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AN ALTERNATIVE APPROACH TO ABSOLUTE-VALUE TEST FOR THE PARAMETERS OF MULTIPLE LINEAR REGRESSION MODELS

CHRYSOGONUS C. NWAIGWE and P. I. UCHE

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ABSTRACT

An alternative approach to absolute-value test statistic M_n is developed for conducting tests simultaneously on all the parameters of multiple linear regression models.
Under certain null and alternative hypotheses, the new test statistic is shown to have limiting central and noncentral chisquare distributions, respectively.
This paper uses a measure of efficiency due to Pitman to compare the asymptotic relative efficiency of the alternative approach to absolute-value test with its classical counterpart (the F-test). Numerical comparison of the alternative approach to absolute-value test with the F-test shows that the present test is slightly more efficient than the classical F-test for $h_{ni} > 20$ for all i, where $h_{ni} = \sum_{j=0}^{p} b_j \chi_{ji}$ for all i.

KEYWORDS: Alternative approach, absolute-value test, multiple linear regression models parameters, asymptotic relative efficiency.

1 INTRODUCTION

Hajek (1962) constructed an asymptotically most powerful rank score statistic for testing hypotheses about slopes in simple linear regression models. Following this, Adichie (1967) developed a sign rank statistic for conducting tests simultaneously on intercepts and slopes in simple linear regression models. Jogdeo (1964) extended Hajek's procedure by constructing a statistic for performing tests simultaneously on all the parameters of multiple linear regression models. Onuoha (1977) also generalized Adichie's technique by using a statistical model similar to that of Jogdeo in obtaining a sign-rank statistic for testing hypotheses about the complete set of parameters in multiple linear regression models. Onuoha (2001) also developed a value-oriented test for testing the parameters of multiple linear regression models. Nwaigwe (2003), in his M.Sc unpublished work, following Onuoha (2001) developed absolute-value tests for testing the parameters of multiple linear regression models. Onuoha and Nwaigwe (2003) also developed an absolute value test for testing the parameters of multiple linear regression models.

The present paper is an extraction and a review of Nwaigwe (2003).

In the above papers and in the present paper, the test statistic is a quadratic form consisting of component test statistics and the inverse of a covariance matrix. Apart from Onuoha (2001) and Nwaigwe (2003), in the papers cited above, the component test statistics are defined in terms of the regression constants as well as the signs and/or ranks of the observations, while, in this paper, they are defined in terms of the absolute values of the observations and the regression constants. Hence, for large observations, computations of values of the component test statistics are easier to obtain in this paper than in the above papers since ranking of large observations is usually very tedious.

2. THE PROPOSED TEST STATISTIC

Instead of using the signs and ranks of the observations as in Onuoha (1997) or the actual observations as in Onuoha (2001), we have used the absolute values of the observations and the regression constants to construct the present statistic. The multiple linear regression model is of the form

$$Y_{ni} = \beta_o + \beta_1 \chi_{1i} + \ldots + \beta_p \chi_{pi} + Z_{ni}$$

CHRYSOGONUS C. NWAIGWE, Department of Statistics, University of Nigeria, Nsukka, Enugu State, Nigeria.
P. I. UCHE, Department of Statistics, University of Nigeria, Nsukka, Enugu State, Nigeria.

$$Y_{ni} = \sum_{j=o}^{p} \beta_j \chi_{ji} + Z_{ni}; 1 \le i \le n < \infty, \chi_{oi} \equiv 1 \text{ for all } i \quad (2.1)$$

where Y_{ni} are independent random variables with distributions

$$p(Y_{ni} \le y \mid \beta_j) = F(y - \sum_{j=o}^{p} \beta_j x_{ji}), \quad (2.2)$$

and where F is a distribution function, x_{ni} are known regression constants because they are assumed to be known without error, Z_{ni} are independently and identically distributed random variables with zero means and unit variances and the β_j's are the unknown parameters under test $(-\infty \angle \beta_j \angle \infty)$.

The problem is to test the following null and alternative hypotheses:

$$H_0 : \beta_j = 0 \text{ for all j} \quad (2.3)$$

$$H_n : \beta_j = n^{-1/2} b_j \quad (2.4)$$

where n is the sample size of the observations and b_j is the estimate of β_j.

$H_n : \beta_j = n^{-1/2} bj$ implies that the β_j's take values other than zero.

This is in line with Onueha (1997). Thus, H_n is seen to tend to H_0 at the rate of $n^{-1/2}$ as n increases. Let the component test statistics and the covariance matrix be defined respectively, as

$$T_{nj} = n^{-1/2} \sum_{i=1}^{n} \chi_{ji} \mid Y_{ni} \mid, 0 \le j \le p, \chi_{0i} \equiv 1 \text{ for all i} \quad (2.5)$$

and

$$\lambda_{njk} = cov(T_{nj}, T_{nk}), 0 \le j,k \le p \quad (2.6)$$

where |. | is the absolute value symbol and $cov(T_{nj}, T_{nk})$ is the covariance of T_{nj} and T_{nk}.
The proposed test statistic is a quadratic form consisting of the component test statistics ($T_{no}, T_{n1}, \ldots, T_{np}$) and the inverse of the covariance matrix $\|\lambda_{njk}\|$ and is given by

$$Mn_a \equiv (T_{n0}, T_{n1}, \ldots, T_{np})' \| \lambda_{njk} \|^{-1} (T_{n0}, T_{n1}, \ldots, T_{np}) \equiv T_n' \|\lambda_{njk}\|^{-1} T_n \quad (2.7)$$

where

$$T'_n \equiv (T_{no}, T_{n1}, \ldots, T_{np}) \quad (2.8)$$

and $\| \lambda_{njk} \|^{-1}$ is the inverse of the (p+1)(p+1) matrix, $\|\lambda_{njk}\|$, with elements in (2.6) while ||.|| is matrix notation.

3. LIMITING DISTRIBUTION OF M_n UNDER H_0

In this section, certain theorems such as the central limit theorem and the concept of convergence in probability shall be employed to prove the limiting distribution of the M_n under H_o.

Let E_o, Var_o, Cov_o and P_o, respectively, denote that the expectation, the variance, the covariance and the probability are being derived under H_o. From (2.1), (2.2) and the assumptions on Z_{ni},

$$E(y_{ni}) = \sum_{j=o}^{p} \beta_j \chi_{ji} \qquad (3.1)$$

Under H_o, we have

$$E_o(y_{ni}) = 0 \qquad (3.2)$$

$$Var_o(y_{ni}) = Var(y_{ni}) = 1 \qquad (3.3)$$

From (2.5)

$$E_o(T_{nj}) = E_o(n^{-1/2} \sum_{i=1}^{n} \chi_{ji} \mid y_{ni} \mid$$

$$= n^{-1/2} \sum_{i=1}^{n} \chi_{ji} E \mid y_{ni} \mid$$

$$|y_{ni}| = y_{ni} \text{ if } y_{ni} > 0$$

$$= -y_{ni} \text{ if } y_{ni} < 0$$

$$E|y_{ni}| = E(y_{ni}) P(y_{ni} > 0) - E(y_{ni}) P(y_{ni} < 0)$$

$$= E(y_{ni}) [1-2 P(y_{ni} < 0)] \text{ that is } P(y_{ni} > 0) = 1-P(y_{ni} < 0)$$

$$E|y_{ni}| = \sum_{j=o}^{p} \beta_j \chi_{ji} [1 - 2F(-\sum_{j=o}^{p} \beta_j \chi_{ji})] \qquad (3.4)$$

Hence

$$E_o(T_{nj}) = n^{-1/2} \sum_{j=o}^{p} \chi_{ji} [\sum_{j=o}^{p} \beta_j \chi_{ji} [1 - 2F(-\sum_{j=o}^{p} \beta_j \chi_{ji})]$$

$$E_o(T_{nj}) = 0, \; 0 \leq_j \leq p \qquad (3.5)$$

Since under H_o, $\beta_j = 0$.

$$= var(y_{ni}) [p(y_{ni} > 0) + p(y_{ni} < 0)]$$

$$= var(|y_{ni}|) = var(y_{ni}) \qquad (3.6)$$

Hence

$$\lambda_{njk} = n^{-1} \sum_{i=1}^{n} \chi_{ij} \chi_{ik} var(y_{ni})$$

$\lambda_{nik} = n^{-1}\sum_{i=1}^{n}\chi_{ij}\,\chi_{ik}$ (since var $(y_{ni}) = 1$) (3.7)

$\text{cov}_o\,(T_{nj}, T_{ni}) = n^{-1}\sum_{i=1}^{n}\chi_{ij}\,\chi_{ik}$

$\chi_{io} = 1$ for all i

Equation (3.7) yields

$$\text{Var}_o\,(T_{nj}) = \lambda^2_{nj} = n^{-1}\sum_{i=1}^{n}\chi^2_{ij} \qquad (3.8)$$

Let $L(T_n \mid p) \underset{n\to\infty}{\to} N(a, b^2)$ denote that the distribution law of $[T_n\text{-}a)/b]$ tends to the

Standard normal distribution under P, since $|y_{ni}|$ are independent random variables,

then T_{nj} in (2.5) is a sum of independent random variables. $n\to\infty$

Consequently, the central limit theorem applies.

Then, from (3.5) and (3.8),

$$L(T_{nj} \mid p_o) \underset{n\to\infty}{\to} N(0, \lambda^2_j),\ 0 \le j \le p \qquad (3.9)$$

where $\lambda^2_j = \lim_{n\to\infty} \lambda^2_{nj}$ is given in (3.8).

To prove the joint asymptotic normality of the T_{nj}, we use a well-known theorem of cramer (1945). To this effect and for arbitrary constants a_j $(0 \le j \le p)$, we have

$$T^*_n = \sum_{j=0}^{p} a_j T_{nj} = \sum_{j=0}^{p} a_j n^{-1/2} \sum_{i=1}^{n}\chi_{ji}\,|y_{ni}|$$

$$T^*_n = n^{-1/2}\sum_{i=1}^{n}\left(\sum_{j=0}^{p} a_j\chi_{ji}\,|y_{ni}|\right) \qquad (3.10)$$

T^*_n is the sum of independent random variables, $|y_{ni}|$, it follows therefore, from (3.2), (3.3) and central limit theorem that

$$L(T^*_n \mid P_o) \underset{n\to\infty}{\to} N\,(o, \lambda^{*2}) \qquad (3.11)$$

where

$$\lambda^{*2} = \lim n^{-1}\sum_{i=1}^{n}\left(\sum_{j}^{p} a_j\,\chi^2_{ji}\right)^2,\ \chi_{oi} \equiv 1 \text{ for all } i \qquad (3.12)$$

Hence, we conclude that

$$\underset{n\to\infty}{L}[T_{no}, T_{n1}, \ldots, T_{np}) \mid P_o] \to N_{p+1}(o, \| \lambda_{jk} \|) \qquad (3.13)$$

where $N_{p+1}(o, \|\lambda_{jk}\|)$ is a (p+1) – variate normal distribution with mean

vector $\underline{0}$ and covariance matrix $\| \lambda_{jk} \| = \lim_{n\to\infty} \|\lambda_{njk}\|$ which has elements

$$n^{-1} \sum_{i=1}^{n} \chi_{ij}\, \chi_{lk},\ \chi_{io} \equiv 1 \text{ for all i.}$$

Using (3.9), (3.13) and the theorems on quadratic forms (Adichie (1967)), the limiting distribution of M_n under H_o is central chisquare.
That is

$$\underset{n\to\infty}{L}(M_n \mid P_o) \to \chi^2 (\nu = p+1).$$

As a direct consequence of (Adichie (1967)), the critical function,

$$\psi (M_n \mid P_o) = 1 \text{ if } M_n > \chi^2 (\alpha, p+1)$$
$$= 0 \text{ if } M_n \leq \chi^2 (\alpha, p+1)$$

provides an asymptotic level α test of H_o, where $\chi^2 (\alpha, p+1)$ is the 100 (1-α) % point of the central chisquare distribution with (p+1) degrees of freedom.

4. LIMITING DISTRIBUTION OF M_n UNDER H_n

Let E_n, Var_n, Cov_n and P_n, respectively denote that the expectation, the variance, the covariance and the probability are derived under the alternative hypothesis, H_n.
We recall that

$$E(y_{ni}) = \sum_{j=0}^{p} \beta_j \chi_{ji}$$

and that

$$H_n: \beta_j = n^{-1/2} b_j$$

Hence,

$$E_n(y_{ni}) = n^{-1/2} \sum_{j=0}^{p} b_j \chi_{ji}$$

$$E_n(y_{ni}) = n^{-1/2} h_{ni} \qquad (4.1)$$

and

$$var_n(y_{ni}) = var_o(y_{ni})$$

$$var_n(y_{ni}) = 1 \qquad (4.2)$$

where

$$h_{ni} = \sum_{j=0}^{p} b_j x_{ji}, 1 \leq i \leq n, \chi_{0i} = 1 \text{ for all i} \qquad (4.3)$$

$$E_n(T_{nj}) = n^{-\frac{1}{2}} \sum_{i=1}^{n} \chi_{ji} E_n |y_{ni}| =$$

$$n^{-\frac{1}{2}} \sum_{j=0}^{p} \chi_{ji} [\sum_{j=0}^{p} \beta_j \chi_{ji} (1 - 2F(-\sum_{j=0}^{p} \beta_j \chi_{ji}))]$$

$$E_n(T_{nj}) = n^{-1} \sum_{i=1}^{n} \chi_{ji} h_{ni} [1 - 2F(-n^{-\frac{1}{2}} h_{ni})] = \mu_{nj} \qquad (4.4)$$

where

$$h_{ni} = \sum_{j=0}^{p} b_j \chi_{ji}$$

and

$$\lambda_{njk} = Cov_n(n^{-\frac{1}{2}} \sum_{i=1}^{n} \chi_{ji} | Y_{ni} |, n^{-\frac{1}{2}} \sum_{i=1}^{n} \chi_{ki} | Y_{ni} |)$$

$$= n^{-1} \sum_{i=1}^{n} \chi_{ik} \cdot \chi_{ik} Covn(| Y_{ni} |, | Y_{ni} |),$$

$$= n^{-1} \sum_{i=1}^{n} \chi_{ij} \cdot \chi_{ik} \, \mathrm{var}_n(| Y_{ni} |)$$

$$= n^{-1} \sum_{i=1}^{n} \chi_{ij} \cdot \chi_{ik} \, \mathrm{var}_n(Y_{ni})$$

$$\lambda_{njk} = n^{-1} \sum_{i=1}^{n} \chi_{ij} \cdot \chi_{ik} \qquad (4.5)$$

$$\mathrm{Var}_n(T_{nj}) = \mathrm{var}_n(n^{-\frac{1}{2}} \sum_{i=1}^{n} \chi_{ji} | Y_{ni} |)$$

$$= n^{-1} \sum_{i=1}^{n} \chi_{ji} \cdot \chi_{ki} \, \mathrm{var}_n |Y_{ni}|$$

$$\mathrm{var}_n(T_{nj}) = \lambda^2_{nj} = n^{-1} \sum_{i=1}^{n} \chi^2_{ij} \qquad (4.6)$$

Using (4.4), (4.6) and the central limit theorem, we have

$$L(T_{nj} | P_n) \underset{n \to \infty}{\to} N(\mu_j, \lambda_j^2), 0 \leq j \leq p, \qquad (4.7)$$

where

$\mu_j = \lim \mu_{nj}$ in (4.4) and

$$\lambda^2_j = \lim_{n\to\infty} \lambda^2_{nj} \text{ in } (4.6)$$

The proof of the joint asymptotic normality of $(T_{no}, T_{n1}, \ldots, T_{np})$ under H_n follows the same pattern as that given for the (p + 1) statistics under Ho.
Hence,

L $(T_{no}, T_{n1}, \ldots T_{np})$ under H_n follows the same pattern at that given for the (p + 1) statistics under H_o, and

$$L((T_{no}, T_{no}, T_{n1}, \ldots, T_{np}) \mid P_n) \underset{n\to\infty}{\rightarrow} N_{p+1}(\underline{\mu}, \|\lambda_{jk}\|), \quad (4.8)$$

where the R H S of (4.8) is the (P + 1)-variate normal distribution with mean vector $\underline{\mu} = \lim \underline{\mu}_n$ and $\|\lambda_{njk}\|$ have their elements defined in (4.4) and (4.5), respectively. For the limiting distribution of M_n under H_n, we again apply (Adichie (1967)) and equation (4.8) and the fact that

$$g(T) = T'\|\lambda_{jk}\|^{-1}T \text{ and } M_n = g(T_n) =$$

$$T'_n\|\lambda_{njk}\|^{-1}T_n,$$

we have

$$L(T_n|P_n) = L[(T_{n0}, T_{n1}, \ldots, T_{np})|P_n] \underset{n\to\infty}{\rightarrow} L(T|P_n)$$

$$= N_{p+1}(\underline{\mu}, \|\lambda_{jk}\|), \text{i.e.,}$$

$$L(T_n|P_n) \underset{n\to\infty}{=} N_{p+1}(\mu, \|\lambda_{jk}\|) \quad (4.9)$$

and

$$L(M_n \mid P_n) \underset{n\to\infty}{=} L((T'\|\lambda_{jk}\|^{-1}T)), \quad (4.10)$$

where T has a (p + 1)- variate normal distribution with mean vector $\underline{\mu}$ and covariance matrix $\|\lambda_{jk}\|$. The limiting distribution of M_n under H_n is completely specified in (Adichie (1967)). By this, the proposed absolute-value test statistic is asymptotically noncentral chisquare with (p+1) degrees of freedom under H_n and noncentrality parameter given by

$$\Delta = \lim_{n\to\infty} \Delta_n, \quad (4.11)$$

where

$$\Delta_n = \underline{\mu}_n'\|\lambda_{njk}\|^{-1}\mu_n \quad (4.12)$$

Hence, (Adichie (1967)) implies that

$$L(M_n|P_n) \underset{n\to\infty}{\to} \chi^2(p+1, \Delta),$$

where Δ is given in (4.11).

5 ASYMPTOTIC EFFICIENCY OF THE M_n-TEST

In this section, we derive the asymptotic relative efficiency (ARE) of the M_n-test, with its classical counterpart the $\hat{M}_n-test$. To do this, we employ a measure of efficiency due to Pitman which is defined as follows : if under the same sequence of alternatives like the one stated in (2.4), two test statistics have noncentral chisquare limiting distributions with the same number of degrees of freedom, it is shown in Noether (1954) that the ARE of the two tests is the ratio of their noncentrality parameters.

Hence, in order to obtain the ARE of the M_n- test with respect to its $\hat{M}_n-test$, we only need to derive the noncentrality parameter, $\hat{\Delta}_n$, of the $\hat{M}_n-test$,.

The classical test of H_o assumes that F is the normal distribution function and uses the least squares (or the maximum likelihood) estimates, $\hat{\beta}_{nj}, of \beta_j (0 \le j \le p)$.

The $\hat{M}_n-test$ statistic (c.f. eg. Adichie (1967)) is

$$\hat{M}_n = n(\hat{\beta}_{no}, \hat{\beta}_{n1}, \ldots \hat{\beta}_{np})' \| \gamma_{njk} \|^{-1} \hat{\beta}_{no}, \hat{\beta}_{n1}, \ldots \hat{\beta}_{np})$$

$$= n \underline{\hat{\beta}}_n' \| \gamma_{njk} \|^{-1} \underline{\hat{\beta}}_n, o \le j,k \le p, \qquad (5.1)$$

where

$$\underline{\hat{\beta}}_n' = (\hat{\beta}_{no}, \hat{\beta}_{n1} ../, \hat{\beta}_{np}), \qquad 5.2)$$

and $\|\gamma_{njk}\|^{-1}$ is the inverse of the $(p+1)(p+1)$

$$\text{matrix, } \|\gamma_{njk}\| = \| \tau_{njk} \|^{-1}, 0 \le j,k \le p \qquad (5.3)$$

$$\text{and where } \tau_{njk} = \lambda_{njk} = n^{-1} \sum_{i=1}^{n} \chi_{ij}\chi_{ik}, 0 \le j,k \le p, \chi_{oi} = 1 \text{ for all } i \qquad (5.4)$$

The estimates $\hat{\beta}_{nj} (0 \le j \le p)$, being linear functions of normal random variables, are themselves normal.

Hence, (Adichie (1967)) again implies that, under H_o $\hat{M}_n$ has a central chisquare distribution with (p + 1) degrees of freedom and

$$L(\hat{M}_n|P_o) \underset{n\to\infty}{\to} \chi^2 (\nu = p + 1). \qquad (5.5)$$

Besides, we have that, under any given alternatives of the form

$$\beta_j = \beta_{oj} \ (0 \le j \le p), \qquad (5.6)$$

$\hat{M}_n$ has a noncentral chisquare distribution with (p + 1) degrees of freedom and noncentrality parameter given by

$\hat{\Delta}_{no} = n(\beta_{oo}, \ldots, \beta_{op})' \| \gamma_{njk} \|^{-1} (\beta_{oo}, \beta_{o1}, \ldots, \beta_{op})$

$= n\underline{\beta}'_o \| \gamma_{njk} \|^{-1} \underline{\beta}_0,$ (5.7)

(C.F. e.g Lehman (1959))

where

$\underline{\beta}'_0 = (\beta_{00}, \beta_{01}, \ldots, \beta_{0p}).$ (5.8)

It follows that under the sequence of near alternatives, H_n, given in (2.4), $\hat{\Delta}_{n0}$, becomes $\hat{\Delta}_n$, = $(b_0, b_1, \ldots b_p)$

$' \| \gamma_{njk} \|^{-1} (b_0, b_1, \ldots, b_p)$ (5.9)

$= \underline{b}' \| \gamma_{njk} \|^{-1} \underline{b}$

where

$\underline{b}'$ = $(b_0\ b_1, \ldots\ b_p)$. (5.10)

The above results show that

$L(\hat{M}_n \mid P_n) \underset{n\to\infty}{\to} \chi^2(p + 1, \hat{\Delta})$ (5.11)

where $\hat{\Delta} = \lim \hat{\Delta}_n$ given in (5.9). Hence, the ARE of the M_n - test with respect to the $\hat{M}_n$*-test*, is given by

ARE $(M_n, \hat{M}_n) = \Delta / \hat{\Delta}$. (5.12)

6. NUMERICAL COMPARISON OF THE M_n AND $\hat{M}_n$ TEST:

In this section, we intend to make a numerical comparison of the M_n – statistic with respect to its classical counterpart, the $\hat{M}_n$ -statistic.

Consider the Younger's Advertising data below, taken from Ronald (1996, p.409).

Table 6.1

Y_{ni}	84	84	80	50	20	68
X_{1i}	13	13	8	9	9	13
X_{2i}	5	7	6	5	3	5

The model for analyzing the data is given by

$Y_{ni} = \beta_0 + \beta_1 \chi_{1i} + \beta_2\, \chi_{2i} + Z_{ni},\ 1 \le i \le 6$ (6.1)

The problem is to test the following hypotheses :

$H_0: \beta_j = 0, 0 \leq j \leq 2.$ (6.2)

$H_n: \beta_j = n^{-1/2} b_j, n = 6.$ (6.3)

For this comparison, we take b_j ($0 \leq j \leq 2$) to be the least squares estimates, $\hat{\beta}_{nj}$, of β_j. Applying $\hat{\beta}_j = (x'x)^{-1}x'y$ as in Ronald (1996) to table 6.1, gives

$b_0 = \hat{\beta}_{n0} = -41.4654, b_1 = \hat{\beta}_{n1} = 2.5444,, b_2 = \hat{\beta}_{n2} = 14.5350$ (6.4)

Using the data from Table 6.1 in (5.4),
we have

$$\|\tau_{njk}\| = 1/6 \begin{pmatrix} 6 & 65 & 31 \\ 65 & 733 & 341 \\ 31 & 341 & 169 \end{pmatrix} \quad (6.5)$$

From (5.3) and (6.5), we obtain

$$\|\gamma_{njk}\|^{-1} = \begin{pmatrix} 1 & 10.8333 & 5.1667 \\ 10.8333 & 122.1667 & 56.8333 \\ 5.1667 & 56.8333 & 28.1667 \end{pmatrix} \quad (6.6)$$

Equations (5.1), (6.4) and (6.6) give rise to

$\hat{M}_n = 24904.8696$ (6.7)

Using the data from table 6.1 in (2.5) and in (4.5), we respectively get,

$T_{n0} = 157.5838, T_{n1} = 1770.9811, T_{n2} = 872.8348$ (6.8)

$\lambda_{00} = 1, \lambda_{01} = 10.8333, \lambda_{02} = 5.1667$

$\lambda_{10} = 10.8333, \quad \lambda_{11} = 122.1667, \quad \lambda_{12} = 56.8333$ (6.9)

$\lambda_{20} = 5.1667, \quad \lambda_{21} = 56.8333 \quad \lambda_{22} = 28.1667$

from (6.9), we have

$$\|\lambda_{njk}\|^{-1} = \begin{pmatrix} 33.3168 & -18154 & -2.4484 \\ -1.8154 & 0.2324 & -0.1359 \\ -2.4484 & -0.1359 & 0.7589 \end{pmatrix} \quad (6.10)$$

Using (6.8) and (6.10) in (2.7), we obtain the value of M_n under H_o as

$M_n = 27452.62.$ (6.11)

Under H_0, both $\hat{M}_n$ and M_n have limiting central chisquare distribution with 3 degrees of freedom. Hence, from the chisquare table, we have
$\chi^2 (0.05,3) = 7.815$ and $\chi^2 (0.01,3) = 11.34$,
where $\chi^2 (\alpha,\upsilon)$ is the 100 (1-α)% point of the central chisquare distribution with υ degrees of freedom.

We observe from (6.7) and (6.11) that, if the two tests, $\hat{M}_n$ and M_n, were to be consistent while tending to the limit, both of them would reject H_0 at the 5% and 1% levels of significance. But the M_n - test would tend to reject H_o more often than its parametric counterpart, the $\hat{M}_n$ - test.

To obtain the noncentrality parameter $\hat{\Delta}_n$, of the M_n-test we use (6.4) and (6.6) in (5.9) to get

$\hat{\Delta}_n = 4150.8116$ (6.12)

In order to get the noncentrality parameter, Δ_n, of the $\hat{M}_n-test$, we apply data in 6.1 to (4.4). To do this, we assume three distributions for the F in (4.4).
The distributions are the standard Normal, the Double Exponential and the Logistic. These are the three commonly assumed distributions in non-parametric inference and were also used in Onuoha (1994).

$-n^{-½}$ h_{ni} in (4.4) gives

$$-n^{-½} h_1 = -26.2450, \quad -n^{-½} h_{n2} = -38.1128$$

$$-n^{-½} h_{n3} = 26.9851, \quad -n^{-½} h_{n4} = -22.0810 \qquad (6.13)$$

$$-n^{-½} h_{n5} = 10.2222, \quad -n^{-½} h_{n6} = -26.2450$$

where

$$h_{n1} = b_0 + b_1X_{11} + b_2X_{21} = 64.2868$$

$$h_{n2} = b_0 + b_1X_{12} + b_2X_{22} = 93.3568$$

$$h_{n3} = b_0 + b_1X_{13} + b_2X_{23} = 66.0998$$

$$h_{n4} = b_0 + b_1X_{14} + b_2X_{24} = 54.1092 \qquad (6.14)$$

$$h_{n5} = b_0 + b_1X_{15} + b_2X_{25} = 25.0392$$

$$h_{n6} = b_0 + b_1X_{16} + b_2X_{26} = 64.2868$$

For $h_{ni} = \sum_{j=0}^{p} b_j\chi_{ji}$ for all i

where F is standard Normal distribution, then, from 4.4, we have

$\mu_{no} = 61.1964, \ \mu_{n1} = 687.7049, \ \mu_{n2} = 339.7713.$ (6.15)

Using (6.10) and (6.14) in (4.12), we have

$\Delta_n = 8.5562$ (6.16)

Applying (6.12) and (6.16) in (5.12),

we obtain the asymptotic relative efficiency of M_n relative to $\hat{M}_n$ as

ARE (M_n, $\hat{M}$) = $(\Delta_n/\hat{\Delta}_n)$x 100% = = 100.2964%

When F is Logistic distribution, LG(X) = $(1 + e^{-x})^{-1}$, $-\infty<X<\infty$ and when F is Double Exponential distribution,

DE (X) = $\frac{e^x}{2}$ if $X\leq 0$ or $1-\frac{e^{-x}}{2}$ if $X\geq 0$. We also observed the same asymptotic relative efficiency of 100.2964% in the Double exponential and Logistic distributions.

However, it is not always true that the same asymptotic relative efficiency of the M_n to the $\hat{M}$ is observed for each of the distributions. The asymptotic relative efficiency of the M_n to the $\hat{M}_n$ depends on μ_{nj}, which further depends on h_{ni} and h_{ni} determines the value of F in 4.4.

Thus for the given data, the 100.2964% asymptotic relative efficiency of the M_n to the classical F implies that, if the two tests were to be consistent while tending to the limit, the present M_n – test would be about 0.3% more efficient than its classical counterpart, the $\hat{M}_n$ – test.

7. CONCLUSION

From the foregoing, we conclude that, if the two tests were to be consistent while tending to the limit, the present M_n - test would be about 0.03% more efficient than its classical F counterpart, the $\hat{M}_n$ – test for h_{ni} >20, for all i. For h_{ni} $\leq$20, the efficiency of the proposed M_n – test is lower than the efficiency of the classical $\hat{M}_n$ – test. For instance, the asymptotic relative efficiency of the M_n –test when applied to the data used in Adichie (1967) is about 60% which is about 40% lower than the efficiency of the classical F–test. This is because of the dependence of the μ_{nj} which determines the efficiency parameter Δ_n on h_{ni}. In general, we recommend M_n – test for cases in which h_{ni} >20, for all i where $h_{ni} = \sum_{j=0}^{p} b_j \chi_{ji}$ for all i.

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REFERENCES

Adichie, J.N., 1967. Asymptotic Efficiency of a Class of Nonparametric Test for Regression Parameters. Ann. Math. Statistics 38: 884.-893.

Cramer, H., 1945. Mathematical Methods of Statistics. P. 299. Princeton University Press.

Hajek, J., 1962. Asymptoticaly Most Powerful Rank Order Tests. Ann. Math. Statistics 33: 1124-1147.

Jogdeo, K., 1964. Nonparametric Methods for Regression. Report 330. Strichting Math. Centrum Amsterdain.

Lehmann, E. L., 1959. Testing Statistical Hypotheses p. 284. Wiley, New York.

Meyer, P.L., 1973. Introductory Probability and Statistical Applications. 2nd ed., p. 251. Addison Wesle/Publishing Company.

Nwaigwe, C.C., 2003. Absolute-Value Tests for Parameters of Multiple Linear Regression Models. M.Sc Thesis. University of Nigeria, Nsukka.

Noether, G.E., 1954. On a Theorem of Pitman. Ann. Math. Statistics 26: 64-68.

Onuoha, C. N., 1977. Asymptctic Efficiency of Class of Rank Tests for Multiple Linear Regressions. South African Statistical Journal. 11: 139-147.

Onuoha, C. N., 2001. A Test Statistic for the Parameters of Multiple Linear Regression Models. Journal of Nigerian Statistical Association (JNSA). 14: 90-101.

Onuoha, C. N. and Nwaigwe, C. C., 2003.. Absolute-Value Tests for Parameters of Multiple Linear Regression Models. Research Communications in Statistics. Lagos, Nigeria. 2: 29-38.

Ronald, C., 1996. Analysis of Variance, Design and Regression. 1st ed., p. 409. Chapman and Hall/CRC. Boca Raton, London, New York Washington, D.C.

Sverdrup, E., 1952. The Limit Distribution of a Continuous Function of Random Variables. Skand. Akturaietidskaift, 35: 1-10.