) should be maintained over a height of many wave lengths.

Table 1: Maiduguri Upper Air Parameters for 11 - 12 - 91

Height (m)	Pressure (mb)	Temp. °C	Temp. °K °	Humidity H%	Vapour Pressure e(mb)
348	973.5	27.0	300.0	23	8.3
549	951.4	22.4	295.4	21	5.7
734	931.2	20.6	293.6	22	5.4
827	921.2	20.5	293.5	20	4.8
928	910.5	22.7	295.7	14	3.9
1140	888.5	23.4	296.4	12	3.5
1456	856.8	22.9	295.9	11	3.1
1837	819.9	21.4	294.4	16	1.1
2921	722.0	13.3	286.3	28	4.3

Table 2: Radio Upper Air Refractivity at Maiduguri on 11-12-91.

Height (m)	N_{dry}	N_{wet}	Refractivity N (N/km)
348	251.8	34.7	286.2
549	250.0	24.4	274.4
734	246.1	23.4	269.5
827	243.6	20.8	264.4
928	238.9	16.6	255.5
1140	232.6	14.9	247.5
1456	224.7	13.2	237.9
1837	216.1	17.6	233.7
2921	195.7	19.6	215.3

Table 3: Vertical Gradients of Meteorological Parameters at Temperature Inversion Altitudes.

S/NO	Height Range (m)	Pressure Gradient $0.27\frac{dp}{dh}$	Temperature Gradient $1.3\frac{dT}{dh}$	Vapour Pressure Gradient $4.5\frac{de}{dh}$	Refractivity Gradient dN/dh (N/km)	Radius of Ray R(km)
1	348 –635	-29.0	+12.2	-175.50	216.7	4614.7
2	348-448	-37.5	+22.1	-363.20	-422.8	2365.2
3	1754-1943	-25.7	+13.8	-119.30	-158.8	6297.2
4	900-1089	-28.1	+25.5	+87.30	-140.9	7097.2
5	348-447	-29.5	-35.5	-22.70	-41.2	24271.8
6	516-875	-29.1	+9.8	-25.6	-64.5	15503.9
7	760-935	-28.4	+12.6	-127.3	-168.3	5941.8
8	547-683	-29.4	+22.5	-115.1	-167.0	5988.0
9	775-928	-26.7	+20.4	-27.1	-74.1	13495.0
10	625-722	-29.0	+28.2	-127.6	-184.7	5114.2
11	881-1210	-27.0	+5.9	-93.8	-127.5	7843.1
12					-40	25000
13					-157	6370.0

4.0 Experimental Studies

In order to establish the empirical relationship between the signal strength and the vapour pressure, two different types of measurements were taken:

(i) Microwave signal level measurement to determine signal level variation.

(ii) Meteorological Upper Air Measurement for the determination of the variation of vertical gradients of radio refractivity, temperature, pressure and humidity (vapour pressure) and then to determine the single effect of vapour pressure on the refractivity gradient. Measurements were taken on NITEL microwave equipments ITT/BTM analogue equipments - at 2 Ghz and 6 Ghz. The transmitter output power was 3 watts (35dBm).

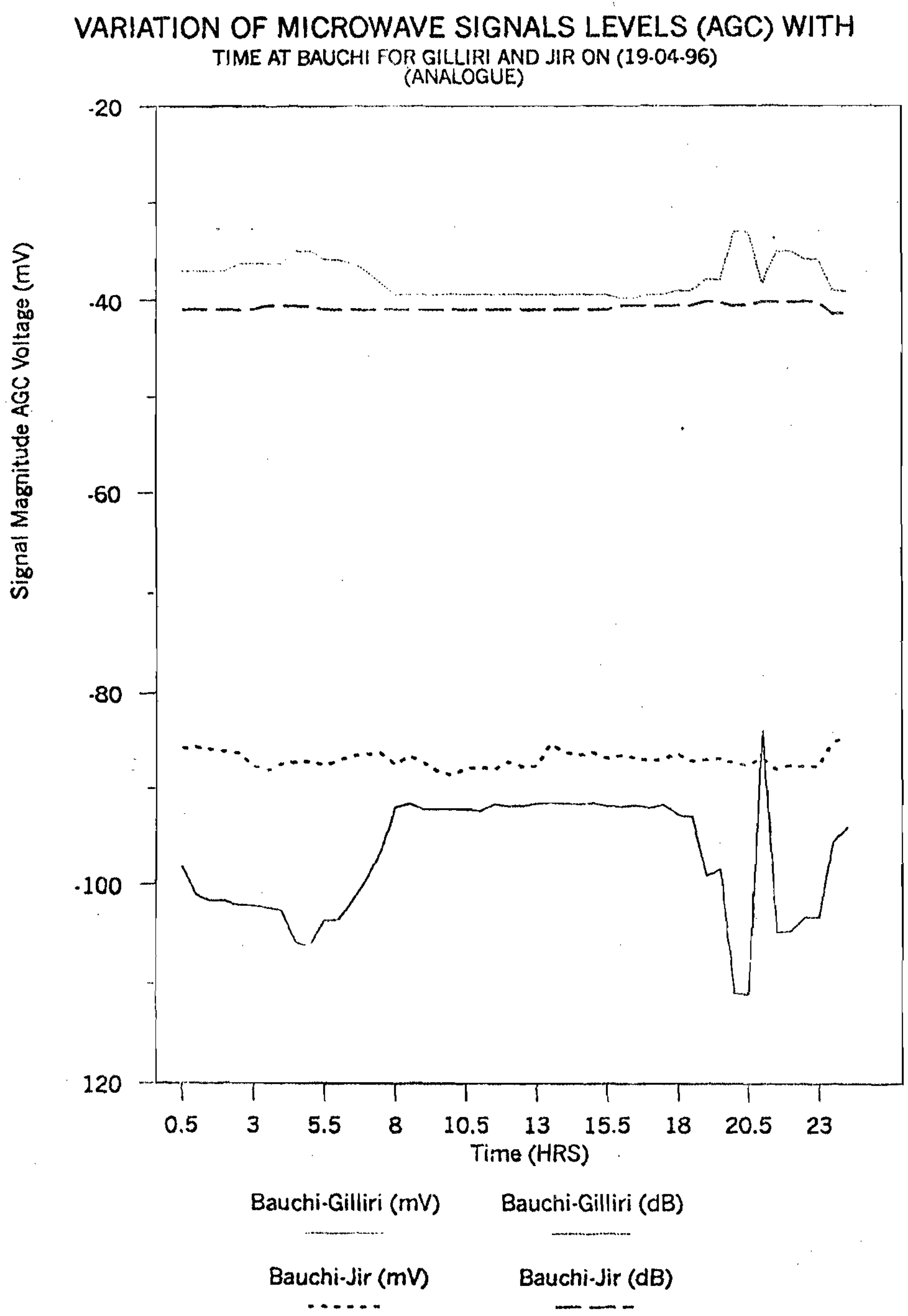


Fig. 1: Variation of Microwave Signal Levels Woth Time at Bauchi

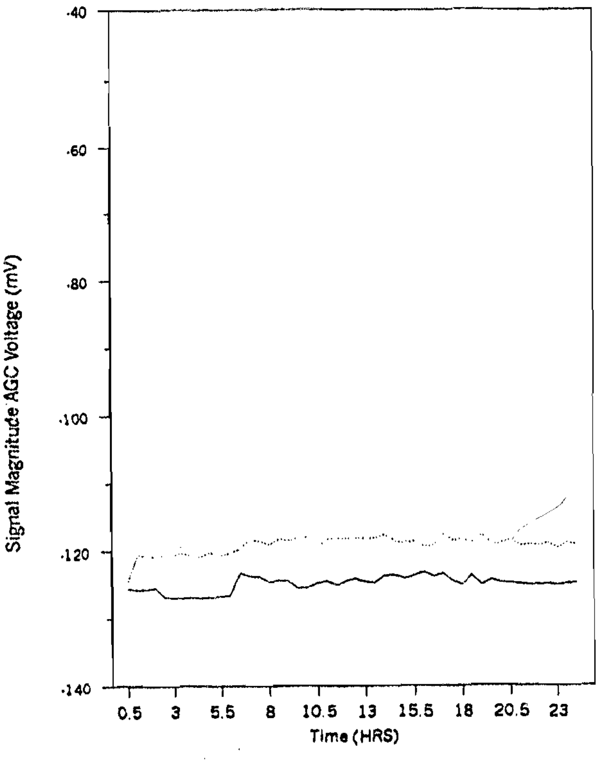


Fig.2 (a): VARIATION OF MICROWAVE SIGNALS LEVELS WITH TIME AT JOS For Mattari and Kassa Hill on (25-05-96) (ANALOGUE)

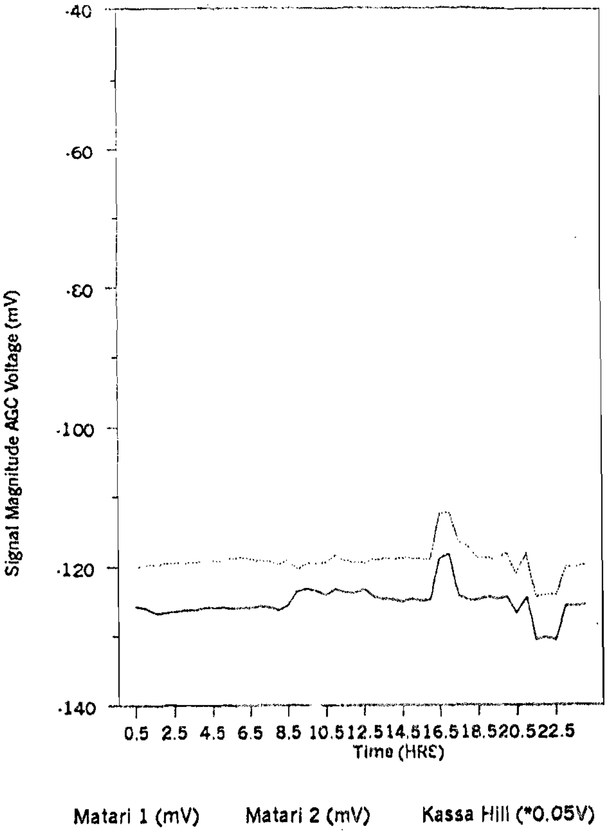


Fig. 2 (b): VARIATION OF MICROWAVE SIGNALS LEVELS WITH TIME AT JOS For Mattari and Kassa Hill on (13-06-96) (ANALOGUE)

Using digital voltmeters the signal automatic gain control (AGC) voltage was measured every 30 minutes daily at Bauchi, Jos and Maiduguri for three consecutive years. The AGC voltage is proportional to the carrier signal amplitude. Figures 1 to 3 show the variation of microwave signal levels with time at Bauchi, Jos and Maiduguri, respectively. The NITEL transmitters utilized for this study are situated at Jir and Gilliri for receivers at Bauchi, while transmitters at Matari and Kassa Hill are for reception at Jos. The transmitters situated at Kesawa and Kumala are for receivers at Maiduguri.

4.1 Meteorological Upper Air Measurements (Radiosonde):

The radiosonde (a balloon-borne radio transmitter) was used to measure pressure, temperature, and relative humidity in percentage up to a height of about 19km at 1200 hours GMT at Maiduguri airport. The measurements taken within the first 2000 metres altitude were meticulously recorded in order to detect the temperature inversion region which is important for terrestrial microwave propagation. The values found were then substituted in equation 12 to determine N and the excess refractive index was finally plotted as a function of height to produce a refractive index profile. See tables 1 and 2 and figs 5 and 6 respectively.

The values of the vertical gradients of pressure, temperature and vapour pressure were determined within the temperature inversion region (0 - 2000m high). These vertical gradients were then substituted into equation (12) to determine the vertical gradient of the refractivity, dN/dh, as indicated in Table 3.

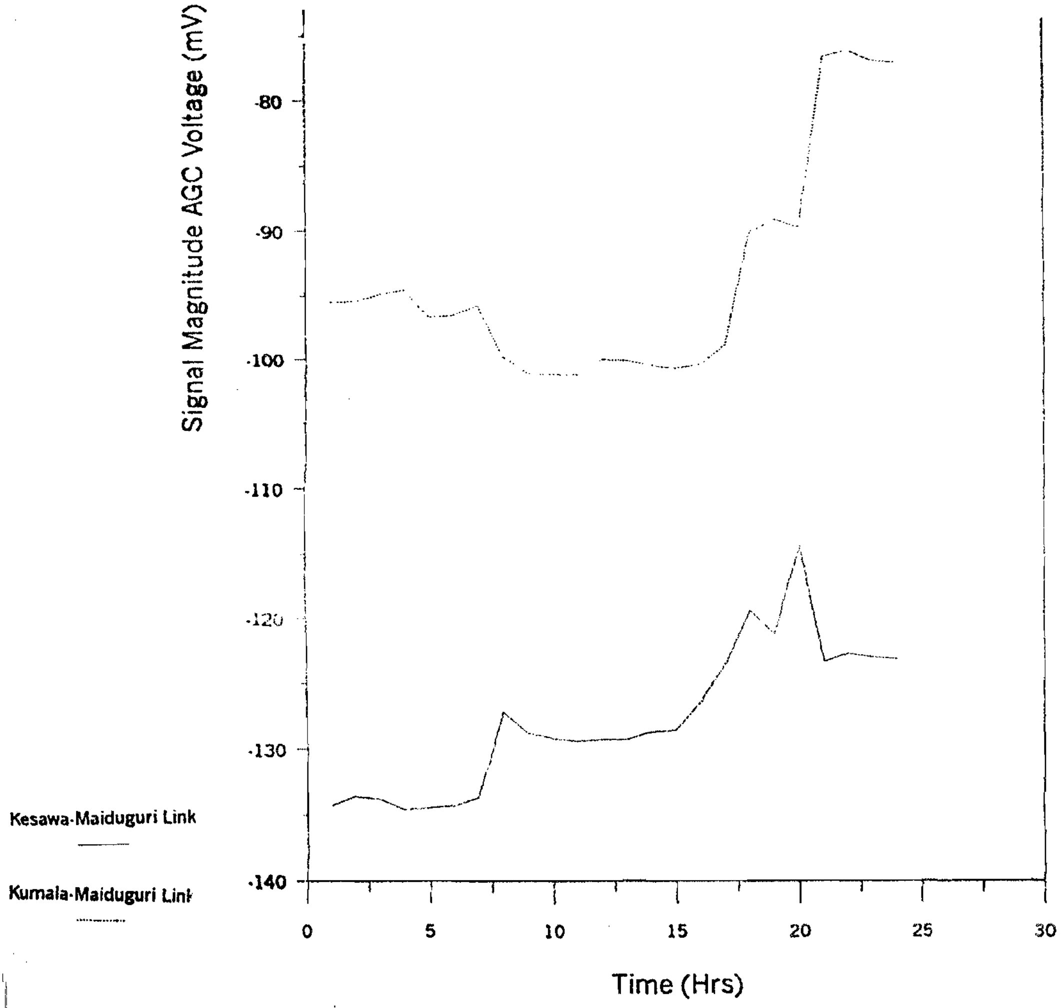
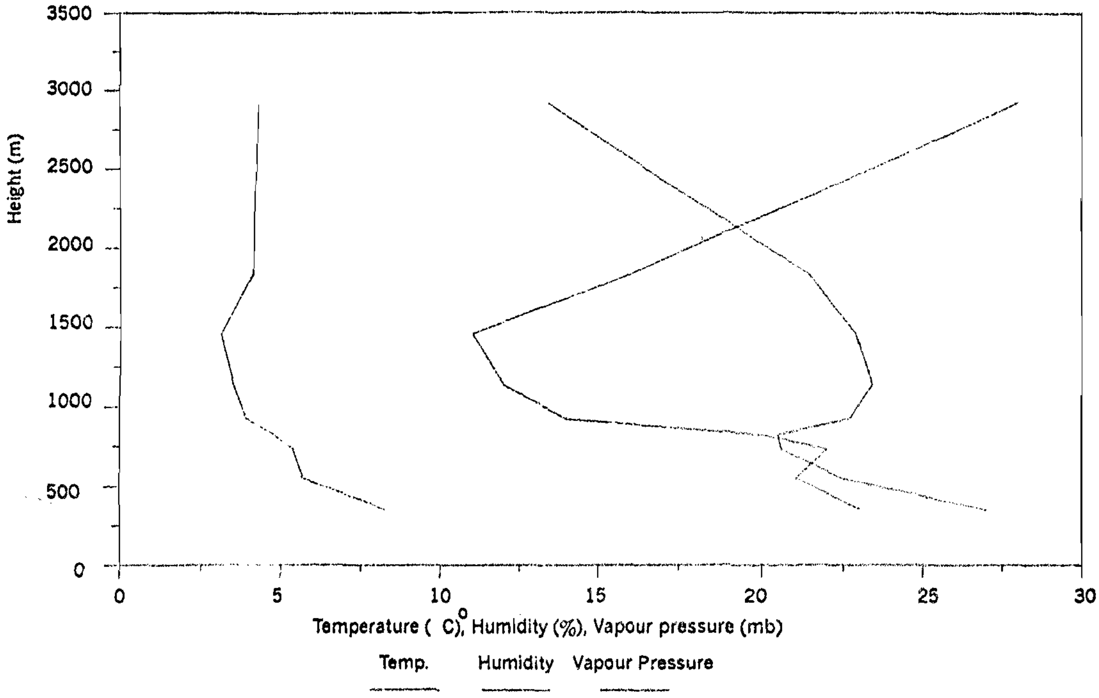
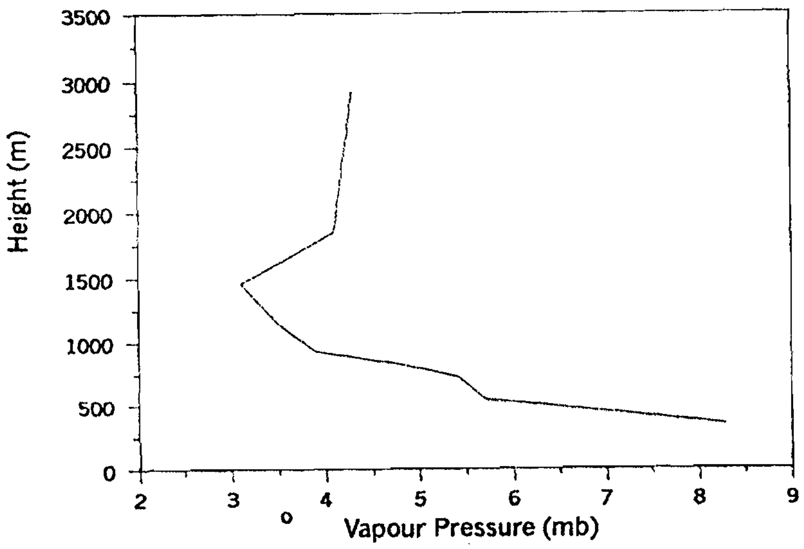


Fig. 3: Variation of Microwave Signal Levels with Time at Maiduguri on 02-01-96

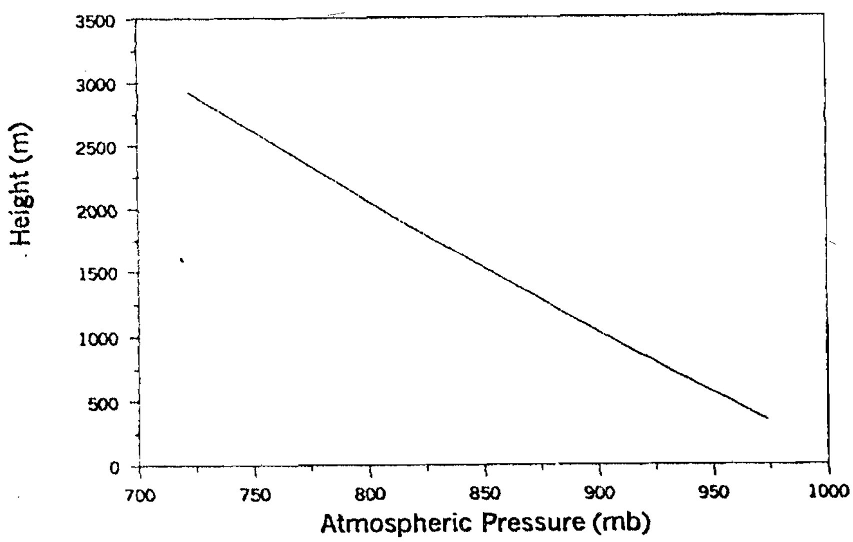


Note: The measured values of Temperature (°C), Humidity (%), Vapour pressure (mb) are numerically within the range of 0 - 30. Hence, for the respective graphs, the x-axis represents:- Temperature (°C), Humidity (%), Vapour pressure (mb)

Fig.4: Maiduguri Upper Air Parameters, 11-12- 91; Variation of Temperature Humidity and Vapour Pressure



(A) Vapour Pressure



(B) Atmospheric Pressure

Fig. 5: Vapour and Atmospheric Pressures (mb) with Height (m) at Maiduguri on 11-12-91within Temperature Inversion Region

DISCUSSION OF RESULTS

From the signal level shown in figures 1 to 3, it is seen that during the daytime, about 0900 hours –1800 hours, the microwave signal amplitude is almost constant. The daytime convective conditions do not affect the microwave signal amplitude. The atmosphere is well mixed and there may be no layer formation. The median signal level therefore remains constant with no amplitude variation. The microwave signal level is higher in the evening, 1800 hours, through the night to early morning hours. The signal enhancement and variation in late evening through the night to early morning hours support the conclusion that there is frequent anomalous propagation. The increase in signal level is believed to be due to duct propagation.

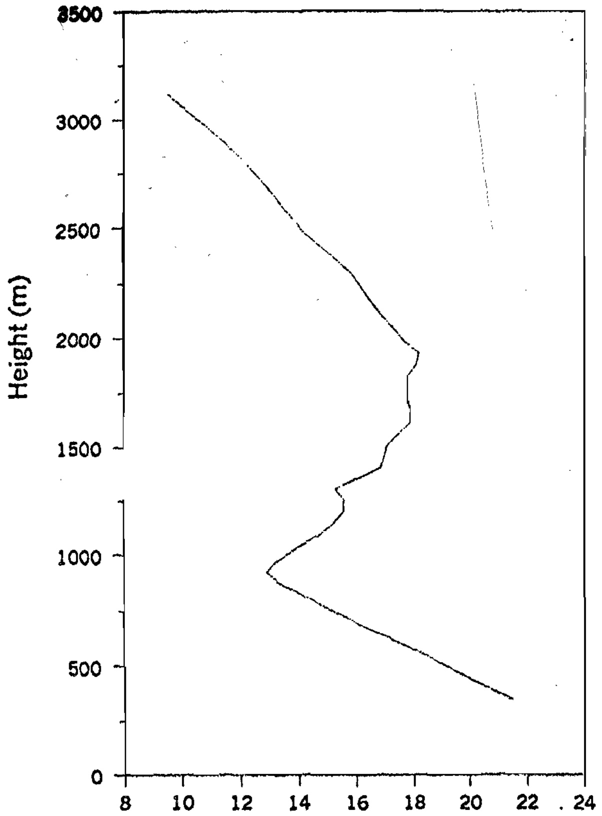


Fig. 6 (a): Variation of Temperature with Height at Maduguri on 15-01-91

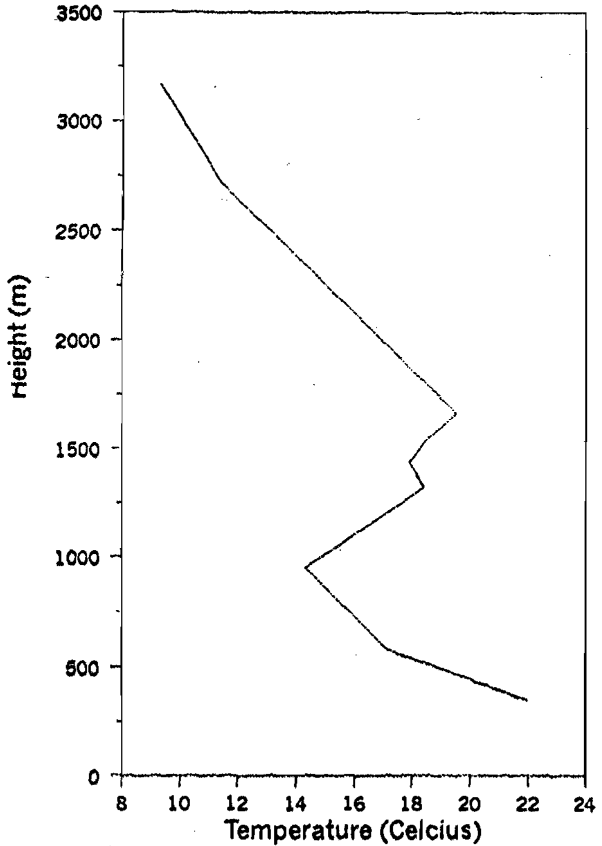


Fig. 6 (b): Variation of Temperature with Height at Maiduguri, 31-12-91

It is seen from Table 3 that the vertical pressure gradient is almost constant. Its effect on the variation of vertical refractivity gradient is therefore very slight. The vertical gradients of temperature and humidity vary greatly because they are markedly dependent on weather conditions. As seen in number 5 of Table 3 the humidity changed sing and became positive. It greatly increased the refractivity gradient. Based on the results, it can be said that the variation of the vertical gradients of temperature and humidity is mainly responsible for the variation of the vertical gradient of refractivity.

The humidity (vapour pressure) gradient can be so low as to be independently responsible for super-refraction and duct formation; the gradients were lower than –157N/km. Radio ducts give rise to duct propagation with consequent signal enhancement and range extension, severe fading and possible interference.

CONCLUSIONS

Based on the variation and enhancement of signal levels during the night, it is concluded that duct propagation takes place at night time. The vertical gradient of pressure is almost constant, and therefore, does not appreciably contribute to the variation of the vertical refractivity gradient. The vertical gradients of temperature and vapour pressure are primarily responsible for low refractivity gradients, less than -157N/km. They are also responsible for the variation in the refractivity gradient because they vary tremendously.

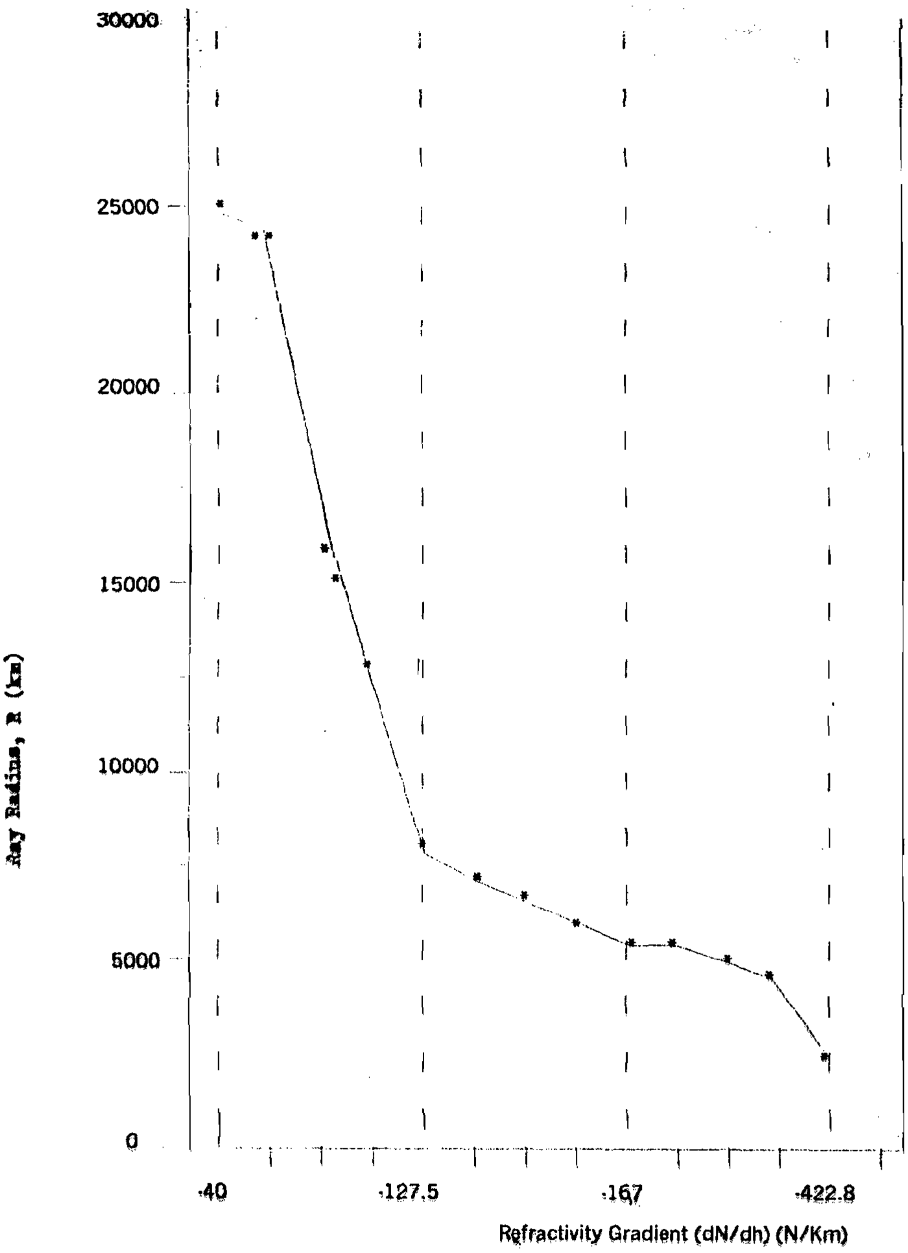


Fig. 7: Variation of Ray Radius with Refractivity Gradient

Humidity (vapour pressure) gradient can be so low, less than -157N/km, as to directly cause super-refraction independently. For the refractivity gradient to be less than -157N/km, the contribution of the vapour pressure gradient, 4.5 $\frac{de}{dh}$ must be less than -100 N/km. The details are shown in Table 3. The lower the refractivity gradient, the smaller the radius of ray curvature.

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