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MODIFIED NGUYEN AND REVOL'S METHOD FOR SOLUTION SET OF LINEAR INTERVAL SYSTEM BASED ON ROHN'S METHOD WITHOUT INTERVAL DATA INPUTS.

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ABSTRACT

The paper is a modificationofNguyen and Revol's method for the solution set to the linear interval system. The presented methoddoes not require solving Kahan's arithmetic which may be a hindrance to that of Nguyen and Revol's method as Nguyen and Revol's method relies mainly on interval data inputs.Our method under consideration first advances solutionusing real floating point LU Factorization to the real point linear system and then solves a preconditioned residual linear interval system for the error term by incorporating Rohn's method which does not make use of interval data inputs wherein, the use of united solution set in the sense of Shary comes in handy as a tool for bounding solution for the linear interval system. Special attention is paid to the regularity of the preconditioned interval matrix. Numerical exampleis used to illustrate the algorithm and remarks are made based on the strength of our findings.

KEY WORDS:refinement of solution, linear interval system, Rohn's method, Hansen-Bliek-Rohnmethod, preconditioned residual linear interval iteration, kahan's arithmetic

AMS subject classifications. 65G20, 65G30.

INTRODUCTION

The paper aims at presenting amodification for the Nguyen and Revol'smethod,Nguyen and Revol(2011), by using classical LU floating point real arithmetic to advance solution to systems of linear equations and then switches to a method due to Rohn (2010a) that is free of interval data inputs,which solves preconditioned residual linear interval system that is guaranteed to enclose all uncertainties as solution set for original linear interval system of equations.

We define linear interval systems of equations in the form

$Ax = b$, (1.1)

Where

$$A = [A_c - \Delta, A_c + \Delta]$$

$$A_c = \frac{1}{2}\left(\underline{A} + \overline{A}\right),$$

$$\Delta = \frac{1}{2} = \left(\overline{A} - \underline{A}\right)$$

$$b = \left(b_c - \delta, b_c + \delta\right),$$

$$\delta = \frac{1}{2}\left(\overline{b} - \underline{b}\right).$$

The $\underline{A}, \overline{A}$ are the lower and upper end points of interval matrix **A** while $\underline{b}, \overline{b}$ are the lower and upper end points for the interval vector **b.The** A_c is the midpoint for the interval matrix **A and** b_c **is** defined similarly for the interval vecto**r b.**

We aim to solve for the unknown interval $x \in IR^n$ in order to obtain the Hull of the solution set

$$\sum(\mathbf{A}, b) = \left(x \in IR^n \mid A \in \mathbf{A} \exists b \in \mathbf{b} \ni \mathbf{A}x = \mathbf{b}\right) \quad . \qquad (1.2)$$

Several characterizations of solution sets to (1.2) exist, for example, Shary (2002), gave three types of such solution sets to include among others as follows:

a tolerable solution set

$$\sum_{\forall\exists}(\mathbf{A}, b) = \left\{x \in IR^n \mid (\forall A \in \mathbf{A})(\exists b \in \mathbf{b})(\mathbf{A}x = b)\right\}, \qquad (1.3)$$

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The controllable solution set

$$\sum_{\forall\exists}(\mathbf{A},b)=\{x\in IR^n \mid (\forall b\in\mathbf{b})(\exists A\in\mathbf{A})(\mathbf{A}x=b)\}, \qquad (1.4)$$

and the united solution set

$$\sum_{\exists\exists}(\mathbf{A},b)=\sum_{\exists\exists}=\{x\in IR^n \mid (\exists A\in\mathbf{A})(\exists b\in\mathbf{b})(\mathbf{A}x=\mathbf{b})\}. \quad (1.5)$$

The terms $\forall,\exists$ appearing above are all quantifiers.

Diagrammatic representation and applications in this regard can be found in Kulpa(2001). For this we will omit.The task of obtaining the Hull of solution set to the system (1.1) is an NP-hard problem, Kreinovich et al (1991).

A great interest has been shown very recently in the use of floating point arithmetic with interval arithmetic operations to form several hybrid methods, Uwamusi and Otunta (2002),Nguyen and Revol (2011) for example.Uwamusi and Otunta (2002) obtained a hybrid method to simultaneously include all zeros of polynomials of single variables.Nguyen and Revol(2011)recently proposed the combination of floating point arithmetic of LU Factorization with the preconditioned residual iteration feasible in interval Gauss-Siedel method.We modify the method presented in Nguyen and Revol(2011) wherein Rohn's method, Rohn(2010a),that is free of interval inputs is used to accelerate the convergence behaviour of the resulting preconditioned residual linear interval system which has an added advantage over that of Nguyen and Revol (2011) approach since the use of Kahan arithmetic is completely ruled out in the constructed algorithm. This is our main focus of studies.

TheMethod

Let $A\in IR^{n\times n}$ be an interval matrix and $b\in IR^n$ be interval vector. We provide the united solution set of the linear interval system (1.1) in the form of equation (1.5) using the procedures we will describe shortly.

Assuming that the interval matrix $A=[A_c-\Delta, A_c+\Delta]$ can be verified for regularity for which $\rho(|A_c^{-1}|\Delta)<1$ holds , let R represent A_c^{-1}. Let the Lebesgue continuity for the function F(x)=Ax-b holds. We present our algorithm under consideration as follows :

Algorithm

(1) Input the coefficient matrix $A\in\mathbf{A}$ and vector **b**
(2) Define ε-order of accuracy of solution
(3) Obtain the inverse midpoint of certain matrix $\mathbf{A}$ using MATLAB 2007 Cholesky Factorization in floating point arithmetic i.e. $B=U^{-1}L^{-1}$
(4) Compute initial estimate of $\hat{x}=U^{-1}L^{-1}b_c$
(5) Precondition the interval matrix $A=[A_c-\Delta, A_c+\Delta]$ to obtain $M=[B\mathbf{A}]$ and verify if M is an H- matrix, (Rump, 2006, 1994 e.g.).
Compute a non negative vector u such that $<M>u>0$ and return ''failed to validate the solution'' if there is inability to deliver a u>0. The system may be singular or too ill conditioned and exit.
(6) Compute the residual interval vector $r=\left(\mathbf{b}-\mathbf{A}\bar{x}\right)$
(7) Precondition the residual interval vector $\mathbf{r}$ to obtain $y=B\mathbf{r}$
(8) If it has not converged, perform the following operations beginning with step 9 else write 'solution obtained'
(9) Solve the matrix inequality in the sense of Rohn(2010a) given by

$$H\left(I-|I-RM_c|-|R|\Delta\right)\geq I$$

Where

$$H=\left(I-|R|\Delta\right)^{-1}\geq 0,$$

$$R=M_c^{-1}$$

(10)Verify that

$$\rho(I-RH)\leq\rho(G)<1$$

where $G=|I-RM_c|+|R|\Delta$ and check the radius of non singularity of the interval matrix, (Rump,2003, e.g.).

(11) Compute the matrix inverse enclosure

for $[M_c - \Delta, M_c + \Delta]^{-1}$

in the form $[e] = [R - (H - I)|R|, R + (H - I)|R|][-y, y]$.

(12) Compute the enclosure

$x = x_c + e$

(13) If $\|x - x_c\| \leq \varepsilon$ stop

(14) Else repeat steps 6 to 13.

end

endif

endif

end

One big advantage in the roles play by Rohn(2010a)'s method in the algorithm is that it is free of interval input data where as that of Nguyen and Revol (2011) involves interval dependencies especially when one has to solve Kahan'sarithmetic,Kahan (1968),Ning and Kearfott(1997) for details.Furthermore once we are able to establish that $\rho(|A_c^{-1}|\Delta) < 1$ one cycle of operation will suffice to give a reasonably good enclosure of the united solution set of the linear interval system of equations.

Let us note that we could as well have used the HBR (Hansen-Bliek-Rohn) formula, (Rohn, 2010b) in the algorithm in place of $[M_c - \Delta, M_c + \Delta]^{-1}$ by the equation

$$[M_c - \Delta, M_c + \Delta]^{-1} = \left[\min\left\{ \underline{B}, T_k \underline{B} \right\}, \max\left\{ \bar{B}, T_\mu \bar{B} \right\} \right]$$

Where from

$$H = (I - |R|\Delta)^{-1},$$

$$\mu = (H_{11}, H_{22}, ..., H_{nn})^{-1},$$

$$T_\mu = (2T_\mu - I)^{-1},$$

$$\underline{B} = -H(|R| + T_\mu (R + |R|))$$

$$\bar{B} = H(|R| + T_\mu (R - |R|))$$

T_μ is the orthant prescribed by the matrix $T_{z \in \mu} = diag(z_{11}, z_{22}, ..., z_{nn})$.

Numerical Example

We consider a linear interval system given in Ning and Kearfott (1997):

Ax=b

Where

$$A = \begin{pmatrix} [3.7,4.3] & [-1.5,-0.5] & [0,0] \\ [-1.5,-0.5] & [3.7,4.3] & [-1.5,-0.5] \\ [0,0] & [-1.5,-0.5] & [3.7,4.3] \end{pmatrix}, \quad b = \begin{pmatrix} [0,14] \\ [0,9] \\ [0,3] \end{pmatrix}$$

We solved the above problem with our method and results are presented in Table 1.

Table 1: Results from modified Nguyen and Revol's method in comparison with Ning and Kearfott (1997)

RESULTS FROMMODIFIED NGUYEN AND REVOL'S METHOD	RESULTS OBTAINED BY NING AND KEARFOTT (1997)
$X = \begin{pmatrix} [0.0692, 6.3800] \\ [0.0939, 6.3990] \\ [0.0655, 3.4061] \end{pmatrix}$	$X = \begin{pmatrix} [0, 6.38] \\ 0, 6.40] \\ [0, 3.40 \end{pmatrix}$

We noted that(Nguyen and Revol,2011) implanted a kind of Hansen- Sengupta method ,Hansen and Sengupta (1981) in their method which they implemented in Moore's interval arithmetic .

One drawback in this approach is that except the preconditioned matrix is an M-matrix, it may happen that there may arise the problem of combinatorial difficulty as the iteration progresses which may compulsorily requires solving Kahan's arithmetic, Kearfott (1991 and 1998).

CONCLUSION

The paper presented algorithm which combines classical real floating point arithmetic feasible in Lu decomposition to solve real point linear systemsas approximate solution. Next a method for refining error term in the form of linear interval system,Rohn(2010a),that is free of interval data inputs capable of delivering enclosure bounds was obtained . The method has the advantage that it does not require solving Kahan's arithmetic where combinatorial difficulty might be a hindrance as may be the case with method ofNguyen and Revol (2011).Furthermore the numerical results presented in Table 1 showed that our method gave tighter bounds than results obtained by Ning and Kearfott(1997) where interval Gaussian algorithm was used.We note that the same results hold instead if we apply the Hansen-Bliek-Rohn bounds, Rohn(2010b) to the part in the algorithm

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