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THE CRISIS IN GEOMETRY AND THE RISE OF RELATIVISTIC LOGIC IN TWENTIETH CENTURY

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ABSTRACT

Every paradigm of thought or action produces a logic and is produced by a logic. In this paper an attempt is made to justify the above aphorism by articulating how the shift from Euclidean geometry to non-Euclidean type gave rise to many-valued logic of the twentieth century.

In this investigation it is discovered that entire Euclidean system is based on the two-valued logic of Aristotle. The fame of the Aristotedian logic was transferred to Euchidean Geometry and the latter thus became authoritarian irrespective of the authoritarian air it enjoyed, it was rejected by a new wave in geometry. It is discovered that its rejection was on the reason that Euclids fifth postulate did not reflect the traditional characteristics of an axiom system. These Characteristics include independence, consistency and completeness. Thus in the nineteenth century non-Euclidean geometry came into being through the works of Gauss Lobachevski, Bolyai and Riemann who replaced the fifth postulate with new postulates. This revolution in geometry was conceived by logicians as an intellectual and emotional process meant to undermine the examplar of absolute knowledge both in mathematic and logic. Thus, from nineteenth to twentieth century a move was made through Pierce, Guthrie, Emil post and Lukasiewicz to introduce a relative logic which rejects the principle of non-contradiction and admits that there are three possible values for any state of affairs. This new logic (many-valued logic) assigns the symbol T for truth and F for falsehood and I for a mixture of (T and F) Truth and falsehood. The general implication of this intellectual revolution is an increase of sympathy for relativism in general epistemology.

KEYWORDS: Crisis, geometry, relativistic logic

Every thought has its presuppositions and implications. The pursuit of these projections of thought is a reserved pre-occupation of philosophy (Collingswood 1939,130,1946,28). Wright particularizes this aphorism to the field of mathematics, when he says that one of the tasks of philosophy of mathematics is to appropriate the implications of any mathematical thought for our generation (Wright 141).

In this essay, therefore, our point of departure is to appropriate the presupposition of non-Euclidean geometry for the development of twentieth century logic.

In this light an aspect of this study is a re-evaluation of Frege's conception of mathematics, namely, as the most objective of all the sciences, which should not be treated in 'a cursory and superficial manner" (Bell, 141). In other words, Frege sees mathematics as an "apriori" and absolute discipline whose associated method accounts for the successes of the sciences and the progress of man.

This is perhaps the reason why by the close of Medieval period and the beginning of modern time mathematics had impact on several fields. This influence was noticed in double-entry bookkeeping, method of philosophy of Rene Descartes, astronomy of Kepler, calendar reform of sixteenth century, the Copernica revolution and the striking researches in physics of Tartaglia which anticipated Galileo's physics. (Barnes vol. 11p. 570-571).

But with the discovery of non-Euclidean geometry in the nineteenth century one would be tempted to assume the impression that mathematics fell to the status of wired and whimsical creature of no possible consequence and influence as it used to be.

This is not entirely an accurate picture. At a radical estimate, non-Euclidean geometry revolutionized human thinking that it may not be an overstatement to say that it caused a new interpretation and new understanding of an overall nature of epistemology (Davis 206).

What are Euclidean and non-Euclidean geometries and their epistemological assumptions? In ancient Egypt, farmer surveyed land in order to lay out boundaries especially to overcome the effects of inundation of the Nile. These farmers used principles concerning lines, angles and figures in their operations (Barker 15).

The Greeks, during the intercultural assimilation with the Egyptians, became acquired with the empirical principles enunciated by the Egyptians. This statement may appear controversial but our findings from history constitutes its vindication; for example Burns and Burnes show that Egyptians laid the foundation of mathematics, for the oldest known treaties in mathematics, *Ahmose Pagyrus*, dates about 2200Bc. It was during the Hellenic period (period of intercultural assimilation) that the Greek made mathematics rigorous and introduced multiplication into the discipline (Burns 37, 50, 165; Cohen and Nagel 129) in other words as Burns conclude, the Greeks started with the mathematical knôwledge of the Egyptians but carried it further (74,152)

In the same vein, Davis and Harsh give us a rough developmental chart of mathematics as follows;

Egyptian:	3000BC........................1600BC
Babylonian:	1700BC........................300BC
Greek:	600BC..........................200BC
Greco Roman:	150AD..........................252AD
Islamic:	750AD..........................145AD
Western:	1100AD........................1600AD
Modern:	1600AD........................Present

And they conclude; "the main system of western mathematics as a systematic pursuit has its origin in Egypt" (9,10). Thus the Greeks like the Egyptians adopted the name, geometry, to mean the use of these principles on the solutions of the problems of measurement.

But the Greeks unlike the Egyptians showed more theoretical enthusiasm in geometry than the Egyptians in that for several years they worked on these principles as to show deductive proofs about measurement of space. Philosophers like Pythagoras, plato and other featured in this exercise.

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About 300 BC Euclid wrote a classic book called, *The Elements* in which he brought together the laws, which other Greeks articulated, in the discipline of geometry.

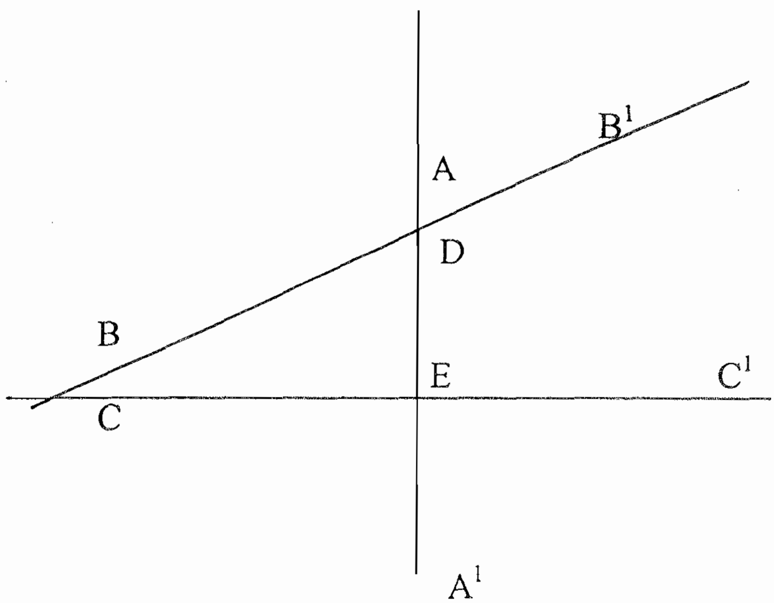
These laws were grouped into two, by Euclid. The first group has fewer elements and is made of laws that cannot be proved but which are accepted as basic truths or premises. The second group is large than the first one. The numbers are provable by appeal to the first basic ones. Members of the first group are called postulates. They are laws about geometrical objects such as lines, angles and figures. They are used to prove other geometrical laws (the second group) called theorems.

There are five postulates of Euclid, namely:

1. A straight line can be drawn from any point to any other point.
2. Any finite straight line can be extend continuously in a straight line.
3. Given any point and any distance, a circle can be drawn with that point as its center and that distance as its radius.
4. All right angles are equal to one another.
5. If a straight line crosses two other straight lines so that the sum of the two interior angles on the side is less than two right angles, then the two other straight lines, if extended far enough, will cross each other on that same side of the first line where those figures are *(Miller and Heeren, 358)*

Reflecting on these postulates one quickly captures the fact that Euclid is not consistently approaching geometry form a purely empirical method as Egyptians did. In actuality, number one definition is practically an impossible for mountains and seas that are between two towns for example will not allow a straight line from one town (point I) to another town (point II). In number two, vertical and horizontal straight lines have limited extensions because of physical obstructions. Even in number three, large circles cannot be drawn in that way. Euclid is not a dummy. He knows of these impediments. The truth is that he is not in this case interested in practical or individual activities or drawings. He is simply talking in principles. Euclid's fourth postulate is a mere tautology; what he is declaring by this, seems to be primarily a truth of logic and not a truth of geometry. As used here, truth of logic is tautology conceptual and relational, while the truth of geometry is empirically measurable hence contingent.

The fifth postulate is complicated. A drawing may help its explanation.



There are three straight lines AA^1, BB^1 and CC^1. The postulate tells us that if AA^1 crosses BB^1 and CC^1 and does so in such a way that angles CEA and BDE add up to a degree less than two right angles then BB^1 and CC^1 must some where cross each other if they are extended enough. This is because the angles CEA and BDE, which are each, less than a right angle, will make it impossible for lines BB^1 and CC^1 to be parallel. They will be inclining to themselves until they eventually intersect.

Apart from Euclid's postulates there are five principles he calls axioms. The difference between postulates and axioms is that the former makes reference to a specific discipline such as geometry, whereas axioms are for general application.

The axioms of Euclid include:

1. Things which are equal to the same thing are equal to one another
   i.e. if Y=X and B=X then Y=B
2. If equals are added to equals the different sums are equal, for example, if x=y and p=q, p+x=y+q.
3. If equals be subtracted from equals the remainders are equal. For example if 20 and (2x10) are equal and (5x2) and 10 are equal then,

$$[20-(5\times2)] = [(2\times10)-10]$$

$$\text{i.e. } (20-10) = (20-10)$$

$$\downarrow \qquad \downarrow$$

$$10 = 10$$

Therefore they are equal:

4. Things, which coincide with one another, are equal to one another.

The question is whether Euclid is talking about physical equality or value equality? For example, one dollar by its size may coincide with one Naira, but this does not make their values equal. Since Euclid is inconsistent in the connotations of the concepts he uses, his positions are liable to variations of interpretations.

5. The whole is greater than any of its part. (Miller and Heeren, 358)

Irrespective of the dissatisfaction that people had about Euclid's geometry, many thinkers up to early nineteenth century still held that Euclid's geometry was absolute. Therefore, to learn geometry was simply to memorize what Euclid presented. Stephen F. Barker captures this authoritarian epistemology when he points out that earlier thinkers and especially the philosopher, Kant agrees that there is no other geometry except that of Euclid. And that the laws of his are necessary and immutable (Barker 37).

From the above, one may conceive that Kant's position has an association with coherence theory. In this theory idealists hold that to be true is to comply with the existing given whole truth. This is the idealist conception of the relationship between truth and consistency. According to this relationship, truth is a reciprocally consistent system of propositions, each of which gets its truth from the whole system (Titus, 204-205). Thus the epistermological assumption on which Euclidean geometrical world-view is based, is that the sum of all knowledge has a pyramidal structure. If one thus finds one kind of true doctrine in geometry, another cannot be said to have knowledge in geometry, " If the another" accepts a contradictory position. Again, if one finds one kind of certainly in geometry, then one could hope for the same kind of certainty in physics, biology, ethics or even in religion (Davis 206). Simply put, Euclidean geometry presupposes authoritarianism in epistemology.

NON-EUCLIDEAN GEOMETRY

Miller captures the correct psychology of the Euclidean geometers before the discovery of non-Euclidean geometry as he says:

> Karl Friedrich Gauss.... Worked out a consistent geometry replacing Euclid's fifth postulate. He never published his work, however, because he feared the ridicule of people who could not free themselves from habitual ways of thinking. Gauss first used the term " non-Euclidean" (393).

From this quotation one can deduce the psychology surrounding the epistemology of Euclidean geometry. In the first instance, Euclidean findings were adopted as 'usual' or 'regular' and therefore should not be broken. Invariably Euclidean position became authoritarian.

However, right from the time of Euclid, the Greek and Arab mathematicians did express dissatisfaction over the fifth postulate. Their complaint was that the postulate was intricate and complicated that it did not carry an air of self-evidence. Therefore, different thinkers did try to displace it by proving that it was a theorem and not a postulate.

Girolomo Saceheri, an Italian Jesuit, used the method of *reduction ad absurdum* to prove it from other postulates but he reached any contradiction. In other words, he failed. Some other mathematicians tried to prove the fifth postulate by assuming the other postulates and axioms of Euclid as premises, yet they did not succeed. In all, their attempts to show that the fifth postulate was logically derivable from the other postulates were futile.

It was during the nineteenth century that the correct logical, positions of the postulates were understood. The understanding is that fifth postulate was supposed to be self-evident, simple and straight forward, according to traditional axiomatic system (Bochenski, 70) but the fifth postulate was not. Therefore the postulate lost its status. Three scholars therefore made independent attempts to replace the postulate without creating any contradiction the system of the geometry.

Karl Friedrich Gauss was the first to work out a consistent geometry that replaced Euclid's fifth postulate and he used the name non-Euclidean geometry to describe his geometry. Independently Nikolai Ivanovich Lobachevski and bolyai worked out a similar system.

The principles of these new geometrics or the postulates that were presented by these geometers as substitutes for the fifth postulates are:

1. Through a point not on a given line always more than one line can be drawn parallel to the given line.
2. The sum of the angles of a triangle may be less than two right angles and the amount by which it is less is proportional to the area of the triangle. Therefore, triangles having unequal areas cannot be similar.
3. The ratio of the circumference of a circle to its diameter is always greater than Π_1 P_1 (Barker 35-36) Later in the nineteenth century, Riemann developed another type of geometry that also discarded (1) the fifth postulate of Euclid and (2) the assumption that a straight line can bee extended indefinitely. In Riemann's geometry the following principles are endorsed:

1. For each straight line there is a maximum length to which the line can be extended
2. Through two given points always more than one straight line can be drawn.
3. The sum of the angles of a triangle may be greater than two right angles. The excess is proportional to the area of the triangle.
4. The ratio of the circumference of a circle to its diameter is always less than a P_1 (Π) and the ratio decreases as the area of the circle increases (Barker, 36-37).

Gauss' tag "non-Euclidean" on the new findings came to be applied to the geometries of Gauss, Lobachevski, Bolyai and Riemann.

What is the significance of this intellectual revolution to twentieth century philosophy?. We may send our back to the descriptioin of the psychological condition of Gauss at the time his work became know. Regarding this psychology Miller and Heeren write:

Early in the nineteenth century Karl Friedrich Gauss, one of the great mathematicians, worked with a consistent geometry, replacing Euclid's Fifth postulate. He never published his work, however, because he feared the ridicule of people who could not free themselves from habitual ways of thinking (393).

This description shows that there was an authoritarian cast on people of his time which made them not to think of abnormal geometry or any geometry that was contrary to Euclid's. even at this, people themselves were not comfortable with the doctrine of Euclid.

The revolution in geometry in the name of non-Euclidean geometry can then be rightly conceived as a test of intellectual and emotional prowess meant to undermine the exemplar of " absolute knowledge" and a very strong reaction to divide the hierarchy of mathematics (Davis 207). If mathematical theories which up to late nineteenth century held disciplines in chains and determined their methods were no more in unity of focus, thinkers in logic at that time, perhaps, because logic was close to geometry, might have started having some urge for liberation and hence started thinking of how human thought could liberate itself from the authoritarianism of the age-long two-valued logic on which Euclidean geometry was based.

Pierce (1838-1919), who wrote between the late 19th and 20th centuries thus introduced in his " multi-methods" the idea of neutral truth-value which is between the determinate truth and the determinate falsehood (Rescher 6). This is a germ of an idea of many-valued logic. Pierce himself might have become unsympathetic with Euclid's authoritarian geometry. This could be one of the reasons why Cokin says that Pierce as a mathematician versed in Kantian geometry, rejected some of the view of Kant (Cokin 207)

It is likely too that Pierce's position, which might have been popularized in America in the early 20th century, influenced, E Guthrie (an America logician at that time). In the early part of the 20th century such an influence might have caused Guthrie to do to two valued-logic what an Austrian, Kurth Godel, did to formalism of Hilbert. Criticizing Frege and other modern two-valued logicians, Guthrie says that it is not enough to say that modern logic includes more than what is found in Aristotelian logic but that it should be clearly stated that the difference is not a matter of degree but that of substance. His argument is that there cannot be a singular judging standard for a logic that obeys the law of noncontradiction and the logic that obeys the law of harmony of contraries (Guthrie 152-158). With this undercutting, Guthrie gives the impression that there could be a logic, alternative to the two-valued type.

It was during the first quarter of the twentieth century that the seed sown in the past concerning many-valued logic came to blossom in the works of Lukasiewicz's and Emil L. post. We shall briefly explain the fundamentals of Lukasirwicz logic.

With a frame of mind saturated with the idea of modality, Lukasiewicz, according to the Kneales, reason in the following way:

> I can assume without contradiction that my presence In Warsaw at a certain moment of next year e.g. at noon on 21 December is at the present time determined neither positively nor negatively. Hence it is possible but not necessary that I shall be present in Warsaw at the given time. On this assumption, the proposition " I shall be in Warsaw at Noon on 21 December of next year can at the present time be neither true nor false". For if it were true, my future presence in Warsaw have to be necessary, which is contradictory to the assumption. If it were false on the otherhand, my future presence in Warsaw would have to be impossible, which is also contradictory to the consumption. Therefore, the proposition considered is at the moment neither true nor false and must possess a third value different from "O" (i.e.) falsity and "T" (which is) truth. This value we can designate by "1/2". It represents the "possible" and joins the true and false as a third value.

For Lukasiewcz, therefore, there are three possible values for any state of affairs. If it is true, it is assigned the symbol "T". If it is false, it is assigned the symbol "F" and if it is a mixture of two contraies such as "T" and "F", it is assigned the symbol 1.

Recently, and in-terms of application, many logicians have developed further what Lukasiewicz started. As Rearcher points out:

An important application of many-valued logic to analysis of the foundational paradoxes in mathematics goes to D.A Bachvar who used the third indeterminate true-value of a three-valued logic to meaningless or undefined (13).

What Bachvar is saying is that in many-valued logic one of the truth values is conceived at a point of paradox. This is where there seems to be a conjunction between truth and falsity. This is a situation in which an individual conceives an antinomy. Equally, Bochenski says that:

> In 1944, Reichenbach showed that quantum mechanics cannot be axiomatized without contractions on the basis of classical logic (two-valued logic) but it can be axiomatized straight forwardly without contradiction in the frame work of Lukasiewicz's three-valued logic (Since then) relative merit of . logic has become a problem for methodology (Bochenki 18).

The implication of this as Bochenski points out is that the so-called heterodox system of logic is applied where signs are not likely to have only two-fold (edetic) meaning, whereas classical logic can operate only where there are two-fold values. No wonder then Rescher says that many-valued logic is of particular interest from philosophical point of view because of its involvement in issues of relativism and conventionalism in logic (Rescher 16).

CONCLUSION

What we have been able to discover is that up to the early nineteenth century, mathematics (geometry) was conceived as a science of absolute truth. Because of such belief, an absolute conception of mathematics came to be considered as an indispensable ingredient in the fabrics of any investigation that had claims to progress.

But the entrance of non-Eulidean geometry dethroned the authoritarianism of mathematic. By association, this dethronement influenced logicians of twentieth century and gave rise to many-valued logics.

The general implication of this intellectual revolution is the increase of sympathetic awareness of relativism in general epistemology. Harris J. F. presents this twentieth century resurgence of relativism, thus:

> Recent decades have witnessed the extraordinary growth of radical relativism, a doctrine which now dominates the entire culture, from popular music to journalism and from religion to school curricula. According to the radical relativist creed, any preposition can be true or false in relation to a chosen frame-work (preface)

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