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THE TOTAL NUMBER OF PARAMETERS IN THE FINITE ELEMENT RECTANGULATION OF A DOMAIN

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ABSTRACT

Rectangular finite elements are important in Finite Element Method. This paper establishes a general formula for obtaining the total number of parameters associated with any finite element rectangulation of a domain. This number is also the dimension of the trial space as well as the size of the associated linear system.

Key words: Rectangular finite elements, Finite Element Method, Finite element Parameters.

INTRODUCTION

Rectangular finite elements are important because they form a basic building block for finite elements in higher dimensions, for example cubes in 3-dimension (Olayi, 1977; Bellingeri, etc, 1995). Also many practical problems are defined on a rectangular domain (Strang and Fix, 1973). For certain plate bending problems, results obtained with rectangular elements are generally better than those with triangular elements (Olayi, 1977 p.64; Clough and Tocher, 1965 pp 525 – 538). With rectangular elements there is a set of transformation relations from global to local coordinates which is simpler than any of those involved with triangular elements (Olayi, 1977, p64; Barry, 1974 pp53 –54).

In this paper we derive a general formula for obtaining the total number of parameters associated with a finite element rectangulation of a domain. This total number of parameters gives the dimension of the trial space for rectangular finite elements.

FINITE ELEMENT RECTANGULATION

The domain is subdivided into rectangles, squares or quadrilaterals. Parameters (nodal parameters) are associated with the nodes which are at the vertices, vertices and edges or vertices, edges and interior of the quadrilaterals depending on the type of basis trial functions to be employed (Olayi, 1977p64-80; Strang and Fix, 1973 p86-90). As it is the case in all finite element methods, the parameters are the unknowns to be determined in solving the given problem and are usually the value for the function or one of its derivatives. The finite element method reduces a partial differential equation problem to that of solving a systems of linear equations of the form

$$KQ = F \quad \ldots\ldots (1)$$

where K and F are known and Q is the vector of unknown nodal parameters.

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THE THEOREM

Let a finite element rectangulation of a domain contains R rectangles, with v parameters on each vertex, e parameters in the interior of each edge and r parameter in the interior of each rectangle. Then the total number, N, of parameters in the rectangulation is given by

$$N = R(2e + v + r) + \frac{B}{2}(e + v) + v \quad \ldots\ldots (2)$$

where B is the number of boundary edges.

Proof :

It can be shown by induction that

$$R + V - E = 1 \quad \ldots\ldots (3)$$

where R is the number of rectangles, V is the number of vertices and E the number of edges in the rectangulation. Also

$$E = I + B \quad \ldots\ldots (4)$$

where I is the number of interior edges and B is the number of boundary edges.
There are four edges per rectangle, each interior edge is in two rectangles while a boundary edge belongs to one rectangle. Thus

$$4R = 2I + B \quad \ldots\ldots (5)$$

The total number N of parameters is

$$N = eE + vV + rR \quad \ldots\ldots (6)$$

Because there are e, v and r parameters respectively per edges, vertex and interior of a rectangle. From (3) and (4) we have

$$R + V - (I + B) = 1 \quad \ldots\ldots (7)$$

And from (5)

$$4R - 2I - B = 0 \quad \ldots\ldots (8)$$

Substitute for I from (8) in (7) and solve for $V(=\frac{2+B+2R}{2})$. Substitute for I in (4) to obtain $E(=\frac{4R+B}{2})$. Substitute for E and V in (6) and simplify to obtain (2). This establishes the theorem.

Even though the theorem and proof use rectangles, the results are still true if rectangles are replaced by squares or quadrilaterals.

APPLICATION OF THEOREM

In finite element method, a basis function is associated with each parameter and the trial space is formed by the basis functions. Therefore the dimension of the trial space is equal of the total number of parameters, N. Also the size of the linear system (1) is equal to N. This can also find application in Biomedical Engineering (Bellingeri, etc. 1995).

For bilinear rectangular elements

$v = 1, e = o = r$ giving $N = R + \frac{B}{2} + 1$

And this is the dimension of the trial space associated with bilinear rectangular elements. The dimension of the trial space associated with the biquadratic rectangular elements is 3R + B + 1 because v = 1, e = 1, and r = 0. Thus when B is small compared to R, the dimension of the trail space of the biquadratic rectangular elements is about three times that of the bilinear rectangular elements.

CONCLUSION

A general formula for obtaining the total number of parameters associated with any element rectangulation of a domain is given. This number is also the dimension of the trial space as well as the size of the associated linear system. This will enable us choose a suitable size of the linear system our computer can handle before embarking on the actual construction of the finite elements and the generation of the system matrices.

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