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LEAKBACK OF PULSATILE FLOW OF PARTICLE FLUID SUSPENSION MODEL OF BLOOD UNDER PERIODIC BODY ACCELERATION

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ABSTRACT

We considered a two-phase model flow of blood subject to both pulsative pressure gradient due to normal heart action and a periodic body acceleration. The leakback of flow due to the fact that the velocity of the red cell is greater than the mean velocity is derived using the method of characteristics. The variation in body acceleration amplitude though affects the velocity profile in the capillary tubes, it has no effect on the leakback in the tubes. Leakback is mainly determined by the balance of the viscous drag and the driving force of the applied pressure gradient.

KeyWords: Leakback, Pulsatile Flow, Fluid Suspension, Periodic Acceleration, Viscous Drag

INTRODUCTION

The desire to quantitatively understand both the microscopic and macroscopic behaviour of blood when it flows through vessels of the circulatory system and other geometries, has supplied the motivation for appropriate investigations. The macroscopic rheological properties of blood have been studied with the ultimate aim of predicting the microscopic flow behaviour of blood in any part of the circulatory system. Recent works have shifted to relating gross rheological behaviour to the behaviour of the constituent of blood. Thus we can now seek mechanistic explanations of the gross properties of blood in terms of red cell aggregation, plasma viscosity and rheological properties of the red cell content. There has also been interest in determining the limits within which the continuum model and macroscopic properties of blood can be successively applied.

Recent experimental results on blood flow indicate that blood may be better treated as a two-phase fluid, that is, a suspension of red blood cells in plasma. (see Usha and Prema (1999)) and the literature cited threin. Usha and Prema (1999) investigated a two-phase model of blood flow subject to both the pulsative pressure gradient due to normal heart action and a periodic body acceleration. They obtained analytical expressions for the axial velocity of both the fluid and the red cell. Usha and Prema (1999) used Laplace and Hankel transforms. However Sperb (1999), showed that for large times when the initial disturbances are forgotten, there exist simple analytical solutions for both velocities.

Experimental results show (as expected) that the velocity of the red cell is greater than the mean velocity of the fluid. This leads to a leakback of flow (see Durchame et al (1991)). In the present paper we examine the leakback of flow.

Prolong exposure to accelerative disturbances that are common in normal life (for instance, while landing and taking off of aircraft, riding a tractor and fast movements of the body during gymnastics and sport activities) may lead to health problems like headache, abdominal pain, loss of vision, dizziness and increased pulse rate even though it has been shown that human body can adapt to changes. It is possible

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that dangerous combinations of body acceleration and pressure gradient of blood flow may be responsible for such health problems. It is therefore, desirable to set a standard for short and long term exposures of human being to such an acceleration.

MATHEMATICAL MODEL AND GOVERNING EQUATIONS

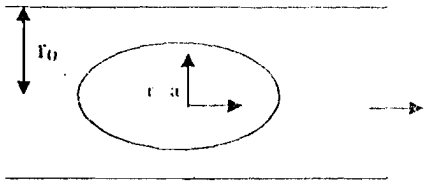

Fig. 1: The passage of Red Blood Cell in a Capillary

We shall restrict our investigation to flow in a capillary under the following assumptions:

1. Blood is considered as a two-phase fluid.
2. Blood is considered as an incompressible Newtonian fluid.
3. The flow is laminar.
4. The variation of velocity along the tube length is small as compared with the rate of change of velocity with respect to time.
5. Interaction between two phases is according to Stekes drag law.
6. The volume fraction occupied by the red cells is taken as a constant.
7. The viscous force depends on velocity along the tube Ayeni and Akinrelere (1984).

Under the above assumptions, we modify Usha and Prema (see Usha and Prema (1999)) model to obtain

$$\frac{du_f}{dt} = \frac{1}{\rho_f}\left(A_0 + A_1 \cos w_p t\right) + a_0 \cos\left(w_h + \phi\right) + \frac{8\pi\mu_s}{A\rho_f(1-c)}\left(u_p - u_f\right) + \frac{cf_0}{\rho_f(1-c)}\left(u_p - u_f\right) \quad \ldots 1$$

$$\frac{du_p}{dt} = \frac{1}{\rho_p}\left(A_0 + A_1 \cos w_p t\right) + a_0 \cos\left(w_h + \phi\right) + \frac{f_0}{\rho_p}\left(u_p - u_f\right) \quad \ldots 2$$

where c is the volume fraction occupied by the red cells, ρ_f and ρ_p are the actual densities of the materials constituting the fluid and the particle phase. The fluid phase density is $\rho_f(1-c)$ and the particle phase density is $\rho_p c$, μ_{s_p} represents the particle- fluid mixture viscosity. $f_0\left(u_p - u_f\right)_p$ is the force exerted by particles. A is the cross-sectional area, A_0 is the constant component of the pressure gradient A_1 is the amplitude of the fluctuating component, the body acceleration is $a_0 \cos\left(w_h + \phi\right)$. u_f and u_p are the

velocities of fluid and particles respectively.

1. Method of solution

From the equations (1) and (2) we obtain

$$\frac{d}{dt}\left(u_p - u_f\right) = \left(\frac{1}{\rho_p} - \frac{1}{\rho_f}\right)\left(A_0 + A_1 \cos w_p t\right) + \left[\frac{f_0}{\rho_p} - \frac{8\pi\mu_s}{A\rho_f(1-c)} - \frac{cf_0}{\rho_f(1-c)}\right]\left(u_p - u_f\right) \quad \ldots 3$$

Let

$$\frac{1}{\rho_p} - \frac{1}{\rho_f} = \alpha, \qquad \frac{f_0}{\rho_p} - \frac{8\pi\mu_s}{A\rho_f(1-c)} - \frac{cf_0}{\rho_f(1-c)} = \beta \quad \ldots 4$$

$$\frac{d}{dt}\left(u_p - u_f\right) - \beta\left(u_p - u_f\right) = \alpha\left(A_0 + A_1 \cos w_p t\right) \quad \ldots 5$$

Hence

$$u_p - u_f = ke^{-\beta t} + \alpha\left[\frac{A_0}{\beta} + \frac{A_1 w_p}{1+\beta^2}\left(\frac{\sin w_p t}{w_p} + \frac{\cos w_p t}{w_p^2}\right)\right] \quad \ldots 6$$

where k is the constant of integration. The leakback of flow is (see Sperb (1999))

$$2\pi r_0\left(u_p - u_f\right) = 2\pi r_0\left\{ke^{-\beta t} + \alpha\left[\frac{A_0}{\beta} + \frac{A_1 w_p}{1+\beta^2}\left(\frac{\sin w_p t}{w_p} + \frac{\cos w_p t}{w_p^2}\right)\right]\right\} \quad \ldots 7$$

Actually r_0 ranges between 2μm and 5μm, β is positive so that the exponential term drops out when the initial distributions are forgotten (see Sperb (1999))

Discussion and Conclusion

It is observed that while the variation in body acceleration amplitude brings in qualitative as well as quantitative changes in velocity profile in the capillary tubes, it has no effect on the leakback in the tube. However the balance of the viscous drag on the particle and the driving force of the applied pressure gradient uniquely determine leakback. This is the dynamical condition stated by Skalak (1992). Equation (7) also demonstrated the well known fact Cokelet (1987) that in capillary flow, the red blood cells generally move more rapidly than the mean velocity in the vessel because they occupy the central portion of flow where velocity is greatest. This is the so called Fahraeaus effect. Analysis of equation (7) also shows that the effect of volume fraction occupied by the particles c on the leakback in the tube is that as c increases the leakback increases.

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