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STOCHASTIC SIMULATION OF HOURLY AVERAGE WIND SPEED IN UMUDIKE, SOUTH-EAST NIGERIA

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ABSTRACT

Ten years of hourly average wind speed data were used to build a seasonal autoregressive integrated moving average (SARIMA) model. The model was used to simulate hourly average wind speed and recommend possible uses at Umudike, South eastern Nigeria. Results showed that the simulated wind behaviour was reproducible and matched well with the characteristics of the experimental values. The model may thus be used to forecast wind speed and its impact on the environment of Umudike, very accurately. Specifically the wind speed values have the capacity of propelling a wind bound generating plant capable of serving as a back-up power to the national grid.

Key Words: simulation; wind speed; SARIMA model; forecasts; back-up power.

INTRODUCTION

Recently, there has been increasing reports of damage due to wind storms in south-eastern Nigeria, especially in Abia State. In June, 1995 for example, windstorm severely damaged a pine plantation and roof tops of several buildings at the forestry Research Institute of Nigeria at Ahieke near Umudike. Again in May 1998, tress were wind thrown and some roof tops blown off at Government College Umuahia. Recently, precisely in February 2002, heavy windstorm coupled with unprecedented hailstones destroyed many house roofs, churches and windscreens of cars at Umudike.

Over the years, there has been the hues and cries of the Nigerian populace to diversify the nation's source of energy supply. Wind energy conversion systems have been suggested as an alternative back-up energy supply in Nigeria with less total reliance on conventional energy source namely:- The National Electric Power Authority (NEPA)

Since wind power is a function of wind speed, forecasts of power are generally derived from forecasts of speed. Dynamic statistical analysis based on hourly average wind speed values is used to simulate and forecast wind speed while the main statistical characteristics of the observed data (e.g average wind speed, variance, autocorrelation coefficients, probability density, persistence etc) are used to build the model.

As at the time of writing this paper, there was little or no research information on the impact of hourly average wind speed on agricultural productivity and the environment in South-eastern part of Nigeria.
The main purpose of this study, therefore, is to build a stochastic model of hourly wind speed in Umudike and recommend possible uses. In Agriculture or the society at large.

LITERATURE REVIEW

Oboli (1964) noted that pressure directly affects the movement of winds. Winds blow from a region of high

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pressure to a region of low pressure. Other factors such as earth's rotation combine to determine the actual direction of the wind. Winds directly affect temperature and rainfall. High wind speed and drought also affect wind erosion, flower abortion, wind throw of trees, dust pollution, flooding, high evapo-transpiration and storms (NEST, 1991; Akeredolu 1989). When the dark top soil is removed or degraded by treeless landscape and high wind speed, soil and crop productivity decline.

The erosive power of wind on soils depends largely on its speed and the size and density of the soil particles (Ogirigiri, 1986). Sandy soils move readily when wind speed 15cm above the ground attain or exceed 4.8 m/sec or 17.28 km/h.

NEDECO (1961) stressed too that high wind speeds generally increase wave-crest and littoral drifts in the Niger Delta region of Nigeria.

Statistical studies of wind speed as discrete random variable began about 1942 (Blanchard and Desrochers, 1984). Blanchard and Desrochers (1984) used the stochastic model directly to fit observed hourly wind speed data. Other reseaches (Brown et al, 1984, Daniel and Chen 1991, and Nfaoui et al 1996) improved on previous studies by using data collected for a period ranging from a few months to up to twelve years.

Other researchers (Gorden and Raddy 1988, Bossanyi 1985; Bensaad 1985; Boulay 1990) have been able to produce a reference year using this model which in turn can be used to start a wind generator in a given location.

In order to predict the output of wind-bound effect, by using the stochastic simulation model, it is necessary to generate a typical year of synthetic data which represents accurately the actual statistic of multiyear. Thus the model is useful to generate one reference year for any location such as Umudike

METHODOLOGY

To build a suitable seasonal autoregressive integrated moving average (SARIMA) model for the Umudike wind Run (UWR) data we proceeded through the three steps recommended by Box and Jenkins (1976). This model, it would be recalled, allows for dependence. It also allows the user to single out, from an entire class of autoregressive moving average, (ARMA) models, one that would best represent the original data.

A tentative model was first identified by examining a time series plot, and plots of the autocorrelation (acf) and partial autocorrelation function (pacf) of the data. The parameters of the tentative model were then estimated. Finally, the adequacy of the model was checked.

It was observed that significant values occurred only at lags 1 and 12. Therefore we tentatively fitted a pure MA model of the form:

$$(1-B^{12})\, X_t = \omega_t = (1-\phi_1 B)(1-\phi_{12}B^{12})a_t \qquad \text{.... 1}$$

where B is the Backshift operator. The pacf of the differenced series helped to confirm our tentative choice of a pure MA model.

Ten years (1987-1996) of hourly average wind speed data (see table 1) were thus collected from the meteorological unit of the National Root Crops Research Institute (NRCRI), Umudike, to build the SARIMA model. Tests were conducted to validate the model. For example, comparison was made between the generated and real series of data to check if the wind behaviour was reproducible. Specifically, the observed and simulated values for every month (monthly average, variance and first two autocorrelation functions) were compared. The probability density function of simulated series was also compared with that of the observed series to check if the model would be able to generate accurate probability density function for certain values of wind speed. This ultimately would indicate that the SARIMA model could generate synthetic time series data. A statistical computer package (MINITAB) was employed in the data analysis. The reference year so produced was used to predict the performance of a wind-bound electric generator in Umudike .

TABLE 1: Daily Average Wind Speed At Umudike (1987-1996)

Umudike Wind Run (km/Day) Date 1987-1996 No:0507.23 Lat. 05°29′N
Long. 07°33′E Alt.122 meters or 400ft (ASL)

Months	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
JAN	103	106	112	74	109	116	121	103	76	98
FEB	123	118	105	86	127	102	109	81	95	125
MAR	119	126	115	97	119	121	111	114	110	124
APRIL	112	118	109	136	104	108	105	95	94	100
MAY	123	103	85	94	88	92	87	80	80	97
JUNE	119	106	106	92	107	118	89	89	93	109
JULY	123	127	97	107	104	110	111	105	105	103
AUG	121	123	105	121	119	126	108	107	115	118
SEPT	117	110	111	110	109	116	88	95	105	116
OCT	109	101	88	87	107	102	86	77	85	98
NOV	93	86	64	71	88	86	84	77	66	75
DEC	82	90	60	91	88	79	78	105	82	78

RESULTS AND DISCUSSIONS

Our data set shown in Table1 shows the Umudike wind run data from January 1987 through December, 1996. The time series plot of the wind run data in figure1, indicates that there is no trend, but that there is seasonal variation. The acf of the wind run data is shown in figure 2.
To determine whether the autocorrelation at lag k is significantly different from zero, the following hypothesis may be tested.

$$\left.\begin{aligned} H_o &: \rho_k = 0 \\ H_1 &: \rho_k \neq 0 \end{aligned}\right\} \quad \text{....(2)}$$

For any k, reject H_o if $|r_k| > 2/\sqrt{n}$, where n is the number of observations. The test is for a 95% confidence interval .

For n = 120 an autocorrelation coefficient greater than $2/\sqrt{120} = 0.1826$ would be considered significantly different from zero. At lag 2 the autocorrelation coefficient is less than 0. 1826 indicating that it is not significantly different from zero, and so we conclude that there is no trend in the wind run data.

Notice that the autocorrelations at lags 12, 24, 36 and 48 are all greater than 0.1826 and are thus significantly different from zero. This is indicative of a seasonal pattern in the data
The data is, therefore, differenced to remove the seasonal component.

Figure 3 shows the autocorrelation function of the differenced series, $\omega_t = (1-B^{12})X_t$. We observe that there is a cut off after lag 1 for the autocorrelation function. A visual observation of the partial autocorrelation function for ω_t also shows that there is a cut off after lag 1 for ϕ_{kk}.. This implies that the nonseasonal pattern of our model will have the following values:
p = 1 , d = 0, q =1

Also, since r_{12} is significantly different from zero (0.32> $2\sqrt{108}$ = 0.1925), while r_{24} is not significantly

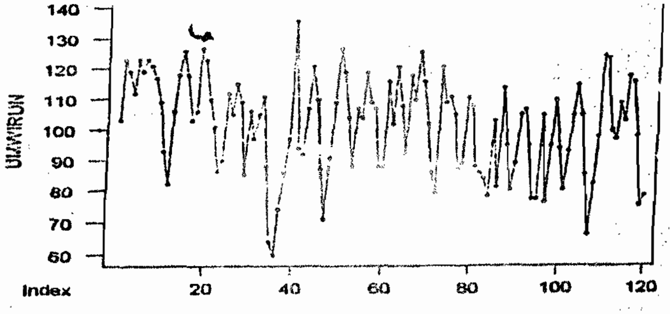


Figure 1.: Time Series Plot of Umudike Wind run.

different from zero (0.10<0.1925), it therefore implies that a seasonal MA(12) term should be included in the model. A tentative model to be fitted to our data set is

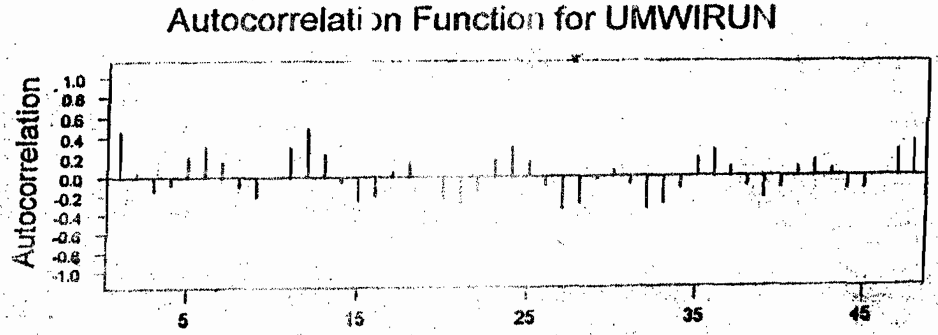
SARIMA (101) x $(011)_{12}$ ....(3)

We used MINITAB to fit the above model to our data set and we obtained the following:

$$(1-0.86B)(1-B^{12})\,X_t = -\,0.17 + (1-0.65B)(1-0.84B^{12})\,e_t$$

(0.1079) (0.0741) (0.1625) (0.082)

(7.97) (-2.33) (4.02) (10.30)



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.47	5.19	27.62	13	0.23	1.58	112.14	25	0.16	0.92	177.85	37	0.10	0.48	272.29
2	0.05	0.45	27.92	14	-0.05	-0.35	112.52	26	-0.10	-0.58	179.41	38	-0.10	-0.52	274.19
3	-0.14	-1.31	30.64	15	-0.26	-1.69	121.63	27	-0.35	-2.02	198.93	39	-0.23	-1.14	283.54
4	-0.09	-0.79	31.51	16	-0.20	-1.28	127.08	28	-0.29	-1.62	212.52	40	-0.13	-0.66	286.75
5	0.21	1.86	37.07	17	0.06	0.40	127.63	29	-0.03	-0.19	212.71	41	0.09	0.45	288.27
6	0.31	2.72	49 71	18	0.16	1.05	131.50	30	0.06	0.35	213.37	42	0.19	0.92	294.76
7	0.16	1.32	53.06	19	0.01	0.09	131.53	31	-0.08	-0.45	214.60	43	0.07	0.36	296.73
8	-0.11	-0.86	54.54	20	-0.25	-1.58	140.71	32	-0.35	-1.89	234.72	44	-0.14	-0.66	299.28
9	-0.21	-1.67	60.27	21	-0.29	-1.77	152.79	33	-0.29	-1.54	249.05	45	-0.13	-0.65	302.79
10	-0.02	-0.12	60.31	22	-0.15	-0.92	156 29	34	-0.13	-0.68	251.88	46	-0.01	-0.06	302.81
11	0.30	2.32	72.13	23	0.17	1.04	160.81	35	0.19	0.97	256.06	47	0.26	1.25	316.20
12	0.49	3.68	104.74	24	0.29	1.75	173.98	36	0.27	1.37	270.70	48	0.35	1.67	340.85

FIGURE 2: Autocorrelation function for Umudike wind run.

The autoregressive coefficient ϕ is estimated to be 0.8601, with a standard deviation of 0.1079 and a t–ratio of 7.97. The moving average coefficient Θ_1 is estimated to be 0.6534, with a standard deviation of 0.1625 and a t-ratio of 4.02. The seasonal moving average coefficient Θ_{12} is 0.8423, with a standard deviation of 0.0818 and a t-ratio of (10.30), the constant term is estimated as – 0.17302, with a standard deviation of 0.07414 and a t–ratio of –2.33.

To determine whether there is linear regression, we test the following hypotheses:

$$\left.\begin{array}{l} H_o: \beta=\beta_0 \\ \text{vs } H_1: \beta\neq\beta_0 \end{array}\right\} \text{---. (4)}$$

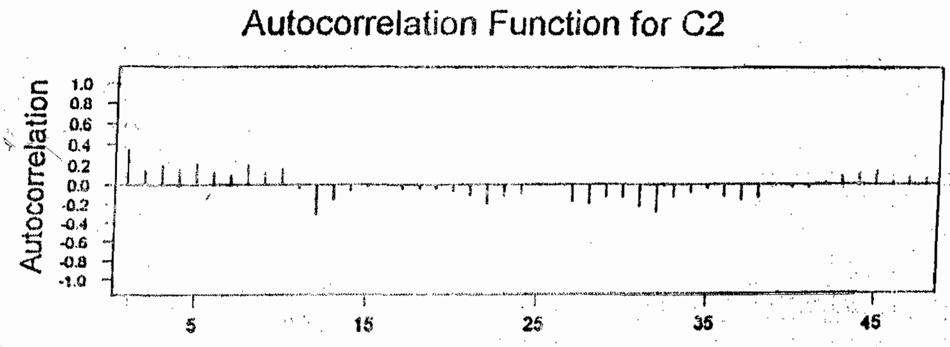
In particular when $\beta_o = 0$, we are testing to see if there is linear regression. The decision rule is to reject H_o if t_{cal} is greater than $t_{tab.}$

Now for ϕ_1 , $t_{cal} > t_{tab}$, we reject H_0 and conclude that there is linear regression. Carrying out similar tests we also conclude that both Θ_1 and Θ_{12} are significantly different from zero since the t–ratios are greater than the tabulated t-values at 5% level of significance.

The last part of the display (the modified Chi-square statistic) measures how well the model fits the d

We now test the hypothesis:

H_0: The specified SARIMA model fits the data.



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.36	3.71	14.15	13	-0.16	-1.16	57.47	25	0.01	0.07	71.26	37	-0.16	-1.01	122.76
2	0.14	1.28	16.27	14	-0.08	-0.56	58.20	26	-0.01	-0.04	71.28	38	-0.13	-0.82	125.74
3	0.19	1.71	20.23	15	-0.03	-0.23	58.32	27	-0.19	-1.32	76.43	39	-0.02	-0.11	125.60
4	0.15	1.37	22.95	16	0.00	0.01	58.32	28	-0.21	-1.42	82.70	40	-0.04	-0.22	126.03
5	0.21	1.86	28.15	17	-0.05	-0.38	58.67	29	-0.14	-0.93	85.64	41	-0.04	-0.25	126.34
6	0.13	1.09	30.08	18	-0.05	-0.33	58.94	30	-0.15	-1.01	88.92	42	0.02	0.13	126.42
7	0.10	0.83	31.22	19	-0.05	-0.36	59.26	31	-0.24	-1.59	97.63	43	0.08	0.50	127.64
8	0.20	1.71	36.20	20	-0.07	-0.51	59.91	32	-0.29	-1.91	110.95	44	0.10	0.64	129.68
9	0.13	1.03	38.15	21	-0.11	-0.83	61.68	33	-0.13	-0.85	113.82	45	0.14	0.84	133.29
10	0.16	1.28	41.24	22	-0.21	-1.51	67.62	34	-0.09	-0.65	115.04	46	0.10	0.59	135.14
11	-0.04	-0.31	41.43	23	-0.12	-0.88	69.74	35	-0.05	-0.31	115.43	47	0.08	0.47	136.34
12	-0.32	-2.57	54.43	24	-0.10	-0.73	71.25	36	-0.13	-0.84	118.38	48	0.06	0.38	137.14

FIGURE :3: Autocorrelation function for the differenced series.

Vs H_1: The specified SARIMA model does not fit the data.

At 5% level of significance $\chi^2_{tab} = \chi^2_a(0.05) = 16.92$ and $\chi^2_{cal} = 4.8$ .... (5)

Now, since $\chi^2_{cal} < \chi^2_{tab}$ we accept H_0. This implies that the specified SARIMA model fits our data set.

CONCLUSIONS

It has been shown from the above discussions that for wind speeds greater than lm/s the main statistical characteristics of observed series are preserved in the simulated values for every month using the SARIMA model. In addition, the model can also generate synthetic time series data. By doing this for each month, it was possible to generate a synthetic year of data which we called the reference year. This reference year could be used to predict the performance of a wind bound generating plant at Umudike. The plant could indeed be capable of serving as a back-up power to the national grid, especially at peak periods when power outage is generally recurrent.

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