

# OBTAINING PROPORTION OF OPTIMUM YIELD WHEN FULL YIELD IS UNATTAINABLE

E. C. NDUKA and T. A. BAMIDURO

(Received 27 February 1998; Revision accepted 12 September 2001)

## ABSTRACT

In a situation where it is not feasible to obtain full yield at harvest time because it is unattainable or simply unavoidable; combined methods of calculus of variation and maximum likelihood estimates are used to obtain a proportion of this yield. This is obtained for both known (desired) and unknown (estimable) proportions of the maximum yield on rectangular and square plot - formations. The reference model is that due to Berry (1967) on intra-row and inter-row spacing experiment.

**Keywords:** Berry's model, Calculus of variations, Maximum likelihood, Plot-formations.

## INTRODUCTION

The desire of any agriculturist is to obtain full yield at harvest time. However, in practice, this is not always realisable. The reason may be due to some natural disasters which may affect survival of all plants at normal harvest time. Economic (market) forces may also persuade him not to wait up to this harvest time to obtain his maximum yield

In either of these circumstances, he may be forced to seek some proportion,  $\lambda$  (say) of this maximum yield because full yield is unattainable or simply unavailable; Mead (1970) obtained the proportion of this maximum yield for known  $\lambda$  in a one-factor experiment using simple method of calculus of variation.

In this study, a two-factor case is considered using basic principles of maximum likelihood estimate and calculus of variations. This is applied for both known (desired) and unknown (estimable)  $\lambda$ , where the reference model is that due to

Berry (1967). The model is considered most plausible for its use in intra-row and inter-row spacing experiments; and also in rectangular and square plot-formations.

### The Reference Model

The reference model, Berry (1967) is given as  $\omega^{-\theta} = \alpha + \beta_1 x_1^{-1} + \beta_2 x_2^{-1} + \beta_{12} (x_1 x_2)^{-1}$ ;  $0 < \theta \leq 1$

where  $x_1$  is the intra-row spacing  
 $x_2$  is the inter-row spacing  
 $\omega$  is the yield per plant

- $\alpha$  is constant yield independent of  $x_1$  and  $x_2$
- $\beta_1$  is the rate of yield per unit of  $x_1$
- $\beta_2$  is the rate of yield per unit of  $x_2$
- $\beta_{12}$  is the rate of yield per unit of density
- $\theta$  is a critical factor which depends on any parts of the plant.

For the one-factor experiment that is,  $\omega^{-\theta} = \alpha + \beta x^{-1}$ , Mead (1970) defined  $W = \rho\omega$  as the total yield per unit area Analogous to Mead (1970), we define in this case.

$W = \rho\omega$  as the total yield per unit area (where  $\rho = (x_1, x_2)^{-1}$  is the density defined as the number of plants per unit area).

**Maximum Yield**

Our interest here is to determine this maximum yield defined by  $W^I = W_{\max}$

where  $W = x_1^{-1} x_2^{-1} [\alpha + \beta_1 x_1^{-1} + \beta_2 x_2^{-1} + \beta_{12} (x_1 x_2)^{-1}]^{\frac{1}{\theta}}$

Mead (1970) obtained this form for a one-factor experiment disregarding  $\theta$  as a variable. Here we postulate that  $W = f(x_1, x_2, \theta)$  and therefore find  $x_1, x_2$  and  $\theta$  which maximize  $W$ . From the principles of Maximum likelihood estimate, Rohatgi (1976)  $\ln W$  is a monotonic function, hence,  $x_1, x_2$  and  $\theta$  which maximize  $W$  also maximize  $\ln W$ .

$$\ln W = -\ln x_1 - \ln x_2 - \frac{1}{\theta} \ln z \text{ where}$$

$$z = \alpha + \beta_1 x_1^{-1} + \beta_2 x_2^{-1} + \beta_{12} (x_1 x_2)^{-1} \dots\dots\dots 1.2$$

Taking partial derivatives with respect to  $x_1, x_2$  and  $\theta$  respectively and equating to zero we have:

$$\frac{-1}{x_1} - \frac{1}{\theta z} \left( \frac{\partial z}{\partial x_1} \right) = 0 \dots\dots\dots 1.3$$

$$\frac{-1}{x_2} - \frac{1}{\theta z} \left( \frac{\partial z}{\partial x_2} \right) = 0 \dots\dots\dots 1.4$$

$$\frac{1}{\theta^2} \ln z = 0 \dots\dots\dots 1.5$$

From (1.5),  $\Rightarrow z = 1$ .

From (1.3) and (1.4), we have that

$$-x_1 \frac{\partial z}{\partial x_1} = 0 \quad \dots\dots\dots 1.6$$

$$-x_2 \frac{\partial z}{\partial x_2} = 0 \quad \dots\dots\dots 1.7$$

Consequently from (1.2), (1.6) and (1.7) respectively will translate to

$$\beta_1 x_1^{-1} + \beta_{12} x_1^{-1} x_2^{-1} = 0 \quad \dots\dots\dots 1.8$$

$$\beta_2 x_2^{-1} + \beta_{12} x_1^{-1} x_2^{-1} = 0 \quad \dots\dots\dots 1.9$$

Therefore, from (1.2) and using (1.8) and (or) (1.9)

$$\alpha + \beta_1 x_1^{-1} + 0 = 1 \quad \dots\dots\dots 1.10$$

$$\Rightarrow x_1 = \frac{\beta_1}{1 - \alpha - 0} \quad \dots\dots\dots 1.11$$

Similarly,

$$x_2 = \frac{\beta_2}{1 - \alpha - 0} \quad \dots\dots\dots 1.12$$

By substituting the values of  $x_1$  and  $x_2$  in (1.8), we have that

$$(1 - \alpha - 0) + \frac{\beta_{12}}{\beta_1 \beta_2} (1 - \alpha - 0)^2 = 0$$

that is

$$\frac{\beta_{12}}{\beta_1 \beta_2} (1 - \alpha - 0)^2 + 2(1 - \alpha - 0) + (\alpha - 1) = 0$$

that is

$$\theta = \frac{(1 + \beta - \alpha\beta) \pm \sqrt{1 + \beta - \alpha\beta}}{\beta}; \beta \quad \dots\dots\dots 1.13$$

for  $\beta \neq 0$ .

where  $\beta = \frac{\beta_{12}}{\beta_1 \beta_2}$

Therefore, the values of  $x_1$ ,  $x_2$  and  $\theta$  which give the maximum yield ( $W_{max}$ ) are as given in (1.11), (1.12) and (1.13) respectively.

**Proportion of Maximum Yield**

In the circumstances described above, where the experimenter may be forced to seek some proportion,  $\lambda$ , ( $0 < \lambda < 1$ ) of this maximum; this can be obtained from the equation

$$W^{II} = \lambda W_{\max}$$

Therefore, for known  $\lambda$ ,

$$W^{II} = \frac{\lambda(1 - \hat{\alpha} - \hat{\theta})^2}{\hat{\beta}_1 \hat{\beta}_2} \dots\dots\dots 1.14$$

For unknown (estimable)  $\lambda$ , we infer that since  $\max(W^{II}) = 1$ ,

$$\hat{\lambda} = \frac{\hat{\beta}_1 \hat{\beta}_2}{(1 - \hat{\alpha} - \hat{\theta})^2} \dots\dots\dots 1.15$$

Therefore, in (1.14) the experimenter obtains the desired (known) proportion of this maximum yield while in (1.15), he obtains an estimate of this proportion.

**Numerical Application**

Below are the results of an experiment carried out by Wiggins as reported in Nduka (1994) to determine the spacing (intra-row and inter-row) for optimum yields of soybeans over a four year period. Intra-row spacing ( $x_1$ ) is 0.5 inches apart, inter-row spacing ( $x_2$ ) is 8 inches apart and yield is measured in bushels per acre. (These can be converted to standard units: 1 bushel = 36.37kg, 1 acre = 0.4 hectares, 1 inch = 2.54cm).

The coded data are shown below.

$x_1$	0.5	0.5	0.5	0.5	0.5	1.0	1.0	1.0	1.0	1.0
$x_2$	8	12	16	24	32	8	12	16	24	32
$y$	-	-	36.6	31.2	32.0	40.1	37.0	36.4	35.0	31.1
	1.5	1.5	1.5	1.5	1.5	2.0	2.0	2.0	2.0	2.0
	8	12	16	24	32	8	12	16	24	32
	40.0	37.0	35.7	33.7	29.6	39.8	36.5	34.2	32.5	26.8
	2.5	2.5	2.5	2.5	2.5	3.0	3.0	3.0	3.0	3.0
	8	12	16	24	32	8	12	16	24	32
	39.6	35.4	33.9	29.1	26.0	36.1	33.6	29.4	25.9	20.9
	3.5	3.5	3.5	3.5	3.5					
	8	12	16	24	32					
	35.3	28.2	27.0	22.6	-					

The model is

$$\omega^{-\theta} = \alpha + \beta_1 x_1^{-1} + \beta_2 x_2^{-1} + \beta_{12} (x_1 x_2)^{-1}; 0 < \theta \leq 1.$$

Using the statistical package SAS, the Gauss-Newton method of non-linear iterative least

squares yields

$$w^{-0.86216} = 0.0164 - 0.00426x_1^2 - 0.06832x_2^2 + 0.00543(x_1x_2)^{-1}$$

From the methods described above (that is from (1.14) and (1.15)).

For  $\lambda$  known,  $W'' = 5.068\lambda$  and for  $\lambda$  unknown  $W'' = 0.197$ .

This implies that we will obtain a yield of growth rate of 5.068 for  $\lambda$  known, while will obtain 19.7% of optimum yield for  $\lambda$  unknown. The ordinary implication is that premature harvest in this experiment will not be profitable or desirable.

### Conclusion

We have demonstrated in this study using the principles of maximum likelihood estimate and calculus of variation how to obtain proportion of maximum yield in such situations where full yield is unattainable or avoidable. This has been established for the cases of known and unknown proportion  $\lambda$ .

In the known case, the experimenter obtains a given proportion of this maximum, while in the unknown case, an estimate of this proportion is established. This case is more fundamental to any agriculturist, for he is now better guided to know what proportion of this full yield is obtainable at any point in time before full harvest.

In this study, numerical application has pointed to the undesirability of premature harvest.

### Acknowledgement

The explanations from Mead, R. of Applied Statistics, University of Reading on his One-dimensional Model derivations of proportion of Maximum yield has helped to improve the quality of our earlier manuscript.

Also, the comments by the referee(s) have helped to improve on this paper particularly in demonstrating its numerical application to experimental data.

### REFERENCES

- Berry, G. , 1967. A Mathematical Model Relating Plant Yield with Arrangement for Regularly Spaced Crops. *Biometrics*, 23: 505-575.
- Mead, R., 1970. Plant Density and Crop Yield. *Appl. Stats.* 19: 64-81.
- Nduka, E. C. , 1994. Inverse Polynomials in The Exploration of Response Surfaces. Unpublished Ph.D. Thesis; University of Ibadan, 80-85.
- Rohatgi, V. K. 1976. *An Introduction to Probability Theory and Mathematical Statistics*. John Wiley to Sons, NY, 377.