GLOBAL JOURNAL OF MATHEMATICAL SCIENCES VOL. 3, NO.2, 2004: 113 - 122
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A COMPARISON OF POISSON-ONE-INFLATED POWER SERIES DISTRIBUTIONS FOR MODELING RURAL OUT-MIGRATION AT THE HOUSEHOLD LEVEL

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(Received 18 December, 2002, Revision accepted 21 July, 2004).

ABSTRACT

A class of Poisson-one-inflated power series distributions (the binomial, the Poisson, the negative binomial, the geometric, the log-series and the misrecorded Poisson) are proposed for modeling rural out-migration at the household level. The probability mass functions of the mixture distributions are derived and fitted to the data in Sharma (1985). Their performance in describing the total number of out- migrants from a household are subsequently compared. It was found that while the Poisson-one-inflated log-series distribution as superior to others for modeling out-migration from semi-urban areas and growth centres, the Poisson-one-inflated geometric distribution was better in the case of out-migration from remote areas.

INTRODUCTION

Evidently, analysis of migration data at micro level has momentous implications for regional planning as well as for formulation of housing policies (Rossi, 1955; Pryor, 1975). Recently, micro level research on both residential mobility and migration has played a decisive role in development of the theory of migration (Dejong and Gardner, 1981; Speare et.al., 1975). The need for collection and analysis of migration data at the household level is based on the fact that the household is the basic socio-economic unit for integrated rural development. The number of migrants from a household has important bearings on the economic and cultural characteristics of the household. Households with at least one migrant are more prone to have new ideas than households having no migrants (Yadava and Singh, 1991).

Yadava and Singh (1991) identified three types of migrants as comprising of (i) adult males ($\geq$ 15 years) who migrate singly to a place, leaving their wives and children in their village homes, (ii) individuals who migrate with their wives and children, and (iii) males who migrate with their wives, children and some members of their households. They opined that the impact of these three types of migrants on the socio-cultural and economic characteristics of the household are different. It is obvious that migration of many members of a household especially females is more likely to affect the economic status and the socio-cultural outlook of the household. This consideration underscores the importance of concentrating attention on the pattern of distribution of total number of migrants from a household.

Yadava and Singh (1991) reviewed in details past efforts at studying the pattern of the total number of migrants at the household level. They pointed out that even though the models proposed by the authors provided satisfactory fits to the observed distribution of total number of migrants from a household, they have some limitations. Firstly, the prior distribution of males aged fifteen years and over is not known, secondly, the models do not take into account those households from where the wives, children and other members of the household migrate i.e. migration in clusters, and thirdly, the distribution of living children to a couple is not known. Taking these limitations into account, Yadava and Singh (1991) proposed a model that describes the variation in the total number of migrants from a household. Their model is based on the following assumptions:

(a) migrants from a household occur in clusters (groups), (b) migration from a household is a rare event, (c) the risk of a cluster of migrants vary from household to household.

Yadava and Singh (1991) assumed that the number of clusters migrating from a household follows the Poisson distribution, while the number of migrants in a cluster follows the one inflated zero truncated geometric distribution, since at least one person is expected to migrate in household exposed to the risk of migration. The use of the one-inflated distributions are justified by the need to reduce the risk of under-estimation of the probability that one person migrates in households that are exposed to the risk of

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migration. The use of zero-truncated distributions is not justifiable since the zeros are real and observable as there is the possibility that nobody migrates in a cluster in a household.

In this paper, we shall assume the Poisson distribution for the number of clusters migrating, and that the number of migrants in a cluster follows each of the members of the class of one-inflated power series distributions namely: the binomial, the Poisson, the negative binomial, the geometric, the log-series and the misrecorded Poisson. The probability mass functions of the mixture distributions proposed for modeling the total number of out-migrants from a household are derived and fitted to the data in Sharma (1985). Their performance in describing the total number of out-migrants from a household are then compared in terms of the goodness of fit.

The Models

Let Z_i, $i = 1, 2, \ldots, N$ denote the number of migrants from the ith cluster in a household and let N denote the number of clusters of potential migrants in a household. Then,

$$X = Z_1 + Z_2 + \ldots + Z_N$$

is the total number of migrants from a household.

Define $g(s) = \sum_{z=0}^{\infty} P_{Z_i}(z)s^z$ as the probability generating function (pgf) of Z_i where $P_{Z_i}(z)$ is the probability mass function (pmf) of Z_i.

Also, $h(s) = \sum_{n=0}^{\infty} P_N(n)s^n$ is the pgf of N where $P_N(n)$ is the pmf of N.

Then, the pgf of X is given as (Feller, 1968).

$$G_X(s) = h(g(s)) \tag{1}$$

from which the pmf of X can be derived. Specifically, $P_X(x)$ is the coefficient of the s^x in the expansion of $G_X(s)$ as a power series in s.

Assuming that N follows the Poisson distribution with pmf given as

$$P_N(n) = \frac{e^{-\theta}\theta^n}{n!}, \quad n = 0, 1, 2, \ldots, \infty \tag{2}$$

The pgf of N is given as

$$h(s) = e^{\theta(s-1)} \tag{3}$$

where θ is the mean number of clusters of migrants per household.

The mean and variance of the number of migrants in a household are respectively given (Feller, 1968) as

$$E(X) = G'_X(1) \; and \; Var(X) = G''_X(1) + G'_X(1) - [G'_X(1)]^2 \tag{4}$$

where $G'_X(1)$ and $G''_X(1)$ are respectively the first and second derivatives of $G_X(s)$ at $s = 1$

Using equation (1), the pmf of X are derived assuming that Z follows:

(i) the one-inflated binomial distribution
(ii) the one-inflated Poisson distribution
(iii) the one-inflated negative binomial distribution
(iv) the one-inflated geometric distribution
(v) the one-inflated log-series distribution
(vi) the mis-recorded Poisson distribution

The resulting mixed distributions are presented below.

(i) The Poisson-One-Inflated Binomial Distribution

$$P(Z=z) = \omega q^n \qquad z = 0$$
$$= (1-\omega) + \omega n p q^{n-1} \qquad z = 1$$
$$= \omega \binom{n}{z} p^z q^{n-z} \qquad z = 2, 3, \ldots, n \tag{5}$$

where n is the cluster size, p is the probability of a person migrating from a cluster, $p + q = 1$.

The pgf of Z_i is given as

$$G(s) = (1-\omega)s + \omega(q + ps)^n \tag{6}$$

Substituting equations (3) and (5) into equation (1) gives the pgf of X as

$$G_X(s) = \exp[\theta(1-\omega)s + \omega\theta(q + ps)^n - \theta] \tag{7}$$

Extracting the coefficient of s^x in equation (7), gives

$$P_X(x) = e^{-\theta(1-\omega q^n)} \qquad x = 0$$
$$= e^{-\theta} \sum_{r=0}^{x} \frac{[\theta(1-\omega)]^{x-r}}{(x-r)!} \sum_{i=0}^{\infty} \binom{ni}{r} \frac{(\omega\theta)^i}{i!} p^r q^{ni-r}, x = 1,2,\ldots,n \tag{8}$$

$$E(X) = \theta(1-\omega) + n\omega\theta p \tag{9}$$

$$Var(X) = \theta(1-\omega) + n\omega\theta pq + n^2\omega\theta p^2 \tag{10}$$

The estimating equations for the parameters θ, ω and p are

$$e^{-\hat{\theta}(1-\hat{\omega}\hat{q}^n)} = f_0 \tag{11}$$

$$f_0[\hat{\theta}(1-\hat{\omega}) + n\hat{\omega}\hat{\theta}\hat{p}\hat{q}^{n-1}] = f_1 \tag{12}$$

$$\hat{\theta}(1-\hat{\omega}) + n\hat{\omega}\hat{\theta}\hat{p} = \overline{X} \tag{13}$$

where

f_0 is the proportion of zero observations

f_1 is the proportion of one observations

$\overline{X}$ is the observed mean of the distribution.

(ii) The Poisson One-Inflated Poisson Distribution

$$P(Z=z) = \omega e^{-\lambda} \qquad z = 0$$
$$= (1-\omega) + \omega\lambda e^{-\lambda} \qquad z = 1$$
$$= \frac{\omega\lambda^z e^{-\lambda}}{z!} \qquad z = 2, 3, \ldots, \infty \tag{14}$$

where λ is the average number of persons migrating from a cluster.

The pgf of Z_i is given as

$$g(s) = (1 - \omega)s + \omega e^{\lambda(s-1)} \quad (15)$$

The pgf of X is given as

$$G_X(s) = \exp[\theta(1 - \omega)s + \theta\omega e^{\lambda(s-1)} - \theta] \quad (16)$$

Hence

$$P_X(x) = e^{-\theta(1-\omega e^{-\lambda})} \qquad x = 0$$

$$= e^{-\theta}\sum_{r=0}^{x}\frac{[\theta(1-\omega)]^{x-r}}{(x-r)!}\sum_{i=0}^{\infty}\frac{(\theta\omega e^{-\lambda})^i}{i!}\frac{(\lambda i)^r}{r!}, x = 1,2,\ldots,\infty \quad (17)$$

$$E(X) = \theta[1 - \omega + \lambda\omega] \quad (18)$$

$$Var(X) = \omega\theta\lambda^2 + E(X) \quad (19)$$

The estimating equations for the parameters θ, ω and λ are

$$e^{-\hat{\theta}(1-\hat{\omega}e^{-\hat{\lambda}})} = f_0 \quad (20)$$

$$f_0[\hat{\theta}(1-\hat{\omega}) + \hat{\theta}\hat{\omega}\hat{\lambda}e^{-\hat{\lambda}}] = f_1 \quad (21)$$

$$\hat{\theta}[(1-\hat{\omega}) + \hat{\lambda}\hat{\omega}] = \overline{X} \quad (22)$$

(iii) The Poisson-One-Inflated Negative Binomial Distribution

$$P(Z = z) = \omega p^m \qquad z = 0$$

$$= (1 - \omega) + \omega mqp^m \qquad z = 1$$

$$= \omega\binom{m+z-1}{z}p^m q^z \qquad z = 2, 3, \ldots, \infty \quad (23)$$

where $p = 1 - q$ is the probability of a person migrating from a cluster.

The pgf of Z_i is given as

$$g(s) = (1 - \omega)s + \omega p^m(1 - qs)^{-m} \quad (24)$$

The pgf of X is given as

$$G_X(s) = \exp[\theta(1 - \omega)s + \omega\theta p^m(1 - qs)^{-m} - \theta] \quad (25)$$

Thus,

$$P_X(x) = e^{-\theta(1-\omega p^m)} \qquad x = 0$$

$$= e^{-\theta}\sum_{r=0}^{x}\frac{[\theta(1-\omega)]^{x-r}}{(x-r)!}\sum_{i=0}^{\infty}\binom{mi+r-1}{r}\frac{(\omega\theta p^m)^i}{i!}q^r, x = 1,2,\ldots,\infty \quad (26)$$

$$E(X) = \theta(1 - \omega) + m\omega\theta qp^{-1} \quad (27)$$

$$Var(X) = \theta(1 - \omega) + m\omega\theta qp^{-2}(1 + mq) \quad (28)$$

The estimating equations for the parameters θ, ω, m and p are

$$e^{-\hat{\theta}(1-\hat{\omega}\hat{p}^{\hat{m}})} = f_0 \quad (29)$$

$$f_0[\hat{\theta}(1-\hat{\omega}) + \hat{m}\hat{\omega}\hat{\theta}\hat{q}\hat{p}^{\hat{m}}] = f_1 \quad (30)$$

$$[\hat{\theta}(1-\hat{\omega})+\hat{m}\hat{\omega}\hat{\theta}\,\hat{q}\hat{p}^{-1}] = \bar{X} \quad (31)$$

$$[\hat{\theta}(1-\hat{\omega})+\hat{m}\hat{\omega}\hat{\theta}\,\hat{q}\hat{p}^{-2}(1+\hat{m}\hat{q}] = \sigma^2 \quad (32)$$

where σ^2 is the observed variance of the distribution

(iv) The Poisson-One-Inflated Geometric Distribution

$$P(Z=z) = \omega p \qquad z = 0$$
$$= (1-\omega) + \omega pq \qquad z = 1$$
$$= \omega pq^z \qquad z = 2, 3, \ldots, \infty \quad (33)$$

where $p = 1 - q$ is the probability of a person migrating from a cluster.

The pgf of Z_i is given as

$$g(s) = (1-\omega)s + \omega p(1-qs)^{-1} \quad (34)$$

The pgf of X is given as

$$G_X(s) = \exp[\theta(1-\omega)s + \omega\theta p(1-qs)^{-1} - \theta] \quad (35)$$

Thus,

$$P_X(x) = e^{-\theta(1-\omega p)} \qquad x = 0$$

$$= e^{-\theta}\sum_{r=0}^{x}\frac{[\theta(1-\omega)]^{x-r}}{(x-r)!}\sum_{i=0}^{\infty}\binom{i+r-1}{r}\frac{(\omega\theta p)^i}{i!}q^r, x = 1,2,\ldots,\infty \quad (36)$$

$$E(X) = \theta(1-\omega) + \omega\theta qp^{-1} \quad (37)$$

$$Var(X) = \theta(1-\omega) + \omega\theta qp^{-2}(2-p) \quad (38)$$

The estimating equations for the parameters θ, ω and p are

$$e^{-\hat{\theta}(1-\hat{\omega}\hat{p})} = f_0 \quad (39)$$

$$f_0[\hat{\theta}(1-\hat{\omega})+\hat{\theta}\,\hat{\omega}\,\hat{p}\hat{q}] = f_1 \quad (40)$$

$$\hat{\theta}(1-\hat{\omega})+\hat{\omega}\hat{\theta}\,\hat{q}\hat{p}^{-1} = \bar{X} \quad (41)$$

Unlike the distribution proposed by Yadava and Singh (1991), this distribution incorporates the possibility that nobody migrates from a cluster in a household.

(v) The Poisson-One-Inflated Log-series Distribution

$$P(Z=z) = (1-\omega) + \omega\alpha p \qquad z = 1$$

$$= \frac{\omega\alpha\, p^k}{k} \qquad z = 2, 3, \ldots, \infty \quad (42)$$

where $p = 1 - q$ is the probability of a person migrating from a cluster.

$\alpha = -[\ln(1-p)]^{-1}$

The pgf of Z_i is given as

$$g(s) = (1-\omega)s - \omega\alpha\ln(1-ps) \quad (43)$$

The pgf of X is given as

$$G_X(s) = \exp[\theta(1-\omega)s - \theta\omega\alpha\ln(1-ps) - \theta] \quad (44)$$

Thus,

$$P_X(x) = e^{-\theta} \qquad x = 0$$

$$= e^{-\theta}\sum_{r=0}^{x}\binom{\beta+r-1}{r}p^r\frac{[\theta(1-\omega)]^{x-r}}{(x-r)!}, \quad x = 1,2,\dots,\infty \quad (45)$$

where $\beta = \theta\omega\alpha$

$$E(X) = \theta(1-\omega) + \theta p\omega\alpha q^{-1} \quad (46)$$

$$Var(X) = \theta(1-\omega) + \theta p\omega\alpha q^{-1} + \theta p^2\omega\alpha q^{-2} \quad (47)$$

The estimating equations for the parameters θ, ω and p are

$$e^{-\hat{\theta}} = f_0 \quad (48)$$

$$f_0[\hat{\theta}(1-\hat{\omega}) + \hat{\theta}\hat{p}\hat{\omega}\hat{\alpha}] = f_1 \quad (49)$$

$$[\hat{\theta}(1-\hat{\omega}) + \hat{\theta}\hat{p}\hat{\omega}\hat{\alpha}\hat{q}^{-1}] = \bar{X} \quad (50)$$

(vi) The Poisson-Misrecorded Poisson Distribution

The misrecorded Poisson distribution takes into account errors in reporting the number of migrants in a cluster (Johnson et.al., 1992)

$$P(Z=z) = e^{-\lambda}(1+\lambda\phi) \qquad z = 0$$

$$= \lambda e^{-\lambda}(1-\phi) \qquad z = 1$$

$$= \frac{\lambda^z e^{-\lambda}}{z!} \qquad z = 2, 3, \dots, \infty \quad (51)$$

where λ is the average number of migrants in a cluster, ϕ is the probability that one migrant recorded in a cluster is not reported. The pgf of Z_i is given as

$$g(s) = \lambda\phi e^{-\lambda} - \phi\lambda e^{-\lambda}s + e^{\lambda(s-1)} \quad (52)$$

The pgf of X is given as

$$G_X(s) = \exp[\lambda\theta\phi e^{-\lambda} - \lambda\theta\phi e^{-\lambda}s + \theta e^{\lambda(s-1)} - \theta] \quad (53)$$

Thus,

$$P(X=x) = A\exp(\theta e^{-\lambda}) \qquad x = 0$$

$$= A\sum_{r=0}^{x}\frac{(-\lambda\theta\phi e^{-\lambda})^{x-r}}{(x-r)!}\sum_{i=0}^{\infty}\frac{(\theta e^{-\lambda})^i(\lambda i)^r}{i!r!} \qquad x = 1,2,\dots,\infty \quad (54)$$

where $A = \exp[-\theta(1-\lambda\phi e^{-\lambda})]$

$$E(X) = \lambda\theta(1-\phi e^{-\lambda}) \quad (55)$$

$$Var(X) = \lambda\theta(1+\lambda-\phi e^{-\lambda}) + \phi^2e^{-2\lambda}(\theta^2\lambda^2-1) \quad (56)$$

The estimating equations for the parameters θ, ϕ and λ are given as

$$\hat{A}\exp(\hat{\theta}\, e^{-\hat{\lambda}}) = f_0 \qquad (57)$$

$$f_0[\hat{\theta}\hat{\lambda}\, e^{-\hat{\lambda}}(1-\hat{\phi})] = f_1 \qquad (58)$$

$$\hat{\theta}\hat{\lambda}(1-\hat{\phi}\, e^{-\hat{\lambda}}) = \overline{X} \qquad (59)$$

Application

The results derived are illustrated by fitting the various distributions and testing their adequacy for each of the village types: Semi-urban, Remote and Growth centre, using the data contained in Sharma (1985). The parameter estimates, the observed and expected frequencies and the χ^2 values for the different types of villages, based on each of the six distributions are presented in Tables 1 to 6 respectively.

Table 1: Observed and Expected Number of Households according to the Number of Migrants and Type of Village: The Poisson-One-Inflated Binomial Distribution

No. of Migrants	Number of Households					
	Semi-urban		Remote		Growth Centre	
	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.
0	1042	1042	872	872	978	978
1	95	95	176	176	154	154
2	19	14.7	59	45.3	47	36.1
3	10	9.9	18	22.6	18	21.0
4	2		6	12.1	9	11.9
5	2		4		1	
6	0	9.4	0	7.0	0	6.9
7	1		0		0	
8	0		0		1	
Total	1171	1171	1135	1135	1208	1208
$\hat{\theta}$	0.1172		0.2647		0.2122	
$\hat{\omega}$	0.2397		0.2531		0.2761	
$\hat{p}$	0.3175		0.2756		0.2923	
χ^2	3.3184		9.4405		7.9061	
df	1		2		2	

Source: The observed frequencies were extracted from Sharma (1985).

Table 2: Observed and Expected Number of Households according to the Number of Migrants and Type of Village: The Poisson-One-Inflated Poisson Distribution

No. of Migrants	Number of Households					
	Semi-urban		Remote		Growth Centre	
	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.
0	1042	1042	872	872	978	978
1	95	95	176	176	154	154
2	19	15.3	59	48.1	47	38.2
3	10	8.7	18	21.0	18	19.1
4	2		6	10.6	9	10.6
5	2		4		1	
6	0	10.1	0	7.2	0	8.1
7	1		0		0	
8	0		0		1	
Total	1171	1171	1135	1135	1208	1208
$\hat{\theta}$	0.1193		0.2762		0.2194	
$\hat{\omega}$	0.2423		0.2925		0.2991	
	2.4242		1.8566		2.0858	
$\hat{\lambda}$	3.6643		6.3171		6.9259	
χ^2	1		2		2	
df						

Source: The observed frequencies were extracted from Sharma (1985)

Table 3: Observed and Expected Number of Households according to the Number of Migrants and Type of Village: The Poisson-One-Inflated Negative Binomial Distribution

No. of Migrants	Number of Households					
	Semi-urban		Remote		Growth Centre	
	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.
0	1042	1042	872	872	978	978
1	95	95	176	176	154	154
2	19	27.6	59	61.1	47	55.7
3	10		18	18.2	18	14.5
4	2		6		9	
5	2		4		1	
6	0	6.5	0	7.6	0	5.7
7	1		0		0	
8	0		0		1	
Total	1171	1171	1135	1135	1208	1208
$\hat{\theta}$	0.1373		3.1306		0.4603	
	0.4765		1.0156		0.8792	
$\hat{\omega}$	3.1887		0.2959		1.3845	
$\hat{m}$	0.6949		0.7051		0.7043	
$\hat{p}$	13.7951		0.8323		7.1318	
χ^2	0		0		0	
df						

Source: The observed frequencies were extracted from Sharma (1985)

Table 4: Observed and Expected Number of Households according to the Number of Migrants and Type of Village: The Poisson-One-Inflated Geometric Distribution

No. of Migrants	Number of Households					
	Semi-urban		Remote		Growth Centre	
	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.
0	1042	1042	872	872	978	978
1	95	95	176	176	154	154
2	19	24.2	59	61.5	47	53.8
3	10		18	17.8	18	15.6
4	2		6		9	
5	2		4		1	
6	0	9.8	0	7.7	0	6.5
7	1		0		0	
8	0		0		1	
Total	1171	1171	1135	1135	1208	1208
$\hat{\theta}$	0.3410		0.6912		0.6226	
	0.9332		0.8966		0.9486	
$\hat{\omega}$	0.7046		0.6900		0.6966	
$\hat{p}$	3.8765		0.7909		4.3441	
χ^2	0		1		1	
df						

Source: The observed frequencies were extracted from Sharma (1985).

The Tables show that the Poisson-one-inflated log-series distribution provides a superior fit (on the basis of the χ^2 test) than all the other distributions for the same degrees of freedom, in modeling out-migration from semi-urban areas. On the basis of the parameters of this distribution, the average number of migrants among semi-urban households is estimated at 1.4.

In the case of households residing in remote areas, the Poisson-one-inflated geometric distribution provides a superior fit, followed by the Poisson-misrecorded Poisson. The Poisson-one-inflated log-series distribution strongly competes with them. The fact that the Poisson-misrecorded Poisson distribution provides a good fit implies that there is the possibility that one migrant reported in a household in remote areas might not have been recorded. Nonetheless, the probability of this occurring is small (0.0980). Based on the Poisson-one-inflated geometric distribution, the average number of migrants in households residing in remote areas is 0.35.

Table 5: Observed and Expected Number of Households according to the Number of Migrants and Type of Village: The Poisson-One-Inflated Log-series Distribution

No. of Migrants	Number of Households					
	Semi-urban		Remote		Growth Centre	
	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.
0	1042	1042	872	872	978	978
1	95	95	176	176	154	154
2	19	19.4	59	52.7	47	44.2
3	10	7.4	18	18.4	18	16.5
4	2		6	8.2	9	7.7
5	2		4		1	
6	0	7.4	0	7.7	0	7.6
7	1		0		0	
8	0		0		1	
Total	1171	1171	1135	1135	1208	1208
$\hat{\theta}$	0.1168		0.2636		0.2112	
$\hat{\omega}$	0.4684		0.4787		0.5289	
$\hat{p}$	0.6483		0.5985		0.6190	
χ^2	1.7001		3.1300		4.6595	
df	1		2		2	

Source: The observed frequencies were extracted from Sharma (1985)

Table 6: Observed and Expected Number of Households according to the Number of Migrants and Type of Village: The Poisson-Misrecorded Poisson Distribution

No. of Migrants	Number of Households					
	Semi-urban		Remote		Growth Centre	
	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.
0	1042	1042	872	872	978	978
1	95	95	176	176	154	154
2	19	28.1	59	62.7	47	56.5
3	10		18	18.0	18	14.8
4	2		6		9	
5	2		4		1	
6	0	6.1	0	6.3	0	4.8
7	1		0		0	
8	0		0		1	
Total	1171	1171	1135	1135	1208	1208
$\hat{\theta}$	0.8369		0.7414		0.8068	
$\hat{\lambda}$	0.3105		0.4955		0.4530	
$\hat{\phi}$	0.5215		0.0980		0.3222	
χ^2	15.9322		2.3914		10.2976	
df	0		1		1	

Source: The observed frequencies were extracted from Sharma (1985).

The χ^2 values also show that there is not much to choose between the Poisson-one-inflated geometric and the Poisson-one-inflated log-series distributions for modeling out-migration from growth centres. However, the fact that the degrees of freedom is more for the latter than for the former for marginal difference in the χ^2 values, gives it an edge. Based on the parameters of this distribution, the average number of migrants in households residing in growth centres is estimated at 1.4. The average number of migrants per household is the same for both semi-urban areas and growth centres. From Table 4, the propensity of a person to migrate from a cluster in a household ($\hat{p}$) is however slightly higher in semi-urban areas. The propensity and hence the incidence of migration is least among persons residing in remote areas. This may be attributed to lack of information and contact in urban areas by these residents.

CONCLUSION

In order to capture the event that at least one person migrates in a household, we have fitted the mixture of the Poisson and some one-inflated distributions. The results of the fit shows that distributions that take into consideration variations in the probability of a person migrating in a cluster in a household (the log-series, and the geometric which are special cases of the negative binomial distribution) performed better. The ability of the Poisson-misrecorded Poisson to model situations where a migrant reported in a cluster in a household is not recorded was demonstrated by its satisfactory fit for out-migration from remote areas only.

REFERENCES

Dejong, D. E. and Gardner, R. W., 1981. Migration Decision Making: Multi-disciplinary Approaches in Developing Countries. New York, Pergamon Press.

Feller, W., 1968. An Introduction to Probability Theory and Its Applications (Third Edition). New York: Wiley.

Johnson, N. C., Kotz, S. and Kemp, A. W., 1992. Univariate Discrete Distributions (2^{nd} ed.), New York, John Wiley.

Pryor, R. J., 1975. Conceptualization of migration behaviour: a problem at micro-demographic analysis, in L.A. Kosinski and J. W. Webb, Population at micro- scale, New Zealand Geographical Society, Auckland, pp. 105 – 120.

Rossi, P. H., 1955. Why Families Move? A study of Social Psychology of Urban Residential Mobility. Free Glencoe Press, New York.

Sharma, H. L., 1985. A probability distribution for out-migration. Janasamkha. A Journal of Demography, 5(2): 95 – 101.

Speare, A., (Jr.), Goldstein, S. and Frey, W. H., 1975. Residential mobility, migration and metropolitan change, Cambridge: Ballinger Publishing Company.

Yadava, K. N. S. and Singh, R. B., 1991. A probability model for the distribution of the number of migrants at the household level. Genus, XLVII (1 – 2): 49 – 62.